2022. 18(5). 458–466

СТРОИТЕЛЬНАЯ МЕХАНИКА ИНЖЕНЕРНЫХ КОНСТРУКЦИЙ И СООРУЖЕНИЙ

STRUCTURAL MECHANICS OF ENGINEERING CONSTRUCTIONS AND BUILDINGS

ISSN 1815-5235 (Print), 2587-8700 (Online)

HTTP://JOURNALS.RUDN.RU/STRUCTURAL-MECHANICS

ГЕОМЕТРИЧЕСКОЕ МОДЕЛИРОВАНИЕ ФОРМ ОБОЛОЧЕК
GEOMETRICAL MODELING OF SHELL FORMS

DOI 10.22363/1815-5235-2022-18-5-458-466
UDC 624:514.8:72.01

REVIEW / НАУЧНЫЙ ОБЗОР

Analytical surfaces for architecture and engineering

Mathieu Gil-Oulbé[1]✉, Tiékolo Daou[2], Ousmane Mariko[2]

[1]*Peoples' Friendship University of Russia (RUDN University), Moscow, Russian Federation*
[2]*National School of Engineering (ENI-ABT), Bamako, Republic of Mali*
✉ gil-oulbem@hotmail.com

Article history
Received: June 17, 2022
Revised: August 13, 2022
Accepted: August 15, 2022

For citation
Gil-Oulbé M., Daou T., Mariko O. Analytical surfaces for architecture and engineering. *Structural Mechanics of Engineering Constructions and Buildings.* 2022;18(5):458–466. http://doi.org/10.22363/1815-5235-2022-18-5-458-466

Abstract. Geometers have proposed more than 600 analytical surfaces for implementation. The largest number of these surfaces is used in architecture and mechanical engineering. Although digital architecture and free form architecture are now increasingly influencing the design of long-span shell structures and curved buildings, the research and application of analytical surfaces continues on an increasing scale. The purpose of the research is to study the state of affairs in the application of analytical surfaces in the construction and engineering industries and to clarify the classes of surfaces that have found application in the study of physical phenomena or in solving purely mathematical problems, but not used in other areas of human activity. Another goal is to find analytical surfaces promising for application in architecture and mechanical engineering, which are still little known to architects and engineers. It has been established that, as before, designers take new analytical surfaces to implement their creative ideas from well-studied classes of surfaces of revolution, transfer and umbrella, minimal, ruled, wavy surfaces.

Keywords: analytical surfaces, surface classification, thin shells, shell architecture, shells, mechanical engineering, parametric architecture

Mathieu Gil-Oulbé, Candidate of Technical Sciences, Associate Professor of the Department of Civil Engineering, Academy of Engineering, Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation; ORCID: 0000-0003-0057-3485, Scopus Author ID: 57209566642, eLIBRARY ID: 27261171; gil-oulbem@hotmail.com

Tiékolo Daou, Candidate of Technical Sciences, senior lecturer, Department of Civil Engineering, National School of Engineering (ENI-ABT), B.P. 242, Avenue Van Vallenhoven, Bamako, Republic of Mali; ORCID: 0000-0003-1726-5382; daout88@gmail.com

Ousmane Mariko, Candidate of Technical Sciences, Associate Professor of the Department of Civil Engineering, National School of Engineering (ENI-ABT), B.P.242, Avenue Van Vallenhoven, Bamako, Republic of Mali; ORCID: 0000-0002-4532-0230; osomariko@gmail.com

Аналитические поверхности для архитектуры и машиностроения

М. Жиль-ульбе[1], Т. Дау[2], У. Марико[2]

[1]*Российский университет дружбы народов, Москва, Российская Федерация*
[2]*Национальная инженерная школа, Бамако, Республика Мали*
gil-oulbem@hotmail.com

История статьи
Поступила в редакцию: 17 июня 2022 г.
Доработана: 13 августа 2022 г.
Принята к публикации: 15 августа 2022 г.

Для цитирования
Gil-Oulbé M., Daou T., Mariko O. Analytical surfaces for architecture and engineering // Строительная механика инженерных конструкций и сооружений. 2022. Т. 18. № 5. С. 458–466. http://doi.org/10.22363/1815-5235-2022-18-5-458-466

Аннотация. Геометры предложили для внедрения более 600 аналитических поверхностей, из них наибольшее число применяется в архитектуре и машиностроении. Несмотря на то что сейчас значительное влияние на проектирование большепролетных оболочечных структур и искривленных зданий оказывают числовая архитектура и архитектура свободных форм, исследования и применение аналитических поверхностей продолжают увеличиваться. Цель исследования – изучение положения дел в применении аналитических поверхностей в строительной и машиностроительных отраслях и выяснение классов поверхностей, нашедших применение в исследовании физических явлений или в решении чисто математических задач, но не используемых в других отраслях деятельности человека. Определяются перспективы применения в архитектуре и машиностроении аналитических поверхностей, пока малоизвестных архитекторам и инженерам. Установлено, что дизайнеры по-прежнему берут новые аналитические поверхности для реализации своих творческих замыслов из хорошо изученных классов поверхностей вращения, переноса и зонтичных, минимальных, линейчатых, волнообразных поверхностей.

Ключевые слова: аналитические поверхности, классификация поверхностей, тонкие оболочки, архитектура оболочек, оболочки, машиностроение, параметрическая архитектура, перспективы применения

Introduction

Spatial structures covering large areas without intermediate supports have been known to man since ancient times [1; 2]. Until the 20th century, shells were used in various technological structures: vertical shafts, horizontal and inclined tunnels, pipelines, furnaces, and defensive and religious buildings. In most cases, cylindrical structures with vertical and horizontal axes and shells of rotation, in particular domes were used.

Then, in connection with the development of analytical and experimental methods for studying shells and shell structures, the shapes of the structures used became more complicated. They began to meet the ever-increasing demands of architects and engineers and found application where structures with a frequent grid of columns, with numerous walls, or composite low-span buildings were previously used.

The greatest enthusiasm for thin-walled shells continued until the 1980s [3], then interest in them began to decrease [4], but due to the demands of society at the beginning of the 21st century, their wider rational use began in small-sized housing construction, in industrial and public buildings [5; 6]. This was caused by the advent of refined numerical methods for calculating strength, stability, and seismicity, the creation of new building materials [7], and a large proposal from geometers of new forms of middle analytical surfaces of thin-walled shells [8] and rod shell structures [9]. In 1970, R. Buckminster Fuller received a gold-medal for his development of the geodesic dome. Eco Camp Patagonia was the world's first geodesic hotel, it was built in 2001. Nowadays geodesic domes are almost everywhere.

Жиль-улбе Матье, кандидат технических наук, доцент департамента строительства, Инженерная академия, Российский университет дружбы народов, Российская Федерация, 117198, Москва, ул. Миклухо-Маклая, д. 6; ORCID: 0000-0003-0057-3485, Scopus Author ID: 57209566642, eLIBRARY ID: 27261171; gil-oulbem@hotmail.com

Тьеколо Дау, кандидат технических наук, старший преподаватель, департамент строительства, Национальная инженерная школа, Республика Мали, Бамако, Ван Валленховен Авеню, П/Я 242; ORCID: 0000-0003-1726-5382; daout88@gmail.com

Усман Марико, кандидат технических наук, доцент департамента строительства, Национальная инженерная школа, Республика Бали, Бамако, Ван Валленховен Авеню, П/Я 242; ORCID: 0000-0002-4532-0230; osomariko@gmail.com

However, builders have changed their attitude to the building construction materials used. Reinforced concrete was rarely used. Only through understanding why concrete shells' loss in popularity over the course of modern history can designers be equipped with the skills to create and apply this type of construction. Through modifications to design processes, construction stages, material understanding and relevant formwork improvements will architects and designers be able to meet the demands of the 21st century [4].

Analytic surfaces are most fully presented and mathematically described in [8]. Here, more than 600 surfaces are divided into 38 classes, which in turn consist of subclasses, groups and subgroups. Analytical surfaces are well represented in electronic libraries.[1]

Many works are devoted to the use of analytical surfaces in architecture [1] and mechanical engineering [10–13], apparently, for the first time tried to find out the analytical surfaces most often used by architects and to establish cases of a single application in practice of some well-known analytical surfaces to geometers.

Commonly used analytical surfaces

The leaders, of course, are cylindrical, conical surfaces, ruled surfaces of negative Gaussian curvature ($K < 0$), as well as surfaces of revolution [14].

Ruled surfaces, including torso surfaces ($K = 0$), cylindroids ($K < 0$), cylindrical helical strips and cylindrical surfaces with aerodynamic profiles, rotative and spiroidal surfaces with straight generatrices are often used in mechanical engineering, shipbuilding and aircraft construction.

Transfer surfaces, especially direct transfer surfaces, are taken as the basis for the formation of median surfaces of hundreds of thin shells on rectangular plans. They can be seen in any city in developed countries.

Umbrella shells and shells of the umbrella type are actively used to block markets, business centers, circuses, religious buildings and to protect the radar [15]. Radar umbrella shells are operated at many airports around the world.

The next class of surfaces according to the frequency of their use in the national economy can be considered the class of cyclic surfaces [16]. They are used in the shaping of civil, industrial, and agricultural structures. They have found the same wide application in mechanical engineering.

Seventeen algebraic surfaces of the second order are usually distinguished into a separate class, but almost all of these surfaces can be distributed among other classes [8; 17]. All surfaces of the second order, except for imaginary ones, can be seen in the outlines of many architectural structures [18] and machine-building products [19]. Many shells, described with analytical functions, such as quadric surfaces, have certain geometric properties that allow for more feasible construction techniques. For example, a shell form based on a hyperbolic paraboloid can be built from straight elements, and a spherical dome has constant Gaussian and mean curvatures, minimizing the number of the components.

From the class of helical surfaces, the group of ruled and circular helical surfaces is most often used. In mechanical engineering, there are examples of the use of helical surfaces of variable pitch [20]. The surface of the screw pillar and the surface of St. Elijah can often be seen in the outlines of the supporting columns of ancient buildings. The tubular helical surface is well known and widely used both in architecture and in mechanical engineering (coils).

More and more architects support the idea of using minimal surfaces in architecture [21]. A new architectural direction, Minimal Surface Architecture, has emerged. However, most of the ideas are implemented only in projects. 15 minimal surfaces are known that have a parametric form of the assignment. There are descriptions of real thin-walled shell structures built in the form of a minimum surface of revolution (catenoid), a minimum ruled surface (straight helicoid) and a minimum transfer surface (the first Sherk surface). The Olympic Stadium in Munich was built in 1972 using minimal surfaces close to the Schwartz surface.

Rarely used analytical surfaces

Very rarely, objects in the form of one-sided surfaces are used. Known for a dozen installations and sculptures in the form of a Mobius strip. There are several wire art objects imitating the Klein surface. Mathematical model of the Boy's surface with a size of 2 × 2 m is available at Smith College, McConnell Hall.

The torso coverings of public buildings, obtained by parabolic bending of a steel sheet, have already been used in several cases (Figure 1). Their shape is not an analytical surface, since they are built experimentally on

[1] Parametrische Flächen und Körper. Available from: www.3d-meier.de/tut3/seite0.html (accessed: 21.06.2021); Weisstein E.W. Wolfram. Available from: http://mathworld.wolfram.com (accessed: 21.06.2021).

two edge curves. The possibility of obtaining the equation of a torso surface containing two predetermined curves was proved by G. Monge, but only about a dozen torso surfaces containing two predetermined plane curves lying in parallel planes are described in the scientific literature [8]. Torso surfaces are used in shipbuilding and in agricultural engineering for shaping plows and augers.

Figure 1. Hotel Marques de Riscal, Spain, arch. Frank Gerhy (Available from: ru.pinterest.com)

Figure 2. The gently sloping reinforced concrete pavement of the Nekrasovsky market in St. Petersburg, 1960 (Available from: www.g2p.ru)

Velaroidal shells are rarely found on a flat rectangular plan. Their median surfaces are outlined along the direct transfer surface with a generating curve, which changes its curvature during movement [22] and becomes a straight line on the contour. The efficient materialization of velaroidal (funicular) shells is particularly difficult. This hinders their application [23]. There is a well-known velaroidal shell for covering an industrial workshop, designed by M. Mihailescu [24], a sloping reinforced concrete velaroidal shell erected over the Nekrasovsky market in St. Petersburg in 1960 (Figure 2) and a fragment of the covering of the Cultural Center in Muscat (Oman) [25]. The group “Velaroidal surfaces” is often included in the class of transfer surfaces.

Surfaces with a spherical directrix curve have a line lying on a sphere as a directrix curve. Of the many surfaces proposed by geometers, only a cylindrical-spherical strip was found on the playground (Figure 3).

Figure 3. Object in the form of a strip with a directrix curve on a sphere (photo by M. Gil-Oulbe)

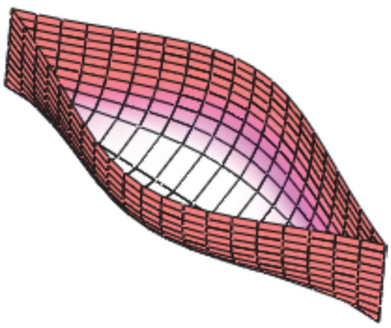

Figure 4. The surface of a ship's hull formed by a family of lines parallel to the waterline [8]

Figure 5. The Pavilion Purr, used as a visitor’s center, Amsterdam’s historic defense line, 2012 (Available from: archdally.com)

The class of continuous topographic surfaces is defined by the continuous frame of its horizontals (level lines) [8]. Surfaces of revolution are also included in this class, but it is generally accepted to single them out in a separate class. Algebraic surfaces given in explicit form $z = z(x, y)$, Catalan surfaces ($K < 0$), surfaces of the same slope ($K = 0$) can also be considered continuous topographic surfaces. Continuous topographic surfaces are mainly used in topography, mining, landscape architecture and shipbuilding. In shipbuilding, they are called hydrodynamic surfaces [26] or algebraic surfaces for ship hulls (Figure 4). In the construction business, they are used very rarely, and they are not intended for use in the construction industry.

Spiral surfaces on circular cones are usually used to form spiral chambers [27]. Architects have found use for a cylindrical-conical spiral strip (second building of the German Historical Museum in Berlin), a spiral conical strip, for example, The Pavilion Purr by Emma Architecten Towers with Wooden Wonder (Figure 5) and a tubular spiral surface.

Spiral surfaces are often confused with spiral surfaces, but spiral surfaces can have any spiral on any surface as a guide curve, and the generating curve can change its shape in the process of movement. An example is a spherical spiral strip (Figure 3) and architectural structures in the form of shells. One of the buildings with a lined spiral roof is given in the article [13].

Surfaces from the "Carved surfaces" class are fully included in the "Surfaces of congruent sections" class. Carved surfaces include circular helical surfaces and helical surfaces with arbitrary flat generating curves. If we take into account all groups of surfaces with rigid flat generating curves that are included in the class of surfaces of congruent sections, then we must assume that these surfaces are popular with architects, especially since all geometric problems for them have already been solved. The author of [28] gives an example of a real structure with congruent curves.

Analytical surfaces that have not found application in practice

Wavy, wavy and corrugated surfaces are very widely represented in architecture, but they are all made in the style of digital architecture, i.e. the surfaces were built according to the given reference points and are not analytical surfaces (Figure 6).

Velaroidal surfaces on a circular plan cannot be included in the wrapping surface class. In [29] they are called velaroidal surfaces. There are several projects of sports facilities and light surfaces (Figure 7), the geometry of these surfaces is well developed, but no real structures of this form have been found.

Architects and machine builders were not interested in surfaces of constant Gaussian curvature (K = const). If we do not take into account the sphere with $K = R^2$ = const, then the remaining surfaces of constant positive Gaussian curvature have not found application in various branches of human activity. Only the Sievert surface (Figure 8) with certain geometric parameters becomes similar to a self-intersecting umbrella-type surface, which can attract architects. Among the surfaces of the subclass "Surfaces of constant negative Gaussian curvature" the pseudosphere is the best known. This surface has found application in garden and park architecture. Several large-scale mathematical models of a pseudosphere made of steel wire, plywood, and aluminum have been made [25; 30].

Figure 6. I. Viner-Usmanova Sports Palace in Luzhniki, Moscow
(photo by L.A. Alborova)

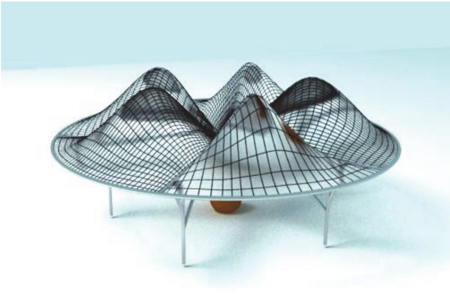

Figure 7. Structure in the form of several identical velaroidal surfaces on a flat circular plan with a singular point in the center (the figure by L. Alborova)

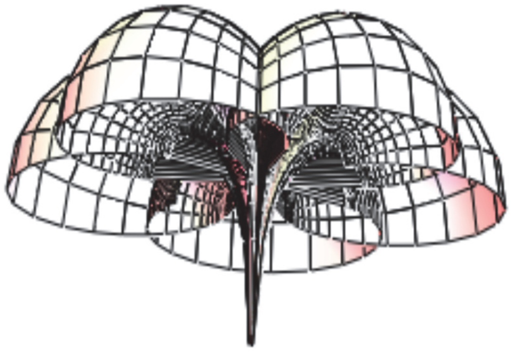

Figure 8. One of the forms of the Sievert surface [8]

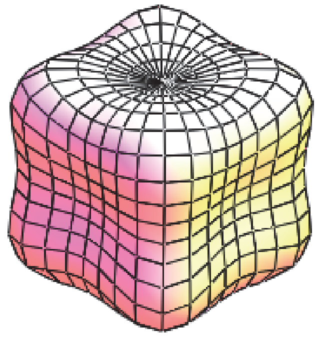

Figure 9. Tooth surface of the 4th order [8]

Algebraic surfaces above the second order may have a good architectural future [8]. The shape of some existing curvilinear structures in the world can be easily approximated by these surfaces. Surfaces such as the parabolic surface of Schroda, Goursat, Euler, the tooth surface (Figure 9), surfaces with two and three double straight lines, T. Nordstrand's surface "Chair", all of the 4th order; an 8th order surface with a 4th order Lame curve, a 4th order Lame curve and an ellipse in 3 principal coordinate sections [8] can easily be used in architecture, or they can be used to solve various technical problems in mechanical engineering.

Among the surfaces of constant mean curvature, the best known are the sphere ($H = R$), the cylindrical surface of revolution ($H = R / 2$), and the minimal surfaces ($H = 0$). These surfaces have found application in both architecture and mechanical engineering. Less well-known are nodoid and unduloid surfaces of revolution, which attract close attention from mathematicians who develop questions of conjugation of two surfaces of revolution [31].

The study of scientific, popular scientific and reference literature, materials contained on the Internet showed that some classes of surfaces have not yet found application in the construction and engineering industries. These are helical and pseudo-minimal [32] surfaces, most of the surfaces from the classes "Athene-minimal surfaces", "Peterson surfaces", except for the transfer surfaces of second-order curves, "Bonnet surfaces", "Blutel surfaces", except for second-order surfaces, "Surfaces Hoshimoto" [33], "Weingarten surfaces", except for surfaces of revolution. These surfaces are used to study some physical processes, solve purely mathematical problems and to determine surfaces isometric to surfaces of revolution.

Possibilities of using new analytical surfaces in engineering structures

The analysis carried out in [34] showed that architects and engineers used only about 5% of more than 600 analytical surfaces proposed by geometer mathematicians. They are used mainly by architects working in the styles of "Parametric architecture", "Geometric high-tech" and "Industrial architecture".

In the 21st century, shell structure design has become dominated by free-form, parametric, and organic architectures. Some architects support this current [35], others doubt its usefulness [36]. E.V. Ermolenko [36] from Moscow Architectural Institute notes: "…postmodernism, deconstructivism, parametric architecture oversaturated the space of human life, a stylistic crisis arose."

Apparently, the architects have already exhausted the set of existing surface classes. Taking into account the demand of society for large-span structures and public buildings of an unusual shape, surfaces from well-studied classes of surfaces of revolution, transfer, umbrella, minimal, ruled and undulating (Figure 6) surfaces will be used. Basically, shell structures will be designed taking into account environmental [37], energy-saving requirements and transforming structures.

Passion for shell structures should not grow into an end in itself. I.A. Bondarenko [38] warns "that in this case one should not slide into populism. It is necessary to observe a sense of proportion in everything. Unfortunately, today all the successes and failures of architects are based mainly on their personal business and human qualities. Individualism and hypertrophied – 'breakthrough' – creative creativity are encouraged too much. <…> This leads to the fact that something untenable can be presented as a professional achievement, as a senseless design originality as an architectural innovation."

Conclusion

In presented paper, it is demonstrated that a number of classes and groups of analytical surfaces used in national economy is increasing in the 21st century.

The authors show that study and information on new analytical surfaces can give impetus to architects and industrial designers for their realization in real shell structures

Voluminous literature containing 40 titles is presented. New surfaces and examples of their single applications are given. For example, a fragment of the covering of the Cultural Center in Muscat (Oman), a cylindrical-spherical strip on child playgrounds in Moscow (Russia), the second building of the German Historical Museum in Berlin in the form of the cylindrical-and-conic spiral strip, and so on are mentioned.

Commonly used classes of analytical surfaces and rarely used analytical surfaces are described. Analytical surfaces that have not found application in practice for the present but having great potential in future are pointed out too.

CAD was already available in the 1970s and 1980s but its use only became common practice starting in the late 1980s with the availability of personal computers. This process greatly facilitated some of the tasks of designing curvilinear structures and attracted young researchers to their design. Materials and information containing in this paper can help to consider a problem on possible wide using of computer modelling of shapes in connection with needs of parametrical, digital, generative, evolutionary, and bionic architecture.

References / Список литературы

1. Krasić S. *Geometrijske površi u arhitekturi.* Niš; 2012.

2. Melaragno M. *An introduction to shell structures*. Springer US; 1991. https://doi.org/10.1007/978-1-4757-0223-1

3. Bradshaw R., Campbell D., Gargari M., Mirmiran A., Tripeny P. Special structures. Past, present, and future. *Journal of Structural Engineering.* 2002;128:691–701. https://doi.org/10.1061/(ASCE)0733-9445(2002)128:6(691)

4. Tang G. An overview of historical and contemporary concrete shells: their construction and factors in their general disappearance. *International Journal of Space Structures*. 2015;30(1):1–12. https://doi.org/10.1260/0266-3511.30.1.1

5. Krivoshapko S.N. Shell structures and shells at the beginning of the 21st century. *Structural Mechanics of Engineering Constructions and Buildings*. 2021;17(6):553–561. https://doi.org/10.22363/1815-5235-2021-17-6-553-561

6. Levy M. From shells to tensile structures: a personal history. *Nexus Network Journal.* 2017;19:565–578. https://doi.org/10.1007/s00004-016-0317-5

7. Bratukhin A.G., Sirotkin O.S., Sabodash P.F., Egorov V.N. *Materials of future and their unique properties*. Moscow: Mashinostroenie Publ.; 1995. (In Russ.)

Братухин А.Г., Сироткин О.С., Сабодаш П.Ф., Егоров В.Н. Материалы будущего и их удивительные свойства. М.: Машиностроение, 1995. 125 с.

8. Krivoshapko S.N., Ivanov V.N. *Encyclopedia of analytical surfaces.* Springer International; 2015. https://doi.org/10.1007/978-3-319-11773-7

9. Vrontissi M. Designing and building a geodesic dome as a bearing structure for an 'artificial sky' lighting installation. *Symposium of the International Association for Shell and Spatial Structures. Evolution and Trends in Design, Analysis and Construction of Shell and Spatial Structures: Proceedings.* Valencia; 2009. p. 1379–1390.

10. Druzhinskiy I.A. *Complex surfaces: mathematical and technological description*. Leningrad: Mashinostroenie Publ.; 1985. (In Russ.)

Дружинский И.А. Сложные поверхности: математическое описание и технологическое обеспечение. Л.: Машиностроение, 1985. 263 с.

11. Podgorniy A.L., Grinko E.A., Solovey N.A. On research of new surface forms as applied to structures of diverse purpose. *RUDN Journal of Engineering Researches.* 2013;(1):140–145. (In Russ.)

Подгорный А.Л., Гринько Е.А., Соловей Н.А. Исследование новых форм поверхностей применительно к конструкциям различного назначения // Вестник Российского университета дружбы народов. Серия: Инженерные исследования. 2013. № 1. С. 140–145.

12. Mamieva I.A., Gbaguidi-Aisse G.L. Influence of the geometrical researches of rare type surfaces on design of new and unique structures. *Building and Reconstruction.* 2019;5(85):23–34. http://doi.org/10.33979/2073-7416-2019-85-5-23-34

13. Grinko E.A. Classification of analytical surfaces as applied to parametrical architecture and machine building. *RUDN Journal of Engineering Researches.* 2018;19(4):438–456. (In Russ.) http://doi.org/10.22363/2312-8143-2018-19-4-438-456

Гринько Е.А. Классификация аналитических поверхностей применительно к параметрической архитектуре и машиностроению // Вестник Российского университета дружбы народов. Серия: Инженерные исследования. 2018. Т. 19. № 4. С. 438–456. http://doi.org/10.22363/2312-8143-2018-19-4-438-456

14. *Кривошапко С.Н.* Упрощенный критерий оптимальности для оболочек вращения // Приволжский научный журнал. 2019. № 4 (52). С. 108–116.

Krivoshapko S.N. A simplified criterion of optimality for shells of revolution. *Privolzhsky Scientific Journal*. 2019;(4):108–116. (In Russ.)

15. Krivoshapko S.N. The opportunities of umbrella-type shells. *Structural Mechanics of Engineering Constructions and Buildings*. 2020;16(4):271–278. https://doi.org/10.22363/1815-5235-2020-16-4-271-278

16. Bock Hyeng Ch.A., Yamb E.B. Application of cyclic shells in architecture, machine design, and bionics. *International Journal of Modern Engineering Research*. 2012;2(3):799–806.

17. Burlov V.V., Nesterenko L.A., Remontova L.V., Orlov N.S. 3D simulation of second-order surfaces. *Geometry and Graphics*. 2016;4(4):48–59. (In Russ.)

Бурлов В.В., Нестеренко Л.А., Ремонтова Л.В., Орлов Н.С. 3D-моделирование поверхностей 2-го порядка // Геометрия и графика. 2016. Т. 4. № 4. С. 48–59.

18. Khmarova L., Usmanova E.A. Second order surfaces in architecture and construction. *IOP Conference Series Materials Science and Engineering*. 2018;451(1):012118. https://doi.org/10.1088/1742-6596/451/1/012118

19. Chempinskiy L.A. *Fundaments of geometrical modelling in machine building.* Samara: Samara University Publ.; 2017. (In Russ.)

Чемпинский Л.А. Основы геометрического моделирования в машиностроении. Самара: Изд-во Самарского университета, 2017. 160 с.

20. Krivoshapko S.N. Geometry and strength of general helicoidal shells. *Applied Mechanics Reviews (USA).* 1999;52(5):161–175. https://doi.org/10.1115/1.3098932

21. Emmer M. Minimal surfaces and architecture: new forms. *Nexus Network Journal.* 2013;15:227–239. https://doi.org/10.1007/s00004-013-0147-7

22. Berestova S.A., Misyura N.E., Mityushov E.A. Geometry of self-bearing covering on rectangular plan. *Structural Mechanics of Engineering Constructions and Buildings.* 2017;(4):15–18. (In Russ.) https://doi.org/10.22363/1815-5235-2017-4-15-18

Берестова С.А., Мисюра Н.Е., Митюшов Е.А. Геометрия самонесущих покрытий на прямоугольном плане // Строительная механика инженерных конструкций и сооружений. 2017. № 4. С. 15–18. https://doi.org/10.22363/1815-5235-2017-4-15-18

23. Rippmann M. *Funicular shell design: geometric approaches to form finding and fabrication of discrete funicular structures* (Dr. sc. thesis). Zürich: ETH; 2016. https://doi.org/10.3929/ethz-a-010656780

24. Mihailescu M., Horvath I. Velaroidal shells for covering universal industrial halls. *Acta Techn. Acad. Sci. Hung*. 1977;85(1–2):135–145.

25. Gbaguidi Aïssè G.L. Influence of the geometrical researches of surfaces of revolution and translation surfaces on design of unique structures. *Structural Mechanics of Engineering Constructions and Buildings*. 2019;15(4):308–314. https://doi.org/10.22363/1815-5235-2019-15-4-308-314

26. Krivoshapko S.N. Hydrodynamic surfaces. *Shipbuilding.* 2021;(3):64–67. (In Russ.) https://doi.org/10.54068/00394580_2021_3_64

Кривошапко С.Н. Гидродинамические поверхности // Судостроение. 2021. № 3. С. 64–67. https://doi.org/10.54068/00394580_2021_3_64

27. Patil Y. Design, fabrication and analysis of Fibonacci spiral horizontal axis wind turbine. *International Journal of Aerospace and Mechanical Engineering.* 2018;5(1):1–4.

28. Grinko E.A. Surfaces of plane-parallel transfer of congruent curves. *Structural Mechanics and Analysis of Constructions*. 2021;(3):71–77. (In Russ.) https://doi.org/10.37538/0039-2383.2021.3.71.77

Гринько Е.А. Поверхности плоскопараллельного переноса конгруэнтных кривых // Строительная механика и расчет сооружений. 2021. № 3 (296). С. 71–77. https://doi.org/10.37538/0039-2383.2021.3.71.77

29. Krivoshapko S.N., Gil-Oulbe M. Geometry and strength of a shell of velaroidal type on annulus plan with two families of sinusoids. *International Journal of Soft Computing and Engineering.* 2013;3(3):71–73.

30. Toda M. Weierstrass-type representation of weakly regular pseudospherical surfaces in Euclidean space. *Balkan Journal of Geometry and Its Applications*. 2002;7(2):87–136.

31. Rubinstein B., Fel L. Stability of unduloidal and nodoidal menisci between two solid spheres. *Journal of Geometry and Symmetry in Physics*. 2015;39:77–98. https://doi.org/10.7546/jgsp-39-2015-77-98

32. Buhtyak M.S. Compound surface as pseudo-minimal one. *Tomsk State University Journal of Mathematics and Mechanics*. 2017;(46):5–13. (In Russ.) https://doi.org/10.17223/19988621/46/1

Бухтяк М.С. Составная поверхность, близкая к псевдоминимальной // Вестник Томского государственного университета. Математика и механика. 2017. № 46. С. 5–13. https://doi.org/10.17223/19988621/46/1

33. Abdel-All N.H., Hussien R.A., Youssef T. Hasimoto surfaces. *Life Science Journal.* 2012;9(3):556–560.

34. Mamieva I.A. Analytical surfaces for parametric architecture in contemporary buildings and structures. *Academia. Architecture and Construction.* 2020;(1):150–165. (In Russ.)

Мамиева И.А. Аналитические поверхности для параметрической архитектуры в современных зданиях и сооружениях // Academia. Архитектура и строительство. 2020. № 1. С. 150–165.

35. Korotich A.V. Innovative solutions of architectural shells: the alternative for traditional building. *Akademicheskij Vestnik UralNIIproekt RAASN*. 2015;(4):70–75. (In Russ.)

Коротич А.В. Инновационные решения архитектурных оболочек: альтернатива традиционному строительству // Академический вестник УралНИИпроект РААСН. 2015. № 4. С. 70–75.

36. Ermolenko E.V. Forms and constructions on the architecture of the soviet avant-garde and their interpretation in modern foreign practice. *Academia. Architecture and Construction.* 2020;(1):39–48. (In Russ.) https://doi.org/10.22337/2077-2020-1-39-48

Ермоленко Е.В. Формы и построения в архитектуре советского авангарда и их интерпретация в современной зарубежной практике // Academia. Архитектура и строительство. 2020. № 1. С. 39–48. https://doi.org/10.22337/2077-2020-1-39-48

37. Mozhdegani A.S., Afhani R. Using ecotech architecture as an effective tool for sustainability in construction industry. *Engineering, Technology & Applied Science Research*. 2017;7(5):1914–1917. https://doi.org/10.48084/etasr.1230

38. Bondarenko I.A. On the appropriateness and moderation of architectural innovation. *Academia. Architecture and Construction.* 2020;(1):13–18. (In Russ.) https://doi.org/10.22337/2077-2020-1-13-18

Бондаренко И.А. Об уместности и умеренности архитектурных новаций // Academia. Архитектура и строительство. 2020. № 1. С. 13–18. https://doi.org/10.22337/2077-2020-1-13-18