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Ruled Shells of Conical Type on Elliptical Base

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Conflicts of interest

The author declares that there is no conflict of interest.

Abstract. The information about main results on geometry of developable surfaces with an edge of regression which have a directrix ellipse in the base is gathered. These surfaces constitute a group called “Ruled surfaces of conical type on elliptical base”. This group includes elliptical cones, torses with two ellipses defined in the parallel planes, equal slope surfaces, and ruled surfaces with the main frame of three superellipses that are ellipses in one coordinate plane and broken straight lines in the other two coordinate planes. The paper presents a method for developing torses onto a plane, approximation of torses by folded surfaces, and parabolic ending of a thin sheet from elastic material into a torse shell. A brief review of the methods of stress-strain and buckling analysis of the considered ruled shells is given, including the displacement-based finite element method and variational energy method. It is shown that analytical methods can be used only in the case of applying the momentless shell theory for ruled thin shells of conical type. The analytical formulae for determining the normal and tangent internal forces in any momentless conic shell with a superellipse in the base are derived. References to forty four scientific articles of other authors, working or having worked on the subject of the paper are given. These references confirm the conclusions of the author and the perspectives of investigations of the considered ruled surfaces and shells.

Keywords: cone, equal slope surface, torse, ellipse, superellipse, approximation of torses by folded surface, stress-strain state of shell, momentless shell theory

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Линейчатые оболочки конического типа на эллиптическом основании

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Заявление о конфликте интересов

Автор заявляет об отсутствии конфликта интересов.

Аннотация. Собраны сведения об основных результатах, полученных автором по геометрии развертывающихся поверхностей с ребром возврата, имеющих в основании направляющий эллипс. Эти поверхности составляют группу «Линейчатые поверхности конического типа на эллиптическом основании», в которую входят эллиптические конусы, торсы с двумя заданными эллипсами в параллельных плоскостях, поверхности одинакового ската и линейчатые поверхности с главным каркасом из трех суперэллипсов в трех координатных плоскостях, один их которых является эллипсом, а два других вырождаются в прямые ломаные линии. Представлены материалы по построению разверток торсов на плоскость, аппроксимации торсов складками, параболическому изгибанию тонкого листа из упругого материала в проектируемую торсовую оболочку. Дана краткая характеристика по методам расчета на прочность и устойчивость рассматриваемых линейчатых оболочек со ссылкой на работы других авторов, которые использовали метод конечных элементов в перемещениях и вариационно-разностный метод. Показано, что аналитические методы применимы только при использовании безмоментной теории расчета тонких линейчатых оболочек конического типа и получены аналитические формулы для определения внутренних нормальных и касательных усилий для любой безмоментной конической оболочки с любым суперэллипсом в основании. Приведены 44 наименования использованных научных источника других авторов, работающих или работавших по теме представленной статьи, подтверждающие выводы, заключения и перспективы исследований, рассмотренных линейчатых поверхностей и оболочек.

Ключевые слова: конус, поверхность одинакового ската, торсовая поверхность, эллипс, суперэллипс, аппроксимация торсов складками, расчет оболочек на прочность, безмоментная теория оболочек

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1. Introduction

Conical surfaces find wide application in civil and mechanical engineering. These are surfaces of zero Gaussian curvature but more precisely, they are degenerated torse surfaces for which the edge of regression deganerates into a point that is the vertex of the cone. Torse surfaces are ruled surfaces [1] and they can be developed onto a plane without ruptures and folds. So, they can be constructed from a single fragment of a plane sheet. This explanes their wide usage in different branches of science and engineering [2]. Circular conical surfaces [3] and conical elliptical surfaces [4; 5] found especially wide application.

Let us introduce a notion of *surfaces of conical type*. These surfaces will include ruled sufaces constructed on two closed curves with two axes of symmetry lying on two parallel planes. In general aspect, superellipses with parallel axes of symmetry can represent these curves. Some surfaces from the class of “Torse surfaces” forming the group “*Equal slope surfaces*” [6–8] can be added to ruled surfaces of conical type.

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One can use different analytical and graphical methods presented earlier in scientific literature [9–12] for constructing the considered ruled surfaces.

Aim of the research

The first aim of the research is to study surfaces of conical type on elliptical base i.e. ruled surfaces constructed on elliptical base

$$(x/a)^2 + (y/b)^2 = 1. \quad (1)$$

The next aim is to suggest thin shells with middle surfaces in the form of surfaces of conical type on elliptical base for implementation, having pointed out the principal publications dealing with the matter and devoted to strength and stability of the offered shells and to investigation of elastic and plastic deformations emerging in the process of bending a metal plane sheet into the designed torse; and having presented simplified methods of strength analysis for consideration.

The momentless theory of shell analysis that was widely used in 1950s and in 1960s [13] is classified as a simplified method of strength analysis and it is not forgotten at present time [14; 15].

2. Surfaces of conical type on an elliptic base

2.1. Elliptical conical surface

If a perpendicular line to the plane of director ellipse (1), dropped from the apex of the cone, passes through the point of intersection of the axes of the director ellipse, then the obtained surface is called the *right elliptical conical surface* [16].

Its canonical equation can be written in the form:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0, \quad (2)$$

where c is the height of the conical surface. Plane $z = h \neq 0$ intersects the cone along an ellipse with semi-axes $a|h|/c$ and $b|h|/c$. Parametrical form of definition of surface (2):

$$x = x(u,v) = au\cos v;\ y = y(u,) = bu\sin v;\ z = z(u) = cu,$$

where $u = z/c$ is a dimetionless parameter, which is equal to the height of the cone divided by parameter c; $0 \le v \le 2\pi.$ Curvilinear coordinates u, v are non-orthogonal but conjugate (Figure 1).

Let us assume that a cone contains two similar ellipses lying in parallel planes (Figure 2):

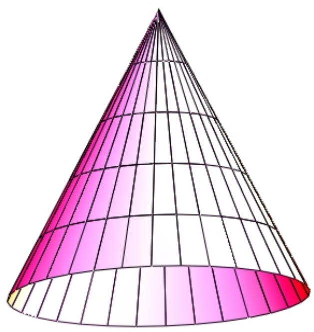


Figure 1. A right elliptical cone
Source: made by the author

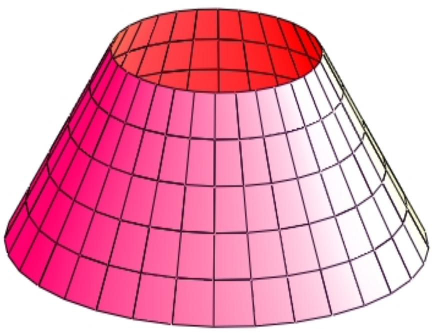


Figure 2. A truncated elliptical cone
Source: made by the author

$$\frac{y^2}{c^2}+\frac{z^2}{d^2}=1, x=l \text{ and } \frac{y^2}{b^2}+\frac{z^2}{a^2}=1, x=0, \quad \text{where } ac=bd.$$

In this case, the implicit equation of the designed surface will be

$$[x(c-b)/l+b]^2-y^2-c^2z^2/d^2=0.$$

Inclined elliptic conical surface is the *second order conical surface* which forms by the movement of a straight line passing through a defined point and intersecting the director ellipse. A perpendicular line to the plane of the director ellipse, dropped from the apex of the cone, does not pass through the center of the director ellipse [17].

2.2. Torses of equal slope with a director ellipse

Torse of equal slope with a director ellipse is a ruled surface having a constant angle α between straight generatricies and the respective principle normals of the director ellipse (1) [18]. Numerical modelling of this surface was described in a paper [19]. It is also possible to find other original sources in which an equal slope surface with director ellipse is investigated [20; 21].

Parametric equations of this surface [22; 23] (Figure 3):

$$z=z(u)=-u\sin\alpha,$$

$$x=x(u,\nu)=a\cos\nu+\frac{ub\cos\alpha\cos\nu}{\sqrt{a^2\sin^2\nu+b^2\cos^2\nu}},\quad y=y(u,\nu)=b\sin\nu+\frac{ua\cos\alpha\sin\nu}{\sqrt{a^2\sin^2\nu+b^2\cos^2\nu}}. \quad (3)$$

This method of defining a torse surface of equal slope supposes that the director ellipse is given by parametric equations

$$x=x(\nu)=a\cos\nu,\quad y=y(\nu)=b\sin\nu. \quad (4)$$

A coordinate line $u = 0$ coincides with the director ellipse, but a family of the u lines are rectilinear generators of a torse of equal slope. A surface of zero curvature (3) is given in terms of principal curvature lines u, ν.

Sections of a torse surface in $z = z_o$ = const planes contain closed curves:

$$x=x(\nu)=\left(a-\frac{z_o b\cos\alpha}{\sin\alpha}\frac{1}{\sqrt{a^2\sin^2\nu+b^2\cos^2\nu}}\right)\cos\nu,$$

$$y=y(\nu)=\left(b-\frac{z_o a\cos\alpha}{\sin\alpha}\frac{1}{\sqrt{a^2\sin^2\nu+b^2\cos^2\nu}}\right)\sin\nu.$$

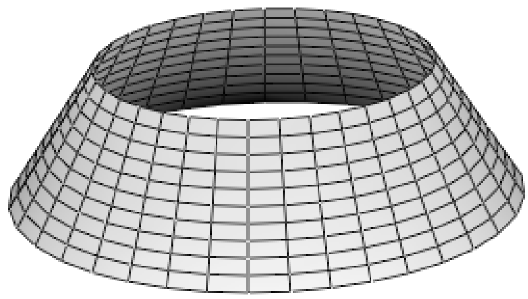


Figure 3. A torse of equal slope with a director ellipse
Source: made by the author

The surfaces of zero Gaussian curvature shown in Figures 2 and 3 are rather like, but the surface shown in Figure 2 is a cone and the surface in Figure 3 is a non-degenerated torse i.e. a torse with an edge of regression.

If the director ellipse is given by parametric equations

$$x=x(\beta)=r\cos\beta,\quad y=y(\beta)=r\sin\beta, \quad (5)$$

then parametric equations of the whole surface take the following form [24]:

$$x = x(u,\beta) = r(\beta)\cos\beta + \frac{ub^2\cos\alpha\cos\beta}{\sqrt{a^4\sin^2\beta + b^4\cos^2\beta}}, \quad \text{where} \quad r = r(\beta) = \frac{ab}{\sqrt{a^2\sin^2\beta + b^2\cos^2\beta}},$$
$$y = y(u,\beta) = r(\beta)\sin\beta + \frac{ua^2\cos\alpha\sin\beta}{\sqrt{a^4\sin^2\beta + b^4\cos^2\beta}}, \quad z = z(u) = -u\sin\alpha, \qquad (6)$$

β is the angle counted off the *Ox* axis in the direction of the *Oy* axis, $\beta \neq \nu$, $r = r(\beta)$ is the distance from the center of the director ellipse to an arbitrary point on it.

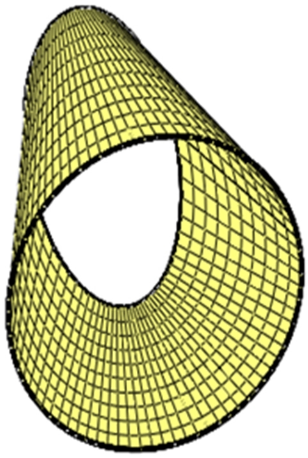


Figure 4. A torse surface with two ellipses in parallel planes
Source: made by the author

2.3. Torse with two ellipses placed in parallel planes and with parallel axes

One can use G. Monge's method [25] for the design of torse of conical type on an elliptical base. This method offers to construct a developable surface by movement of a straight line along two space curves. The directrix of the torse will be passing through the corresponding points of plane director curves where the tangent lines are parallel and these two director curves are plane curves and lie in parallel planes (Figure 4). Additional information can be found in [26].

2.4. Surfaces of conical type on plane elliptical base with inclined lines in other two coordinate planes

Surfaces on a plane base defined by a main frame from three arbitrary plane curves in three main coordinate planes are used, in general, in shipbuilding [22]. For the first time, at the Peoples Friendship University of Russia, these surfaces were offered for application in architecture and in construction. Some of these surfaces can be included in the group of ruled surfaces of conical type.

Ruled surfaces with a main frame from three superellipses

$$|y|^r = W^r\left(1 - \frac{|x|^t}{L^t}\right), \qquad |z|^n = T^n\left(1 - \frac{|y|^m}{W^m}\right), \qquad |z|^s = T^s\left(1 - \frac{|x|^k}{L^k}\right),$$

are studied in paper [27] where one, two, or all three superellipses degenerate into straight lines, i.e. $r = t = 1$, or $n = m = s = k = 1$, or $r = t = n = m = s = k = 1$. Assuming that the ellipse is placed in the *xOy* plane, then $r = t = 2$, but the other two superellipses degenerate into straight lines, i.e. $n = m = s = k = 1$, so we can derive three surfaces of conical type on elliptical base, defined by parametric equations:

$$x = x(u) = \pm uL,\ y = y(u,\nu) = \nu W[1 - u^2]^{1/2},\ z = z(u,\nu) = T[1 - u][1 - |\nu|] \text{ (Figure 5, } a\text{)}; \qquad (7)$$

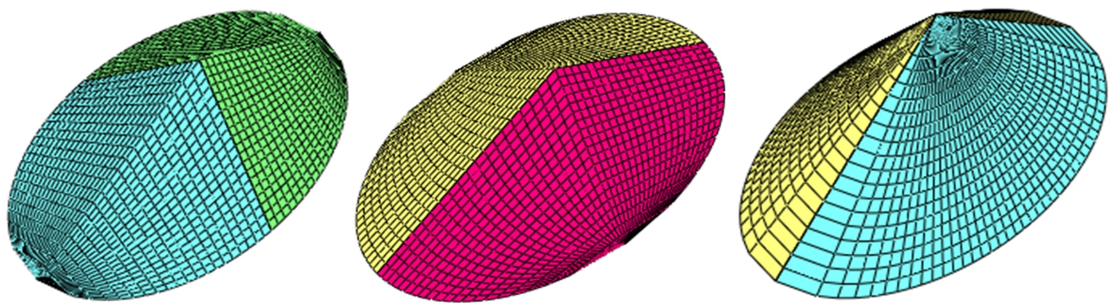
$$x = x(u,\nu) = \nu L[1 - u^2]^{1/2},\ y = y(u) = \pm uW,\ z = z(u) = T[1 - u][1 - |\nu|] \text{ (Figure 5, } b\text{)}; \qquad (8)$$

$$x = x(u,\nu) = \nu L[1 - u],\ y = y(u,\nu) = \pm W[1 - u][1 - |\nu|^2]^{1/2},\ z = z(u) = uT \text{ (Figure 5, } c\text{)}, \qquad (9)$$

where $-L \le x \le L$, $-W \le y \le W$, $0 \le z \le T$; $2L$, $2W$, T are overall dimentions of the considered surfaces. The surface shown in Figure 5, *a* forms by a family of cross sections x = const (straight lines), the surface shown in Figure 5, *b* was formed by a family of cross sections y = const (straight lines), and the surface pictured in Figure 5, *c* was generated by a family of cross sections z = const (ellipses).

Let us introduce a new parameter $0 \ge \beta \ge 2\pi$ in the form $\nu = \sin\beta$, hence $1 - \nu^2 = \cos^2\beta$, then parametric equations (9) become

$$x = x(u,\nu) = L[1 - u]\sin\beta,\ y = y(u,\nu) = W[1 - u]\cos\beta,\ z = z(u) = uT \text{ (Figure 5, } c\text{)}.$$



a *b* *c*

Figure 5. The **s**urfaces on a plane oval base with the same main frame:
a — surface formed by a family of cross sections x = const (straight lines);
b — surface formed by a family of cross sections y = const (straight lines);
c — surface formed by a family of cross sections z = const (ellipses)
Source: made by the author

The vector equation of any analytical surface can be written as

$$\boldsymbol{r} = \boldsymbol{r}(u,\nu) = x(u,\nu)\boldsymbol{i} + y(u,\nu)\boldsymbol{j} + z(u,\nu)\boldsymbol{k}.$$

In this case, according to the theory of surfaces, Gaussian coefficients of the first order (E, G, F) and Gaussian coefficients of the second order (L_0, M, N) for the ruled surface of conical type (7), were obtained in the following form:

$$E = A^2 = \boldsymbol{r}_u^2 = L^2 + \nu^2u^2W^2/(1-u^2) + T^2(1-\nu)^2,$$

$$G = B^2 = \boldsymbol{r}_\nu^2 = W^2(1-u^2) + T^2(1-u)^2 = B^2(u),$$

$$F = \boldsymbol{r}_u\,\boldsymbol{r}_\nu = -\nu uW^2 + T^2(1-u)(1-\nu),$$

$$L_0 = -LTW\nu\,(1-\nu)/(A^2B^2 - F^2)^{1/2},$$

$$M = LTWu(1-u)^{1/2}/(1+u)^{1/2}, \quad N = 0.$$

where A and B are the Lame coefficients in the theory of surfaces. The value of the coefficient of the second fundamental form $N = 0$ shows that curvilinear coordinate ν coincides with the rectilinear generatrix of surface (7), Figure 5, *a*. The value $F \neq 0$ shows that curvilinear coordinates u, ν are non-orthogonal coordinates. The total Gaussian curvature of the examined surface

$$K = \left(L_0N - M^2\right)/\left(A^2B^2 - F^2\right) = -M/\left(A^2B^2 - F^2\right) < 0$$

shows that the ruled surface presented in Figure 5, *a* is a surface of negative Gaussian curvature.

Analysing parametric equations (7) and (8) of the ruled surfaces of conical type presented in Figure 5, *a* and Figure 5, *b*, formed by the method of construction of surfaces with a main frame from arbitrary plane curves lying in main coordinate planes, one can draw a conclusion that these surfaces are identical surfaces. Hence, taking two algebraic curves of the main frame with the same exponents, we obtain two identical algebraic surfaces from three surfaces. By assuming three arbitrary algebraic plane curves of the main frame with the same exponents, we obtain three identical algebraical surfaces of the same order coinciding with the order of the curves of the frame. Having assumed three algebraical curves of main frame with different exponents, we shall have three different algebraic surfaces [27].

Coefficients of the fundamental forms in the theory of surfaces for the ruled surface of conical type (9) were obtained in the following form

$$\begin{aligned}
&E = A^2 = \mathbf{r}_u^{\,2} = W^2\left(1-\nu^2\right)+\nu^2 L^2 + T^2 = A^2(\nu),\\
&G = B^2 = \mathbf{r}_\nu^2 = (1-u)^2\left[L^2 + \nu^2 W^2 / \left(1-\nu^2\right)\right] = (1-u)^2 f_1(\nu),\\
&F = r_u \mathbf{r}_\nu = -\nu(1-u)\left[L^2 - W^2\right] = (1-u) f_2(\nu),\\
&A^2B^2 - F^2 = (1-u)^2\left[T^2L^2 + W^2\left(L^2+\nu^2T^2\right)/\left(1-\nu^2\right)\right] = (1-u)^2 f_3(\nu),\\
&L_0 = 0, M = 0,
\end{aligned} \tag{10}$$

$$N = LTW(1-u)^2 / \left[\left(A^2B^2-F^2\right)^{1/2}\left(1-\nu^2\right)^{3/2}\right] = LTW(1-u)/\left[f_3^{1/2}(\nu)\cdot\left(1-\nu^2\right)^{3/2}\right] = (1-u) f_4(\nu). \tag{11}$$

Coefficients of the fundamental forms (10), (11) of surface (9) show that the coordinate lines u are straight lines and surface (9) is a surface of zero Gaussian curvature

$$N = LTW(1-u)^2 / K = \left(L_0 N - M^2\right)/\left(A^2B^2 - F^2\right) = 0.$$

Hence, curvilinear coordinate lines u, ν are non-orthogonal conjugate coordinates. It is obvious that surfaces (7) and (8) are cylindroids [28], and surface (9) is a right elliptical cone. All of the three ruled surfaces belong to the group of surfaces of conical type on elliptical base.

2.5. Developments of torses onto a plane

It should be noted that torse surfaces permit their development onto a plane without breaks and folds. There are several analytical, graphical, and numerical methods of construction of developments [29]. US Patent for a method for designing a development drawing was even issued [30]. Till present time, only the method of step-by-step calculation of lengths of director curves, rectilinear directrixes, and angles between them was used for the considered torse surfaces. An algorithm of developing a torse using this method was described and realized for particular examples a paper [31]. As applied to the considered ruled surfaces, a development of the torse with a circle and an ellipse on parallel opposite ends was carried out (Figure 6).

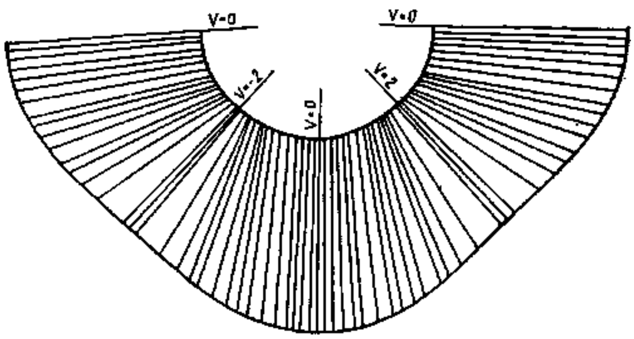


Figure 6. A development of the torse surface with a circle and an ellipse on the parallel ends
Source: made by the author

2.6. Approximation of torses on elliptical base by folds

Substitution of curvilinear surfaces by folded circumscribed surfaces following the form of the original surfaces has certain practical importance [32].

Such substitution is easily realized just for torses, because they are formed by a single-parametric family of planes tangent to the torses along the directrix lines. An example of the substitution by folds of a torse with a circle and ellipse on opposite ends is given in paper [33]. The torse, considered in a paper [33], was constructed on the basis of the torse shown in Figure 4 with the substitution of one director ellipse with a circle (Figure 7).

In Figure 7, the equation of single-parametric family of the planes forming the specified torse surface is denoted by $M(\nu) = 0$. The development of the folded surface is presented in Figure 8. Having the coordinates of the corner points of the fold, one can easily obtain all necessary lengths and angles of the corresponding development. The same method of substitution was used in paper [34], but for a torse with parabolas on parallel ends.

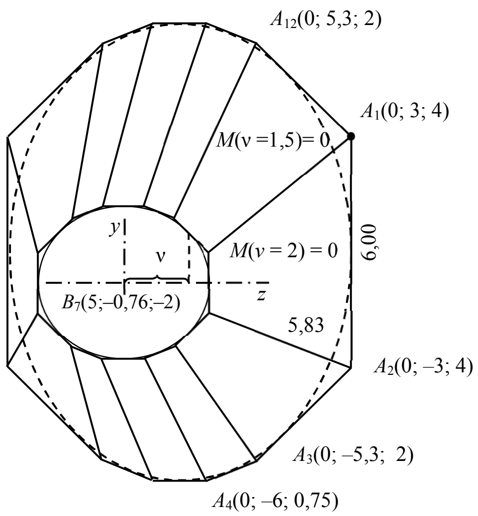


Figure 7. Substitution of the torse surface with a circle and an ellipse on parallel ends
S o u r c e : made by the author

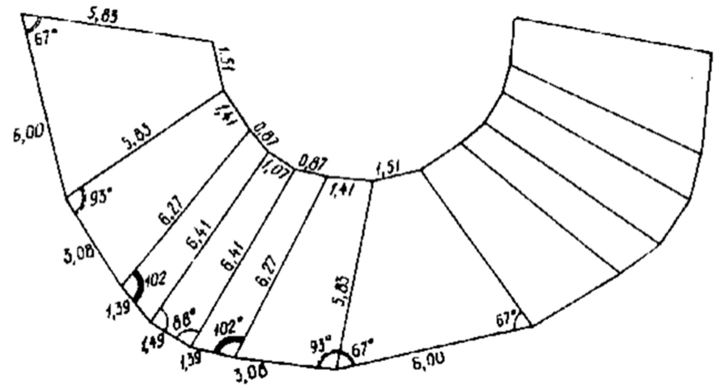


Figure 8. The development of the folded surface shown in Figure 7
S o u r c e : made by the author

3. Shells of conical type on elliptic base

3.1. Parabolic bending of plane thin sheet into a torse shell

In the process of parabolic bending of plane sheet from elastic material into a torse work piece or into a torse shell, internal normal stresses will be emerging in them. These normal stresses can reach the yield limit of the material. Parabolic bending preserves rectilinear generators of developable middle surfaces, length of curves on the surface, and angles between the curves. Hence, parabolic bending can be called isometric bending. Analytical formulae for the determination of the emerging normal stresses are given paper [35].

Equations

$$\kappa_1 = \frac{1}{R_1'} - \frac{1}{R_1} + \frac{\varepsilon_1}{R_1} = 0, \quad \kappa_2 = \frac{1}{R_2'} - \frac{1}{R_2} + \frac{\varepsilon_2}{R_2}, \quad \kappa_{12} = 0, \tag{12}$$

where R_1 and R_2 are the principal radii of curvature of the surface before bending; R_1' and R_2' are the principal radii of curvature of the deformed middle surface after bending, give the opportunity to find the changes in curvature κ_2, κ_1 and in twist κ_{12} of torse middle surface. Normal and tangent forces will be equal to zero (Figure 9, *a*), and the bending M_1, M_2 and the twisting H moments (Figure 9, *b*) for a shell formed by bending and given in lines of principal curvatures can be calculated with the help of well known Hooke's law formulae for shells

$$M_2 = D(\kappa_2 + \nu\kappa_1) = D\kappa_2;$$

$$M_1 = D(\kappa_1 + \nu\kappa_2) = D\nu\kappa_2 = \nu M_2; \quad H = M_{su} = 0,$$

where $D = Eh^3/[12(1 - \nu^2)]$ is the flexure rigidity (bending stiffness) of the shell; ν is Poisson's ratio, h is the thickness of the shell.

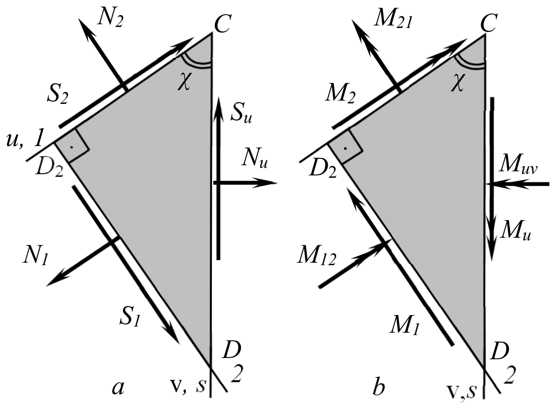


Figure 9. Internal forces and moments:
a — internal forces;
b — internal moments (per unit of length)
S o u r c e : made by the author

S.M. Hollister [36] rightly notes that to bend a real plane product from plywood, aluminium, or steel into the designed torse accurately with preservation of rectilinear directrixes is practically impossible owing to the Poisson's ratio of the used material.

3.2. Problems of strength analysis of thin shells of conical type on elliptical base

Practically all the discussed surfaces of conical type on elliptical base are given in curvilinear non-orthogonal conjugate coordinates. That is why, for strength analysis, one must use a more complex system of governing equations for this system of curvilinear coordinates not in lines of principal curvatures. Hitherto, there are no analytical solutions with the moment theory for the offered shells in scientific literature.

The system of governing equations in lines of principal curvature is suitable only for torse shells of equal slope. A system of 17 equations for the determination of stress-strain components of a thin elastic shell with middle surface shown in Figure 3 and given by the equations (3) is written out in paper [37]. Despite of seemingly simple form of these equtions, no one has been able to solve any particular example.

The problem about strength and stability of equal slope shells on elliptical base is solved much more easily with the help of numerical methods. M.A. Timoshin [38] examined a steel shell subjected to a vertical dead load using Lira 9.4 software with the help of finite element method in displacements. Boundary conditions are fixed or hinged support of lower edge. The first two buckling shapes for the first case of fixed and three forms of hinge supported shell were determined. It was established that shell buckling takes place before exhausting its strength. For comparison, a shell in the form of a truncated elliptic cone shown in Figure 2 was analyzed. It was supposed that the shells shown in Figures 2 and 3 have the same overall dimensions, thickness, and material.

O.O. Aleshina [39] researched the stress-strain state of a cast reinforced concrete cap 5 cm thick, the middle surface of which is defined by parametric equations (3) with geometric parameters of the ellipse $a = 2$ m, $b = 1$ m and the slope angle of rectilinear generatrix equal to $\alpha = 60°$. The analysis was performed with the help of SCAD Office software, representing an integrated system of strength analysis and design of structures on the basis of FEM. The model of the approximated middle surface with quadrilateral and triangle plane elements is shown in Figure 10. The vertical displacement of the cantilever part of the cap is less than 1mm.

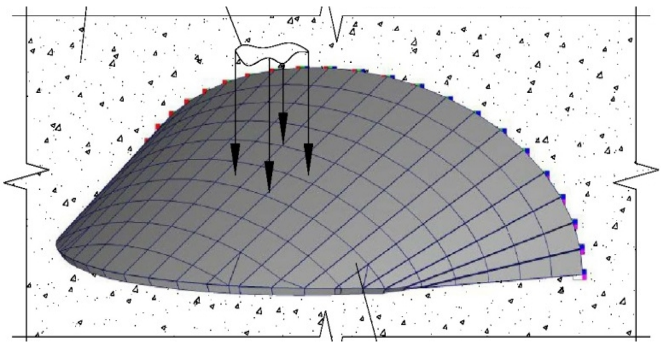


Figure 10. The model of the cap
Source: made by O.O. Aleshina [39]

In paper [15], the investigation of the equal slope shell on elliptical base is described. This investigation was performed using FEM and finite difference energy method. The first calculation was made in SCAD Office software, the second calculation was made with SHELLVRM computer program. The comparison of the obtained results using FEM, finite difference energy method, and momentless shell theory show good agreement. Maximum deviation of the results in the examined nodes is a little more than 18 %.

3.2.1. Momentless theory of shells of conical type on elliptical base

The momentless (membrane) theory assumes that equilibrium in the shell is achieved by having the in-plane membrane forces resist all applied loads without any bending moments. The momentless theory of shells with middle surface of zero Gaussian curvature gives an opportunity to obtain analytical solution and the results can be used for preliminary analysis of technical decisions. The membrane forces by themselves cannot resist local concentrated loads. In order to derive the governing equations for the momentless theory of shells, we need to reject internal bending, twisting moments, and shearing forces. For the application of the momentless theory, we may use only the equilibrium equation. The problem for equal slope shell with the middle surface (3) defined by curvilinear coordinates in lines of principal curvature is solved easier. In that case, the equilibrium equations of a shell element subjected to weight q can be written as

$$\frac{\partial}{\partial u}\left(BN_u\right)-\frac{\partial B}{\partial u}N_{\mathrm{v}}+\frac{\partial S}{\partial \mathrm{v}}+XB=0,$$

$$\frac{\partial N_{\mathrm{v}}}{\partial \mathrm{v}}+\frac{1}{B}\frac{\partial}{\partial u}\left(B^2S\right)=0,$$

$$N_{\mathrm{v}} = ZR_{\mathrm{v}},$$

where $X = -q\sin\alpha$, $Y = 0$, $Z = -q\cos\alpha$. The tangent force S may be found from the second equilibrium equation. Then from the first equilibrium equation, one can obtain normal force N_u [40].

For a torse shell presented in Figure 4, the equations of momentless shell theory can be written as

$$\frac{\partial S_{\mathrm{v}}}{\partial \mathrm{v}}+\frac{\sqrt{B^2-F^2}}{u}(N_u-N_{\mathrm{v}})+F\frac{\partial S_u}{\partial u}+\sqrt{B^2-F^2}\frac{\partial N_u}{\partial u}+\sqrt{B^2-F^2}X=0,$$

$$\frac{\partial N_{\mathrm{v}}}{\partial \mathrm{v}}+\frac{\sqrt{B^2-F^2}}{u}(S_u+S_{\mathrm{v}})-F\frac{\partial N_u}{\partial u}+\sqrt{B^2-F^2}\frac{\partial S_u}{\partial u}+\sqrt{B^2-F^2}Y=0,$$

$$\frac{B^2}{\sqrt{B^2-F^2}}\frac{N_{\mathrm{v}}}{R_{\mathrm{v}}}-\sqrt{B^2-F^2}Z=0,$$

$$(S_u-S_{\mathrm{v}})\sqrt{B^2-F^2}+F(N_{\mathrm{v}}-N_u)=0. \quad (13)$$

In equations (13), curvilinear non-orthogonal conjugate coordinates are denoted by u,v; B, F are the coefficients of the first fundamental form for the torse surface; X, Y, Z are distributed external forces per unit area in the direction of moving coordinate axes (Figure 11). Figure 11 shows the positive directions of the resultant internal forces tangent to the midsurface.

In view of equations (13), one can obtain the values of desired forces in explicit form

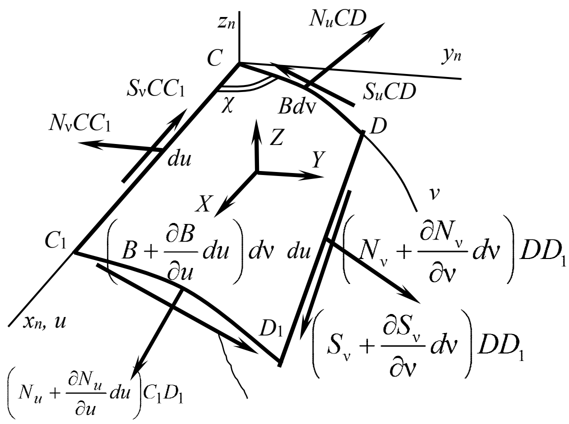


Figure 11. A fragment of momentless shell
Source: made by the author

$$N_{\mathrm{v}}=-\frac{B^2-F^2}{N}Z, \quad N_u=\frac{\sqrt{B^2-F^2}}{F}(S_u-S_{\mathrm{v}})+N_{\mathrm{v}},$$

$$S_u=\frac{1}{B^2}\left[uC_1+u^2C_2+u^3C_3+\frac{F}{u}\frac{\partial V_1(\mathrm{v})}{\partial \mathrm{v}}+V_2(\mathrm{v})\right],$$

$$S_{\mathrm{v}}=C_4+uC_5+\frac{V_1(\mathrm{v})}{u^2}, \quad (14)$$

where $C_i = C_i(\mathrm{v}, X, Y, Z)$, i =1÷5. Values of coefficients C_i in expanded form are given in monography [41]. Arbitrary functions of integration $V_1(\mathrm{v})$ and $V_2(\mathrm{v})$ can be determined by satisfying boundary conditions. Two examples of analysis of a thin torse shell with ellipse and circle on opposite parallel ends subjected to weight or linear load distributed along the top end ($X = Y = Z = 0$) using formulae (13) are given in monography [41].

Bajoriya G.Ch. [42] used the equilibrium equation containing pseudo-forces (forces with an asterisk) for the analysis of the same torse shell:

$$\frac{\partial}{\partial u}\left(BN_u^*+FS_u^*\right)+\frac{\partial}{\partial \mathrm{v}}\left(-S_{\mathrm{v}}^*+\frac{F}{B}N_{\mathrm{v}}^*\right)-\frac{\partial B}{\partial u}N_{\mathrm{v}}^*+\sqrt{B^2-F^2}\left(X+\frac{F}{B}Y\right)=0,$$

$$\frac{\partial}{\partial u}\left(BS_u^* + FN_u^*\right) + \frac{F}{B}\frac{\partial B}{\partial u}N_u^* + \frac{\partial N_v^*}{\partial v} - \frac{F}{B}\frac{\partial S_v^*}{\partial v} - \frac{\partial B}{\partial u}S_v^* + \sqrt{B^2 - F^2}\left(Y + \frac{F}{B}X\right) = 0,$$

$$\frac{BN_v^*}{R_v} + \sqrt{B^2 - F^2}Z = 0,$$

$$\frac{\sqrt{B^2 - F^2}}{B}(S_u^* + S_v^*) = 0. \tag{15}$$

Equations (15) are a particular case of the general equations of equilibrium offered by A.L. Goldenveiser [43] for shells given in curvilinear arbitrary coordinates.

It is possible to obtain values of the unknown pseudo-forces from equations (15) in explicit form

$$S_u^* = -S_v^* = S^*,$$

but the third equation gives

$$N_v^* = \frac{\sqrt{B^2 - F^2}}{N}BZ.$$

On the other hand

$$u^2 S^* = C_1^* u^3 + u^2 C_2^* - \frac{u^3 B}{(B^2 - F^2)^{3/2}}\left(\frac{B^2}{3} - F^2\right)Y + V_1^*(v),$$

$$BN_u^* + FS^* = u^2 C_3^* + uC_4^* - BC_5^* +$$

$$+\frac{\partial}{\partial v}\left[\frac{u^3 Y}{(B^2 - F^2)^{3/2}}\left(\frac{B^3}{6u} + \frac{BF^2}{2u} - \frac{F^2\sqrt{B^2 - F^2}}{2u}\ln\left|u + \frac{uB}{\sqrt{B^2 - F^2}}\right|\right)\right] + +\frac{1}{u}\frac{dV_1^*}{dv} + V_2^*(v), \tag{16}$$

where $C_i^* = C_i^*(v, X, Y, Z)$, $i = 1 \div 5$. Values of coefficients C_i^* in expanded form are presented in monography [41]. Arbitrary functions of integration $V_1^*(v)$ and $V_2^*(v)$ can be determined by satisfying boundary conditions. An example of analysis of a torse shell with ellipse and circle on opposite parallel ends under linear load distributed along the top end ($X = Y = Z = 0$) using formulae (16) are given in [41].

3.2.2. Momentless shell theory of elliptical cone

The general equilibrium equations for thin shells given in curvilinear non-orthogonal conjugate system of coordinates, offered in monography [41], may be used for the analysis of elliptical cone (9) with the help of the momentless theory. In this case, it is necessary to reject the internal shearing forces, bending and twisting moments:

$$\frac{\partial}{\partial v}(AS_v) + \frac{N_u - N_v}{\sin\chi}\left(\frac{\partial B}{\partial u} - \frac{\partial A}{\partial v}\cos\chi\right) + \frac{\partial A}{\partial v}S_u + B\frac{\partial S_u}{\partial u}\cos\chi + B\frac{\partial N_u}{\partial u}\sin\chi + ABX\sin\chi = 0, \tag{17}$$

$$\frac{\partial}{\partial v}\left(AN_v\right) + \frac{S_u + S_v}{\sin\chi}\left(\frac{\partial B}{\partial u} - \frac{\partial A}{\partial v}\cos\chi\right) - \frac{\partial A}{\partial v}N_u + B\frac{\partial S_u}{\partial u}\sin\chi - B\frac{\partial N_u}{\partial u}\cos\chi = 0, \tag{18}$$

$$\frac{N_v}{R_v\sin\chi} - Z\sin\chi = 0, \tag{19}$$

$$(S_u - S_\nu)\sin\chi + (N_\nu - N_u)\cos\chi = 0. \tag{20}$$

So, equations (17)–(20) contain normal N_u, N_ν , and tangent $S_u \neq S_\nu$ forses per unit of length of the corresponding coordinate line. X and Z are the components of external distributed load per unit of area. Let us assume $Y = 0$, taking into account that in this part, only distributed load such as weight q will be considered. Let us also take into account that

$$\cos\chi = F/(AB),\ \sin\chi,\ \partial B/\partial u$$

are functions of dimensionless parameter v only.

External surface loads X, Z are given by the formulae:

$$X = -q\cos\varphi, \quad Z = q\sin\varphi,\ Y = 0, \tag{21}$$

where φ is the angle between the direction of load (weight q) and the direction opposite to the direction of the coordinate line u, and

$$\cos\varphi = T / A, \quad \sin\varphi = \left[A^2 - T^2\right]^{1/2} / A \tag{22}$$

are the functions of the parameter v only.

In the considered case, the Lame parameter $A = A(\nu)$ is equal to the length of rectilinear coordinate line u from the vertex of the cone to the plane $z = 0$.

Equation (19) gives a value of the normal force N_v:

$$N_\nu = Z\sin^2\chi R_v = \frac{Z\sin^2\chi B^2}{N} = \frac{Z\left(A^2B^2 - F^2\right)}{A^2N} > 0. \tag{23}$$

The equation (20) gives a value of the normal force N_u :

$$N_u = (S_u - S_\nu)\mathrm{tg}\chi + N_\nu\,. \tag{24}$$

Taking into account that

$$F = -A(1-u)\frac{\partial A}{\partial \nu},$$

$$\frac{\partial B}{\partial u} - \frac{\partial A}{\partial \nu}\cos x = -\frac{(A^2B^2 - F^2)}{A^2B(1-u)},$$

let us put these variables and normal force N_u determinated by formula (24) into equation (18). After integration of the obtained result, one can have

$$S_\nu = -\frac{A}{(1-u)\sqrt{A^2B^2 - F^2}}\int(1-u)\left(A\frac{\partial N_\nu}{\partial \nu} - \frac{F}{A}\frac{\partial N_\nu}{\partial u}\right)du + \frac{V_1(\nu)}{(1-u)^2}, \tag{25}$$

where $V_1(v)$ is an arbitrary function of integration.

Substituting the same expressions and $(N_u - N_v)$, taken from the formula (20), into the equation (17), and integrating the obtained result, one can find

$$S_u = \frac{A^2B^2 - F^2}{A^2B^2}S_\nu - \frac{F}{AB^2}\int\frac{\partial}{\partial \nu}\left(AS_\nu\right)du - \frac{\sqrt{A^2B^2 - F^2}}{2AB^2}F\left[\frac{N_\nu}{A} - (1-u)X\right] + \frac{F}{AB^2}V_2(\nu), \tag{26}$$

where $V_2(\nu)$ is an arbitrary function of integration. Unknown functions $V_1(\nu)$ and $V_2(\nu)$ are found from boundary conditions, suitable for the momentless shell theory.

Thus, the momentless shell theory gives an opportunity to obtain approximate values of internal normal forces N_u and N_ν from formulae (23) and (24), and values of tangent forces S_u and S_ν from formulae (25) and (26). The formulae (25), (26) are integratable and they can be written in the expanded form.

So, write the formulae (23), (24), (25), and (26) in more detail:

$$N_\nu = \frac{q(1-u)\sqrt{A^2-T^2}\left[L^2\left(W^2+T^2\right)+\nu^2T^2\left(W^2-L^2\right)\right]^{3/2}}{LTWA^3} = (1-u)f_5(\nu),$$

$$S_\nu = \frac{(1-u)A^3}{3\sqrt{A^2B^2-F^2}}\frac{\partial}{\partial\nu}\left(\frac{N_\nu}{A}\right)+\frac{V_1(\nu)}{(1-u)^2} = (1-u)f_6(\nu)+\frac{V_1(\nu)}{(1-u)^2},$$

$$S_u = \frac{A^2B^2-F^2}{A^2B^2}S_\nu - \frac{(1-u)F}{2AB^2}\frac{\partial}{\partial\nu}\left(AS_\nu\right) - \frac{\sqrt{A^2B^2-F^2}}{2AB^2}F\left[\frac{N_\nu}{A}-(1-u)X\right]+\frac{F}{AB^2}V_2(\nu) =$$

$$= \frac{(1-u)^3}{2AB^2}\left\{2\frac{f_3f_6}{A}-\frac{\partial A}{\partial\nu}\left[A\frac{\partial A}{\partial\nu}f_6+A^2\frac{\partial f_6}{\partial\nu}-\sqrt{f_3}f_{5+}\sqrt{f_3}qT\right]\right\}+$$

$$+\frac{1}{B^2}\left[\frac{f^3}{A^2}V_1(\nu)-\frac{\partial A}{\partial\nu}\left(\frac{1}{2}\frac{\partial A}{\partial\nu}V_1(\nu)+\frac{A}{2}\frac{V_1(\nu)}{\partial\nu}\nu+(1-u)V_2(\nu)\right)\right]=$$

$$= (1-u)f_6(\nu)+\frac{1}{B^2}\left[\frac{f^3}{A^2}V_1(\nu)-\frac{\partial A}{\partial\nu}\left(\frac{1}{2}\frac{\partial A}{\partial\nu}V_1(\nu)+\frac{A}{2}\frac{V_1(\nu)}{\partial\nu}+(1-u)V_2(\nu)\right)\right],$$

$$N_u = \frac{\sqrt{A^2B^2-F^2}}{A^2B^2}F\left[-\frac{3}{2}S_\nu+\frac{(1-u)}{2F}A^2\frac{\partial S_\nu}{\partial\nu}-\frac{(1-u)}{2F}qT\sqrt{A^2B^2-F^2}+\frac{A^2}{F}V_2(\nu)\right]+\frac{A^2B^2+F^2}{2A^2B^2}N_\nu,$$

where $f_i = f_i(\nu)$ are the known functions, but $V_1(\nu)$, $V_2(\nu)$ are arbitrary functions of integration, which are found from the boundary conditions in forces.

4. Results of the Research

1. Firstly, the results of geometrical investigations of ruled surfaces of zero Gaussian curvature both with the edge of regression and with the point in the vertex in the top, formed on elliptical base, are grouped. The need to form a group of ruled surfaces of conical type on elliptical base has arisen. Elliptical cones, torses with two director ellipses lying in parallel planes, equal slope surfaces with director ellipse, ruled surfaces with a main frame from ellipse and two broken straight lines, and cylindroid with the same main frame were included in this group.

2. An example of construction of developments of the examined developable surfaces is presented and a method of substitution of torses by folds with construction of their developments is offered.

3. The second part of the paper was devoted to a review of methods of stress analysis of thin shells of conical type on elliptical base. It was established that strength and stability analysis of these shells was performed with the help of Lira 9.4, SCAD Office, and SHELLVRM software, using displacement-based FEM and finite difference energy method. The analysis was carried out for the dead and linear loads.

4. The majority of specific examples of determining normal and tangent forces emerging in the examined ruled shells is solved using the approximate momentless theory of analysis. It was established that under fulfilment of the conditions of the momentless shell theory it gives acceptable results. These results may be used, for example, for setting shell thickness or for detection of the most dangerous sections with maximum internal force resultants. The search of the most optimal shapes of shells, where the moment components are hardly noticeable, is still in progress [44].

5. For the first time, analytical formulae (23)–(26) for the determination of tangent forses in a momentless conical shell on the base in the form of any superellipse were derived.

5. Conclusion

Ruled surfaces of zero Gaussian curvature with an edge of regression and shells with ruled middle surfaces were selected for the investigation. In some examined ruled surfaces, an edge of regression degenerates into a point. This choice was fueled by the wish of designers, civil engineers, and mechanical engineers to create the most simple products, structures, and buildings in terms of manufacturing.

1. Thin-walled structures and products offered for implementation, judging by given references, have most demand in shipbuilding, aircraft building, and in applied physics.

2. By using superellipses of general type instead of ellipses of the second order as the bases of surfaces of conical type, it is possible to considerably widen the number of ruled surfaces offered for research.

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