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Strength of Normal Sections of Flexural Reinforced Concrete Elements Damaged by Corrosion and Strengthened with External Composite Reinforcement

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Abstract. The aim of the study is to develop a methodology for calculating the strength of normal sections of flexural reinforced concrete elements, which suffered corrosion damage and were strengthened with external composite reinforcement. The objects of the study are reinforced concrete elements used in various structures that are exposed to aggressive chloride environment that causes corrosion of concrete and rebars. The research method is based on the use of a diachronic model of deformation of corrosion-damaged elements. This model takes into account changes in the mechanical characteristics of concrete and reinforcement during corrosion and includes equations based on analytical relationships for determining the initial load-bearing capacity of intact structures. An important aspect of the method is taking into account external polymer composite reinforcement, which allows to increase the flexural rigidity and strength characteristics of damaged elements. The Picard's iterative method, which is designed for approximate solutions of differential equations, was used to ensure the accuracy of calculations. The results of the study showed that the proposed method allows to effectively assess the strength of normal sections of reinforced concrete elements subjected to corrosion. It was found that the methodology, which takes into account the changes in strength and deformation characteristics of materials, as well as the effect of aggressive chloride environment, ensures high accuracy and reliability of the analysis. The use of external polymer composite reinforcement significantly increases the stability and durability of structures. Thus, the developed methodology is an important tool for increasing operational reliability and extending the service life of reinforced concrete structures exposed to aggressive environments, which is a relevant problem in the construction industry.

Keywords: strength, reinforced concrete, chloride corrosion, composite materials, strengthening of building structures

Conflicts of interest. The authors declare that there is no conflict of interest.

Authors' contribution. Undivided co-authorship.

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Прочность нормальных сечений изгибаемых железобетонных элементов, поврежденных коррозией и усиленных внешним композитным армированием

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Аннотация. Исследование направлено на разработку методики расчета прочности нормальных сечений изгибаемых железобетонных элементов, подвергшихся коррозионным повреждениям и усиленных внешним композитным армированием. Объектом исследования являются железобетонные конструкции, используемые в различных сооружениях, которые подвергаются воздействию хлоридной агрессивной среды, вызывающей коррозию бетона и арматурных стержней. Метод исследования базируется на применении диахронной модели деформирования коррозионно-поврежденных элементов. Эта модель учитывает изменения механических характеристик бетона и арматуры в процессе коррозии и включает в себя расчеты, основанные на аналитических зависимостях для определения первоначальной несущей способности неповрежденных конструкций. Важным аспектом методики является учет внешнего полимеркомпозитного армирования, которое позволяет повысить изгибные жесткости и прочностные характеристики поврежденных элементов. Для обеспечения точности расчетов использован итерационный метод Пикара, предназначенный для аппроксимации решений дифференциальных уравнений. Результаты исследования показали, что предложенная методика позволяет эффективно оценивать прочность нормальных сечений железобетонных элементов, подверженных коррозии. Установлено, что методика, учитывающая изменения прочностных и деформационных характеристик материалов, а также воздействие хлоридной агрессивной среды, обеспечивает высокую точность и надежность расчетов. Применение внешнего полимеркомпозитного армирования значительно увеличивает устойчивость и долговечность конструкций. Таким образом, разработанная методика служит важным инструментом для повышения эксплуатационной надежности и продления срока службы железобетонных конструкций, подвергающихся воздействию агрессивных сред, что является актуальной задачей в строительной отрасли.

Ключевые слова: прочность, железобетон, хлоридная коррозия, композитные материалы, усиление строительных конструкций

Заявление о конфликте интересов. Авторы заявляют об отсутствии конфликта интересов.

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1. Introduction

1.1. Problems of Corrosion of Reinforced Concrete Structures

Buildings and structures may contain flexural reinforced concrete elements that are exposed to aggressive corrosion loads, which leads to deterioration of concrete and reinforcement, causing premature onset of limit states [1–4]. Corrosion of reinforced concrete is a complex set of chemical processes, as a result of which the strength and deformation properties are significantly changed [5–7].

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One of the key problems in construction is exposure to chloride environments, which is recognized as hazardous and is widespread in practice. Studies conducted by various methods, including expert surveys, analysis of project documentation and field observations, show that about 3/4 of enterprises in chemical, metallurgical and other industries, as well as in the field of transportation and in coastal areas, are exposed to aggressive environments containing chlorides [8–10].

1.2. Existing Models of Corrosion Damage

At the moment, the processes of determining the stress-strain state of structures damaged by corrosion under the combined action of service and environmental loads have been sufficiently investigated. The works of V.M. Bondarenko, V.I. Rimshin, N.K. Rosental, A.I. Popesko, I.G. Ovchinnikov, G.A. Smolyago, V.P. Selyaev, V.P. Chirkov, P.S. Mangat, G.C. Gaal, R. Al-Hammoud, C. Andrade and others [1–16] are devoted to this subject.

In the model of corrosion damage of concrete, it is possible to partially apply the dissipative resistance theory of V.M. Bondarenko [5], according to which the cross-section of the element is divided into three zones. The first zone represents the area of complete material failure of thickness Z^*. The second zone is a transitional zone of partial concrete damage of thickness σ. The third zone is the area of concrete undamaged by corrosion of thickness p (Figure 1).

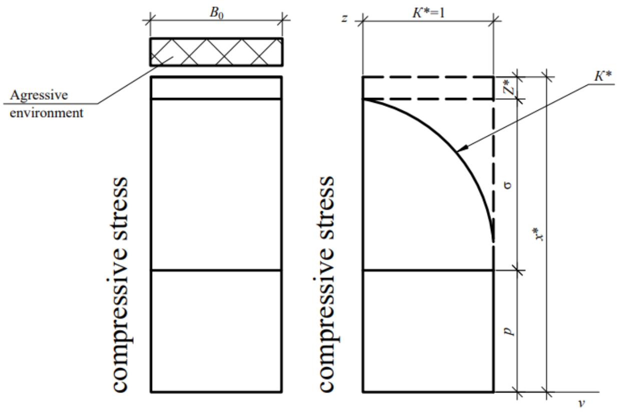


Figure 1. The change of strength characteristics of concrete along the cross section and the relationship between the depth of corrosion and stresses
S o u r c e: made by V.I. Rimshin, L.A. Suleymanova, P.A. Amelin

According to the theory, the stress in corrosion-damaged concrete is described by the following relationship [4–6]:

$$\sigma_{\text{corr}} = \sigma_b\left(t\right) K(z), \tag{1}$$

where $\sigma_b\left(t\right)$ is the stress-strain model of undamaged concrete; $K(z)$ is the function of damage for layer of thickness z.

Coefficient K varies between 0 and 1 and is defined in general form as:

$$K(z) = \sum_{i=0}^{i=3} a_i z^i, \tag{2}$$

where z is the vertical coordinate measured from the stress axis of the corrosion-damaged concrete element; a_i are the coefficients of the power series, which are found at fixed values of K_i.

According to the model, the conditions for determining parameters a_i are the following:

$$\text{at} \quad z = p \quad K(p) = 1 \quad \frac{dk^*}{dz}\Big|_{z=p} = 0, \tag{3}$$

$$\text{at} \quad z > p \quad K^*\left(p + \delta\right) = K_1. \tag{4}$$

Given that $z = p + \sigma \rightarrow K\left(p + \sigma\right) = 0$, coefficients a_0, a_1, a_2 are equal to:

$$a_0 = 1 - \left(\frac{p}{\delta}\right)^2; \tag{5}$$

$$a_1 = 2\frac{p}{\delta^2}; \tag{6}$$

$$a_2 = -\frac{1}{\delta^2}. \tag{7}$$

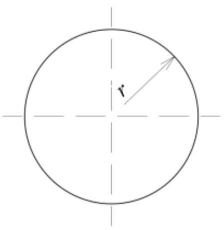
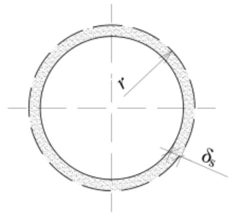
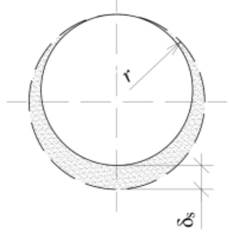
Rebar corrosion changes the geometric characteristics of reinforcement to a greater extent than the physical and mechanical characteristics of steel. The calculated cross-sectional area of damaged steel reinforcement is presented as [15]:

$$A_s^* = A_s - A_s, \tag{8}$$

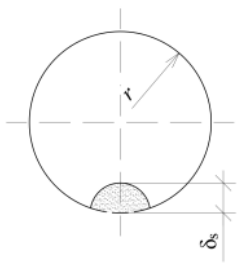
where A_s is the cross-sectional area of rebar before corrosion; A_s^{cor} is the calculated area of corrosion damage of rebar cross-section determined from Table 1.

Table 1

Models of corrosion development along the rebar cross-section and calculated area of corrosion damage

Rebar corrosion type	**Illustration**	**Function for determining the corrosion area of rebar, A_s^{cor}**
Absence of critical concentration		0
Continuous uniform corrosion		$\pi\delta_s\left(2r - \delta_s\right)$
Continuous non-uniform corrosion		$r\delta_s - \frac{\delta_s^2}{4}$

Ending of the Table 1

Rebar corrosion type	Illustration	Function for determining the corrosion area of rebar, A_s^{cor}
Local and pitting corrosion		$\delta_s^2\sqrt{m-m^2}+\delta_s^2\arcsin\sqrt{m}-2r\delta_s\sqrt{m}-2r\delta_s\sqrt{m}+$ $+\delta_s\sqrt{mr^2-\delta_s^2m^2}+r^2\arcsin\frac{\delta_s\sqrt{m}}{r}$, where $m=1-\frac{\delta_s^2}{4r^2}a$

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Note. r is the original radius of rebar, δs is the depth of corrosion damage.

The depth of corrosion of steel reinforcement in the studies of I.G. Ovchinnikov is determined by the following relationship [16]:

$$\delta_s=\begin{cases}0,\ t\le t_{\text{inc}}\\ \dfrac{\delta_{s,0}\left(t-t_{\text{inc}}\right)}{T+\left(t-t_{\text{inc}}\right)},\ t>t_{\text{inc}}\end{cases},\tag{9}$$

where $\delta_{s,0}$, T are experimental constants of damage; t_{inc} is the corrosion initiation time.

The time duration of the incubation period is determined based on the Fick’s law [15]:

$$t_{\text{inc}}=\frac{1}{12D}\left(\frac{a}{1-\dfrac{C_{\text{cr}}}{C_s}}\right)^2,\tag{10}$$

where D is the coefficient of chloride diffusion, a is the thickness of the protective layer of concrete, C_s is the chloride concentration at concrete surface.

1.3. Existing Prerequisites for Composite Strengthening of Reinforced Concrete Elements

A relevant problem in long-term operation of buildings and structures is the extension of the remaining service life of damaged reinforced concrete elements. In construction practice, the process of inspection of technical condition of reinforced concrete structures is performed simultaneously with the aim to increase the bearing capacity of the elements for resisting higher load values.

Along with the existing methods of strengthening of structures, such as increasing the cross-section and reinforcement, application of steel casings, the method of external polymer composite reinforcement is used. The development and application of strengthening methods for intact reinforced concrete structures with external composite reinforcement became possible owing to theoretical and experimental works of V.I. Rimshin, D.R. Mailyan, V.I. Morozov, S.I. Merkulov, P.P. Polsky, J.F. Bonacci, A.H. Al-Saidy and others [17–23].

The tensile behavior of composite materials is characterized by their elastic deformation up to fracture. Composite fibers differ from steel in that they are not ductile and their failure is brittle. The stress of composite materials depends on their strain according to Hooke’s equation:

$$\sigma_f = E_f \varepsilon_f \delta_{s,0}, \quad (11)$$

where E_f is the elastic modulus of the composite material; ε_f is the fiber strain at a specific moment in time.

This affects the strengthening design of reinforced concrete structures with external reinforcement made of composites, as restrictions are imposed on the magnitude of elastic deformations of concrete and steel.

However, at the moment, the problem of determining the load-bearing capacity of previously damaged flexural reinforced concrete elements due to contact with aggressive chloride environment during operation is studied to a small extent. Which is the subject of this scientific study.

The objects of this study are flexural reinforced concrete elements damaged by aggressive chloride environment and strengthened with external composite reinforcement.

The aim of the study is to develop a methodology for calculating the strength of the normal sections of flexural reinforced concrete elements subjected to aggressive chloride environment and strengthened with external polymer composite reinforcement.

2. Methods

In this paper, a mathematical iterative method is used for determining the unknowns, which is reduced to the sequential application of the Picard's method.

The Picard's method, when used as a tool for partitioning the cross-section of a reinforced concrete element, is an iterative approach used for analysis and modeling of damaged structures. It involves successive refinement of the cross-section characteristics by dividing it into sub-elements and iteratively calculating their properties. In general, the formula of the iterative Picard's method is defined by the following relationship:

$$y(t) = y_0 + \int_{t_0}^{t} f\left(x, y(x)\right) dx. \quad (12)$$

Below is the solution algorithm:

- Initial conditions such as cross-sectional configurations of the reinforced concrete element, including dimensions, shape, position and number of rebars, geometric parameters of damage such as depth and length of cracks are specified.
- The cross-section is divided into several zones (sub-elements), taking into account both damaged and undamaged areas.
- An initial approximation of y is selected (adopted at first as a constant) and substituted in the right-hand side of the differential equation: $dy / dx = f(x, y)$.
- The equation is integrated with respect to x, which yields y in terms of x in the second approximation, and the specified numerical values are substituted into it. The result is rounded to the specified number of decimal places or significant digits. Initially, the mechanical properties of each sub-element are evaluated. Iterative calculation of the stress-strain state is performed for each sub-element, taking into account the interaction between them. The sub-element characteristics are updated based on the calculation results, gradually refining the model.
- The iterative process is repeated until convergence is achieved, when changes in the characteristics become insignificant. The results are verified to ensure that the model is adequate and the calculations are correct.
- Integral characteristics of the entire section, such as moment of inertia, static moment, stiffness and bearing capacity, are determined on the basis of the iterative calculations.

The Picard's method in this context provides accurate modeling and analysis, which is essential for assessing the condition of a structure and making decisions about its repair or strengthening.

3. Results and Discussion

The methodology of strength analysis of flexural elements assumes initial determination of flexural strength M of the undamaged structure using the following analytical relationships:

$$N + \sigma_s^{'} A_s^{'} + \sigma_s A_s + b\int_{-h/2}^{z_0} \sigma_p dz + b\int_{z_0}^{h/2} \sigma_c dz = 0; \tag{13}$$

$$M + \sigma_s^{'} A_s^{'} z_c + \sigma_s A_s z_p + b\int_{-h/2}^{z_0} \sigma_p z dz + b\int_{z_0}^{h/2} \sigma_c z dz = 0. \tag{14}$$

Further, to calculate the strength of normal sections damaged by corrosion and elements with cracks, a special case of the diachronic deformation model of V.V. Belov and S.Ye. Nikitin is used [24; 25]. This model takes into account:

- changes in strength and deformation characteristics of concrete in compression and tension;
- development of corrosion in rebars;

For the normal section, the resultant systems of equations of the diachronic model of deformation for corrosion-damaged reinforced concrete elements comprise the conditions of static equivalence $\Sigma N_{\text{cor}} = 0$ and $\Sigma M_{\text{cor}} = 0$, as well as kinematic relationships under all-round aggression:

$$N_{\text{cor}} = b\int_{h/2}^{\delta_1} \sigma_b^{\deg r}(y)\partial y + 2\delta_2 \int_{\delta_1}^{x} \sigma_b^{\deg r}(y)\partial y + (b - 2\delta_2)\int_{\delta_1}^{x} \sigma_b(y)\partial y + A^{`}_{s,\text{cor}} E^{`}_s \varepsilon_b - A_{s,\text{cor}} E_s \varepsilon_s; \tag{15}$$

$$M_{str} = b\int_{h/2}^{\delta_1} \sigma_b^{\deg r}(y)\, y\, dy + 2\delta_2 \int_{\delta_1}^{x} \sigma_b^{\deg r}(y)\, y\, dy +$$
$$+ (b - 2\delta_2)\int_{\delta_1}^{x} \sigma_b(y)\, y\, dy + A^{'}_{s,\text{cor}} E^{`}_s \varepsilon_b \left(\frac{h}{2} - a^{`}\right) - A_{s,\text{cor}} E_s \varepsilon_s \left(\frac{h}{2} - a^{`}\right); \tag{16}$$

$$\frac{\varepsilon_b^m}{\varepsilon_s^m} = \frac{x}{h - x - a}, \tag{17}$$

where h is the height of the rectangular section; b is the width of the rectangular section; ε_b^m is the strain of the reinforcement in tension, in the middle section; E_s is the elastic modulus of the reinforcement; $a^{`}$ is the distance from the resultant force in the reinforcement to the closest section edge; ε_b^m is the concrete strain, in the middle section; $\sigma_b^{\deg r}, \sigma_b$ are the concrete stresses in the damaged and undamaged zones respectively; δ_1, δ_2 are the depths of damage of the concrete section in the compression zone and side edges.

The calculation methodology is based on a number of assumptions and hypotheses, including the assumption of constant external load on the element and concentration of aggressive environment around the section throughout the entire observation period. Corrosion of reinforcement by section reduction and polynomial stress-strain relationship of concrete are also taken into account. It is assumed that the axis of the center of gravity passes through the middle of the beam section height.

The contribution of external composite strengthening of flexural element is determined by adding the forces in the composite reinforcement at a distance $h/2$ from the location of the axis of symmetry to the equilibrium equations (15) and (16), as shown in Figure 2.

As a result, the following relationships are obtained:

$$N_{str} = b\int_{h/2}^{\delta_1} \sigma_b^{\deg r}(y)\partial y + 2\delta_2 \int_{\delta_1}^{x} \sigma_b^{\deg r}(y)\partial y + (b - 2\delta_2)\int_{\delta_1}^{x} \sigma_b(y)\partial y +$$
$$+ A^{`}_{s,\text{cor}} E^{`}_s \varepsilon_b - A_{s,\text{cor}} E_s \varepsilon_s - A_f E_f \varepsilon_f; \tag{18}$$

$$M_{str}=b\int_{h/2}^{\delta_1}\sigma_b^{\deg r}(y)\,y\,\partial y+2\delta_2\int_{\delta_1}^{x}\sigma_b^{\deg r}(y)\,y\,\partial y+\left(b-2\delta_2\right)\int_{\delta_1}^{x}\sigma_b(y)\,y\,\partial y+$$
$$+A'_{s,\mathrm{cor}}E'_s\varepsilon'_b\left(\frac{h}{2}-a'\right)-A_{s,\mathrm{cor}}E_s\varepsilon_s\left(\frac{h}{2}-a\right)-A_{s,\mathrm{cor}}E_s\varepsilon_s\frac{h}{2}; \qquad (19)$$

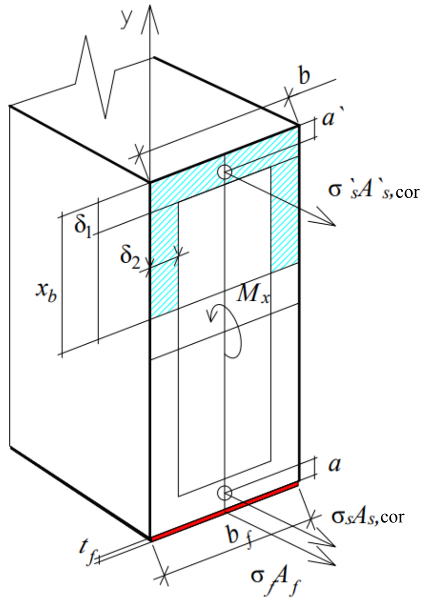

Figure 2. Model of the normal section of strengthened damaged rectangular element

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$$\frac{\varepsilon_b^m}{\varepsilon_f^m}=\frac{x}{h-x}, \qquad (20)$$

where A_f is the cross-sectional area of the composite material; E_f is the elastic modulus of the composite material; ε_f^m is the strain of the composite material, in the middle section.

In order to perform the iterative calculation, equations (17) and (18) defining axial force N_{str} and bending moment $M_{x,str}$ are expressed in terms of flexural stiffness D, curvature $\frac{1}{r_x}$ and fiber strain at the starting point ε_0:

$$N_{str}=D_{13}\frac{1}{r_x}+D_{33}\varepsilon_0; \qquad (21)$$

$$M_{str}=D_{11}\frac{1}{r_x}+D_{13}\varepsilon_0\,. \qquad (22)$$

Taking into account the forces of the polymer composite material, the flexural stiffness values are equal to:

$$D_{11}=b\int_{h/2}^{\delta_1}E_b^{\deg r}(y)\nu_b y^2\partial y+2\delta_2\int_{\delta_1}^{x}E_b^{\deg r}(y)\nu_b y^2\partial y+\left(b-2\delta_2\right)\int_{\delta_1}^{x}E_b(y)\nu_b y^2\partial y+$$
$$+A'_{s,\mathrm{cor}}\alpha'_s E'_s\nu_s\left(\frac{h}{2}-a'\right)^2-A_{s,\mathrm{cor}}\alpha_s E_s\nu_s\left(\frac{h}{2}-a\right)^2-A_f\alpha_f E_f\left(\frac{h}{2}\right)^2; \qquad (23)$$

$$D_{13}=b\int_{h/2}^{\delta_1}E_b^{\deg r}(y)\nu_b y\,\partial y+2\delta_2\int_{\delta_1}^{x}E_b^{\deg r}(y)\nu_b y\,\partial y+\left(b-2\delta_2\right)\int_{\delta_1}^{x}E_b(y)\nu_b y\,\partial y+$$
$$+A'_{s,\mathrm{cor}}\alpha'_s E'_s\nu_s\left(\frac{h}{2}-a'\right)-A_{s,\mathrm{cor}}\alpha_s E_s\nu_s\left(\frac{h}{2}-a\right)-A_f\alpha_f E_f\left(\frac{h}{2}\right); \qquad (24)$$

$$D_{33}=b\int_{h/2}^{\delta_1}E_b^{\deg r}(y)\nu_b\partial y+2\delta_2\int_{\delta_1}^{x}E_b^{\deg r}(y)\nu_b\partial y+\left(b-2\delta_2\right)\int_{\delta_1}^{x}E_b(y)\nu_b\partial y+$$
$$+A'_{s,\mathrm{cor}}\alpha'_s E'_s\nu_s-A_{s,\mathrm{cor}}\alpha_s E_s\nu_s-A_f\alpha_f E_f, \qquad (25)$$

where $E_b^{\deg r}, E_b$ are the elastic moduli of concrete in damaged and undamaged zones respectively; E_s, E'_s are the elastic moduli of reinforcement in tension and compression zones respectively; E_f is the elastic modulus of the polymer composite material; $\alpha'_s, \alpha_s, \alpha_f$ are the adjustment factors of geometrical characteristics of compressed, stretched and polymer composite reinforcement; ν_b, ν_s are the coefficients of the secant modulus of concrete and reinforcement of a particular region.

4. Conclusion

In this study, the general relationships of stress-strain state changes in corrosion-damaged flexural reinforced concrete elements, which are strengthened with external polymer composite reinforcement, were determined. The results of the study allowed to formulate the following conclusions:

1. A methodology for calculating the strength of flexural reinforced concrete elements has been developed, including the initial determination of the load-bearing capacity of undamaged structure using analytical relationships.

2. To estimate the strength of normal sections of corrosion-damaged reinforced concrete elements, a diachronic deformation model was used, which takes into account the changes in strength and deformation characteristics of concrete and reinforcement due to corrosion.

3. The developed methodology includes stress-strain models of corrosion-damaged reinforced concrete elements and application of external polymer composite reinforcement, which makes it possible to accurately determine flexural stiffness and strength characteristics of structures.

4. The proposed methodology is based on a number of assumptions, including constant external load and concentration of aggressive environment around the cross-section throughout the observation period, which ensures the stability of the calculations.

5. It is established that exposure to aggressive chloride environment is a critical factor contributing to the corrosion of concrete and reinforcement, which leads to premature failure of reinforced concrete structures.

6. In order to obtain accurate results, the Picard's mathematical iterative method, designed for approximate solutions of differential equations, was applied, which provides high accuracy and reliability of the results.

These conclusions emphasize the significance of the developed methodology and its potential for improving practices of protection and strengthening of reinforced concrete structures in aggressive environments.

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