2022. 18(4). 375–386

СТРОИТЕЛЬНАЯ МЕХАНИКА ИНЖЕНЕРНЫХ КОНСТРУКЦИЙ И СООРУЖЕНИЙ

STRUCTURAL MECHANICS OF ENGINEERING CONSTRUCTIONS AND BUILDINGS

ISSN 1815-5235 (Print), 2587-8700 (Online)

HTTP://JOURNALS.RUDN.RU/STRUCTURAL-MECHANICS

DOI 10.22363/1815-5235-2022-18-4-375-386
UDC 69:624.074:624.012.4

RESEARCH ARTICLE / НАУЧНАЯ СТАТЬЯ

Comparative analysis of the stress state of an equal slope shell by analytical and numerical methods

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Article history
Received: June 29, 2022
Revised: August 20, 2022
Accepted: August 21, 2022

For citation
Aleshina O.O., Ivanov V.N., Cajamarca-Zuniga D. Comparative analysis of the stress state of an equal slope shell by analytical and numerical methods. *Structural Mechanics of Engineering Constructions and Buildings.* 2022;18(4):375–386. http://doi.org/10.22363/1815-5235-2022-18-4-375-386

Abstract. Works on the study of the stress-strain state of the shell of an equal slope with an ellipse at the base have not been widely performed. The present paper is a part of a series of articles on the analysis of the geometry and stress state of torses of an equal slope with a directrix ellipse by various methods under different loads and support conditions. The derivation of the differential equations of equilibrium of the momentless theory of shells for determining internal forces in the torse with a directrix ellipse under the action of internal pressure is presented. The analytical results are compared with results obtained by the finite element method (FEM) and the variational difference method (VDM). The advantages and disadvantages of three calculation methods are determined, and it is established that VDM results are more accurate compared to FEM, but FEM-based software is a more powerful tool to perform the structural analysis.

Keywords: thin shell theory, analytical method, momentless state, torse shell, surface of equal slope, finite element method, variational-difference method, SCAD Office, computing system, Mathcad system

Сравнительный анализ напряженного состояния оболочки одинакового ската аналитическим и численными методами

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История статьи
Поступила в редакцию: 29 июня 2022 г.
Доработана: 20 августа 2022 г.
Принята к публикации: 21 августа 2022 г.

Аннотация. Исследование напряженно-деформированного состояния оболочки одинакового ската с эллипсом в основании не получило широкого распространения. Настоящая работа является частью серии статей, посвященных анализу геометрии и напряженного состояния торсов одинакового

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ската с направляющим эллипсом различными методами при различных нагрузках и условиях опирания. Представлен вывод дифференциальных уравнений равновесия безмоментной теории оболочек для определения внутренних сил в торсе с направляющим эллипсом под действием внутреннего давления. Аналитические результаты сравниваются с результатами, полученными методом конечных элементов (МКЭ) и вариационно-разностным методом (ВРМ). Определены преимущества и недостатки трех методов расчета и установлено, что результаты ВРМ точнее по сравнению с МКЭ, но программное обеспечение на основе МКЭ является более мощным инструментом для выполнения расчета конструкции.

Ключевые слова: теория тонких оболочек, аналитическое решение, безмоментное состояние, торсовая оболочка, поверхность одинакового ската, метод конечных элементов, вариационно-разностный метод, вычислительный комплекс, SCAD Office, система Mathcad

Для цитирования
Aleshina O.O., Ivanov V.N., Cajamarca-Zuniga D. Comparative analysis of the stress state of an equal slope shell by analytical and numerical methods // Строительная механика инженерных конструкций и сооружений. 2022. Т. 18. № 4. С. 375–386. http://doi.org/10.22363/1815-5235-2022-18-4-375-386

Introduction

The present paper is one more of a series of research articles on the study of the geometry and stress-strain state of torses of equal slope with a directrix ellipse by various methods of analysis under different loads and support conditions. To date, the authors have reviewed, analyzed, and drawn conclusions on the tensional state of the torse under the action of a linear uniformly distributed load directed along the generatrix at the upper edge of the shell [1], uniformly distributed load on the middle surface along straight generatrixes [2], and the shell self-weight [3]. The works [1–3] study the problem with simple (movable) supports of the ellipse at the base. The article [4] considers a rigid (fixed) support under the action of self-weight of the torse. A new structure in the shape of a torse of an equal slope is proposed in [5], and new results in geometric studies are shown in [6; 7].

The development of modern technologies and innovative structural design and construction methods is impossible without scientifically based methods of analysis, and research of mathematical and experimental models [8–10]. Along with numerical methods, there are also analytical methods for structures analysis, which engineers use, due to their complexity, only for a narrow class of thin-walled structures and elements [11].

The finite element method (FEM) is a numerical method for calculating the stress-strain state (SSS) of various types of structures. Due to the variety of finite element types and the possibility of modifying their sizes and shapes, this method has undeniable advantages for the analysis of structures of complex shapes, with holes or with stress concentration zones. The paper [12] proposes a method of shell design using triangular finite elements to increase the accuracy of the solutions. The work [13] reports an algorithm developed for strength analysis of large span thin-walled structures in the geometric nonlinear formulation. However, the FEM in comparison with the variational-difference method (VDM) does not consider the external and internal geometry for the determination of the stress-strain state of thin-shell spatial structures of complex shapes with rapidly changing geometrical characteristics [14].

The variational-difference method [15–17], also known as finite-difference energy method (FDEM) [15; 18–20], also belongs to the numerical calculation methods [21]. The VDM allows to consider the geometric parameters of the middle surface of shells for a more accurate determination of the SSS of the thin-shell structures. The history of VDM development begins with Courant's proposal in 1943 [15; 22; 23]. Houbolt in 1958 [18; 24], Griffin and Varga in 1963 [24; 25], Bushnell in 1973, and Brush and Almroth in 1975 [26] continued the development of this method. In the early 2000s Professor V.N. Ivanov and his PhD students developed SHELLVRM, a computer software based on the VDM for determining the SSS of certain types of plates and shells with middle surfaces described by analytical equations [14; 21; 27].

In 2015, Krivoshapko and Ivanov published the encyclopedia [28], where described over 600 analytical surfaces. Among an extensive variety of analytical surfaces, the torse shells of equal slope have a distinctive characteristic of unfolding onto a plane without folds [27]. This class of surfaces is used in many areas of industry [29; 30].

Method

Torse shell of equal slope with an ellipse at the base

A straight line moving in the normal plane of a flat directrix curve with a constant angle of inclination to the normal plane of the directrix forms a ruled surface of equal slope. The torse surface of equal slope with an ellipse at the base (Figure 1) is formed when the ellipse is set as a flat directrix curve. The basic properties of the surfaces of an equal slope are described in [11; 27]. These surfaces are surfaces of zero Gaussian curvature ($K = 0$) and also belong to the Monge surfaces [27].

The directrix ellipse is defined by parametric equations [11]:

$$x = x(v) = a\cos v\,, y = y(v) = b\sin v\,. \tag{1}$$

The parameters a and b are the dimensions of the semi-axes of the directrix ellipse, and the parameter v is within $0 \le v \le 2\pi$.

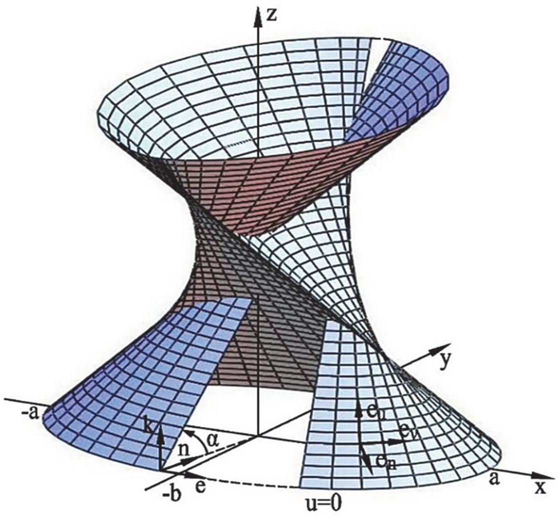

Figure 1. Torse shell of equal slope with an ellipse at the base

The parametric equations of the torse of equal slope with an ellipse at the base are [11]

$$x = x(u,v) = a\cos v - \frac{ub\cos\alpha\cos v}{\sqrt{a^2\sin^2 v + b^2\cos^2 v}}\,;$$

$$y = y(u,v) = b\sin v - \frac{ua\cos\alpha\sin v}{\sqrt{a^2\sin^2 v + b^2\cos^2 v}}\,;$$

$$z = z(u) = u\sin\alpha. \tag{2}$$

The coefficients of the basic quadratic forms of this surface and its main curvatures are [11]

$$A = 1; \qquad B = \mu^{1/2} - u\frac{\beta}{\mu}\,; \qquad F = 0; \qquad L = M = 0;$$

$$N = B\frac{ab\sin\alpha}{\mu}\,; \qquad k_1 = k_u = 0; \qquad k_2 = k_v = \frac{ab\sin\alpha}{\mu B}\,, \tag{3}$$

where $\mu = \mu(v) = a^2\sin^2 v + b^2\cos^2 v$; $\beta = ab\cos\alpha$.

In this research the momentless theory (MLT) of shell analysis, the variational-difference method and the finite element method are applied to study a thin torse of equal slope with a directrix ellipse under the action of a uniformly distributed load $q = 1$ kN/m^2 directed along the normal to the middle surface of the torse (internal pressure) (Figure 2). Consider the torse with the following geometric parameters $a = 3$ m, $b = 2$ m, $\alpha = 60°$ and $u = 2$ m. Boundary condition at the level $u = 0$ m is simple (movable) support and free edge is at the level $u = 2$ m.

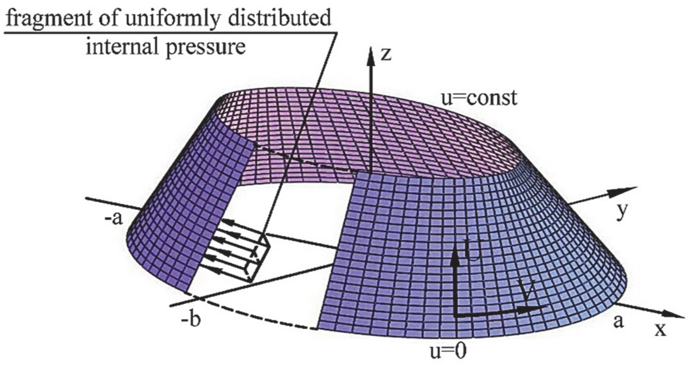

Figure 2. Torse under the action of internal distributed surface load

To determine the parameters of the stress state of the torse (Figure 2) the momentless theory of shell analysis, the SHELLVRM program based on the VDM and the SCAD Office software based on the FEM are used.

Differential equations of equilibrium of a momentless torse shell

To determine the normal and tangential forces under the action of a uniformly distributed load acting in the direction normal to the middle surface of the torse (Figure 2), we obtain differential equations of equilibrium of the momentless theory in orthogonal curvilinear curvature lines [11]:

$$\begin{gathered}\frac{\partial}{\partial u}(BN_u)-\frac{\partial B}{\partial u}N_v+\frac{1}{A}\frac{\partial}{\partial v}(A^2S)+ABX=0;\\ \frac{\partial}{\partial v}(AN_v)-\frac{\partial A}{\partial v}N_u+\frac{1}{B}\frac{\partial}{\partial u}(B^2S)+ABY=0;\\ \frac{N_v}{R_v}+\frac{N_u}{R_u}-Z=0.\end{gathered} \tag{4}$$

For this type of applied load on the studied torse of equal slope (Figure 2), we have $X = Y = 0$ and $Z = q$. The differential equations of equilibrium (4) are simplified as follows:

$$\begin{gathered}\frac{\partial}{\partial u}(BN_u)-\frac{\partial B}{\partial u}N_v+\frac{\partial S}{\partial v}=0;\\ \frac{\partial N_v}{\partial v}+\frac{1}{B}\frac{\partial}{\partial u}(B^2S)=0;\\ \frac{N_v}{R_v}-Z=0.\end{gathered} \tag{5}$$

The forces S, N_u are equal to zero, i.e. $S = 0$ and $N_u = 0$ at the level $u = 2$ m.

From the third differential equation of the system (5) we obtain an expression for the normal force N_v:

$$N_v=\frac{q}{\tan\alpha}\left(\frac{\mu^{\frac{3}{2}}}{\beta}-u\right). \tag{6}$$

From the second differential equation of system (5) we obtain the expression for the tangential force S:

$$S=\frac{1}{B^2}\left[-\int B(u,v)\frac{\partial N_v}{\partial v}\mathrm{d}u+X_1(v)\right]. \tag{7}$$

Here $X_1(v)$ is an arbitrary function of integration.

$$\frac{\partial N_v}{\partial v} = \frac{3q}{2\sin\alpha}\varphi\mu^{\frac{1}{2}}\sin 2v\,;\;\; \varphi = \frac{(a^2-b^2)}{ab}\;; \tag{8}$$

$$\int B(u,v)\mathrm{d}u = u\mu^{\frac{1}{2}} - \frac{u^2 B}{2\mu} = \frac{u}{2}\left(B(u,v) + \mu^{\frac{1}{2}}\right) = \frac{\mu}{2\beta}\left(\mu - B^2(u,v)\right). \tag{9}$$

The arbitrary function of integration $X_1(v)$ under the boundary condition at the upper edge $S = 0$ at $u = \eta = 2$ m must be equal to

$$X_1(v) = \frac{3q}{4\sin\alpha}\varphi\sin 2v\left(2\eta\mu - \eta^2\beta\mu^{-\frac{1}{2}}\right). \tag{10}$$

Equation (7) for obtaining values of the tangential forces *S* taking into account the value of the arbitrary integration function $X_1(v)$ takes the following form:

$$S = -\frac{1}{B^2(u,v)}\frac{3q}{4\sin\alpha}\varphi\sin 2v\left[\mu^{\frac{5}{2}}\beta^{-1} - 2\eta\mu + \eta^2\beta\mu^{-\frac{1}{2}} - B^2(u,v)\mu^{\frac{3}{2}}\beta^{-1}\right]. \tag{11}$$

From the first equation of the system (5) we obtain an expression for the normal force N_u:

$$N_u = \frac{1}{B(u,v)}\left[\int\left(\frac{\partial B(u,v)}{\partial u}N_v - \frac{\partial S}{\partial v}\right)\mathrm{d}u + X_2(v)\right]. \tag{12}$$

Here $X_2(v)$ is an arbitrary function of integration.

Performing the integration by parts of the terms of expression (12) we have

$$\int\frac{\partial B(u,v)}{\partial u}N_v\,du = -\frac{q}{\tan\alpha}\frac{\mu}{2\beta}\left(\mu - B^2(u,v)\right). \tag{13}$$

$$\int\frac{\partial S}{\partial v}\mathrm{d}u = \frac{3q\varphi}{4\beta\sin\alpha}\left[2\cos 2v\left(\frac{1}{B(u,v)}\xi_1(v) + u\mu^{\frac{3}{2}}\right) + \right.$$

$$\left. +\sin^2 2v\frac{(a^2-b^2)}{2}\left(\frac{1}{B^2(u,v)}\xi_2(v) + \frac{1}{B(u,v)}\xi_3(v) + 3u\mu^{\frac{1}{2}}\right)\right], \tag{14}$$

where

$$\xi_1(v) = -\left[\frac{\mu^{\frac{7}{2}}}{\beta} - 2\mu^2\eta + \mu^{\frac{1}{2}}\eta^2\beta\right];$$

$$\xi_2(v) = -\left[6\mu^{\frac{3}{2}}\eta - \frac{3\mu^3}{\beta} - 3\eta^2\beta\right];$$

$$\xi_3(v) = -\left[\frac{9\mu^{\frac{5}{2}}}{\beta} - 12\mu\eta + \frac{3\eta^2\beta}{\mu^{\frac{1}{2}}}\right]. \tag{15}$$

The arbitrary function of integration $X_2(v)$ under the boundary condition $N_u = 0$ at the upper edge $u = 2$ m must be equal to

$$X_2(v) = \frac{q}{\text{tg}\alpha}\frac{\mu}{2\beta}(\mu - B^2(\eta, v)) + \frac{3q\varphi}{4\beta\sin\alpha}\left[2\cos 2v\left(\frac{1}{B(\eta, v)}\xi_1(v) + \eta\mu^{\frac{3}{2}}\right) + \right.$$

$$\left. + \sin^2 2v\frac{(a^2 - b^2)}{2}\left(\frac{1}{B(\eta, v)^2}\xi_2(v) + \frac{1}{B(\eta, v)}\xi_3(v) + 3\eta\mu^{\frac{1}{2}}\right)\right]. \qquad (16)$$

Taking into account the value (16) of the arbitrary integration function $X_2(v)$, the equation (12) for calculating the values of normal forces N_u takes the form

$$N_u = \frac{1}{B(u, v)}\left\{\frac{q}{\text{tg}\alpha}\frac{\mu}{2\beta}\left(B^2(u, v) - B^2(\eta, v)\right) + \right.$$

$$+ \frac{3q\varphi}{4\beta\sin\alpha}\left[2\cos 2v\left(\xi_1(v)\left(\frac{1}{B(\eta, v)} - \frac{1}{B(u, v)}\right) + \mu^{\frac{3}{2}}(\eta - u)\right) + \right.$$

$$+ \sin^2 2v\frac{(a^2 - b^2)}{2}\left(\xi_2(v)\left(\frac{1}{B^2(\eta, v)} - \frac{1}{B^2(u, v)}\right) + \right.$$

$$\left.\left.\left. + \xi_3(v)\left(\frac{1}{B(\eta, v)} - \frac{1}{B(u, v)}\right) + 3\mu^{\frac{1}{2}}(\eta - u)\right)\right]\right\}. \qquad (17)$$

To find results of forces N_v (6), S (11) and N_u (17), we use the software Mathcad.

Stress state investigation of the torse by the FEM and the VDM

For the first analysis, the SCAD Office computing complex is used. Figure 2 shows the computational model of the torse with approximation of the middle surface by quadrangular plane elements with maximum side size of 0.228 m.

For the second analysis, the program SHELLVRM is used. This program also allows us to implement the conditions for the momentless state of the shell and the grid is similar to the grid in SCAD.

Results and discussion

The results of the analytical analysis (MLT) are compared with the results of two numerical analysis (by FEM and VDM) for 11 cross-sections (Figure 3).

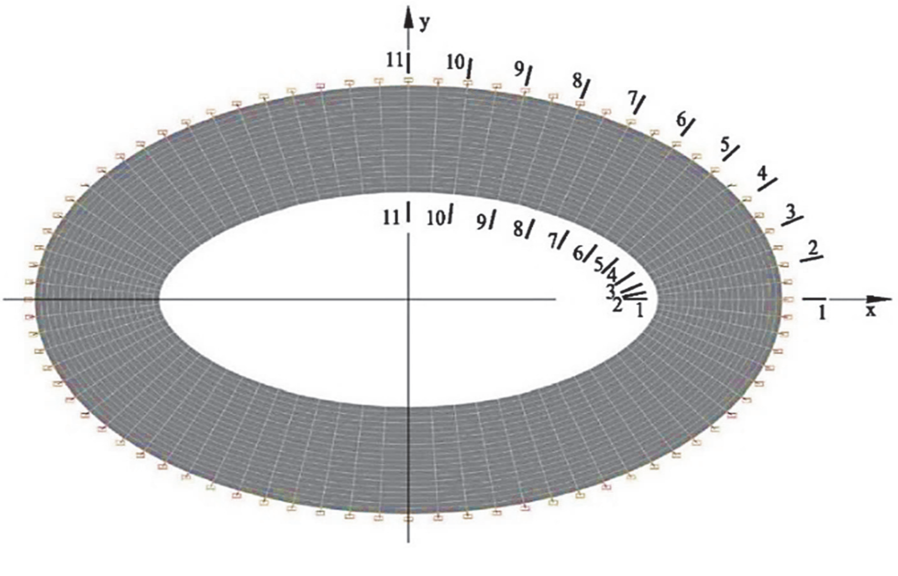

Figure 3. Cross-sections of the torse to compare the results

The maximum deviations of the results of normal force N_v by MLT from the results by FEM and VDM in section 1-1 are 26.7% (Figure 4), in section 2-2 – 21.7%, in section 3-3 – 28.3%, in section 4-4 – 34.7%, in section 5-5 – 22.4%, in section 6-6 – 9.7% (Figure 5), in section 7-7 – 2.6%, in section 8-8 – 2.8%, in section 9-9 – 4.1%, in section 10-10 – 4.5% and in section 11-11 – 4.7%. It should be noted that the maximum deviations are in the nodes with coordinates u = 0 m, u = 1.8 m and u = 2 m, in other nodes the deviations do not exceed 5.7%.

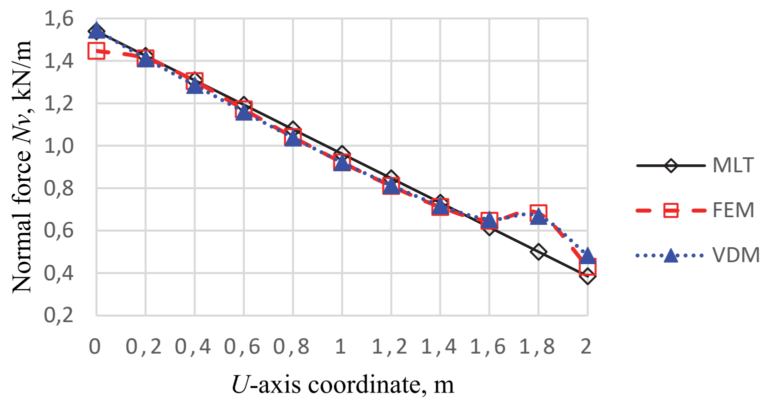

Figure 4. Comparison of numerical results for normal force N_v in section 1-1

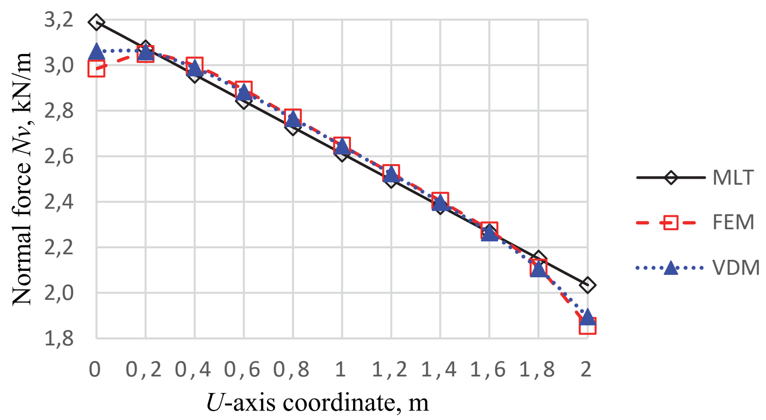

Figure 5. Comparison of numerical results for normal force N_v in section 6-6

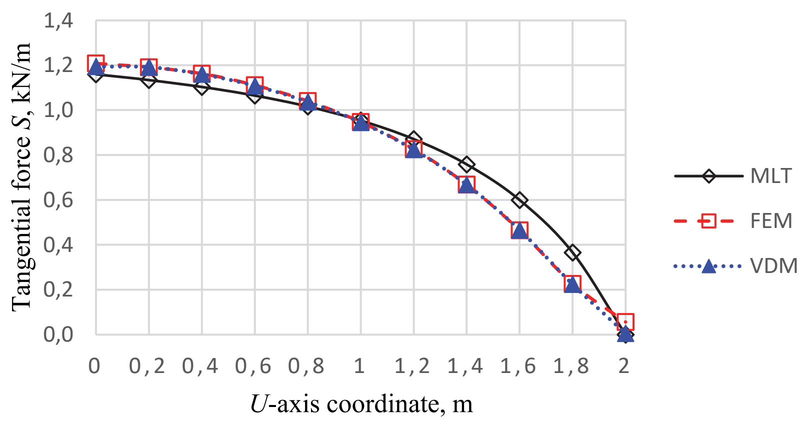

Figure 6. Comparison of numerical results for tangential force S in section 3-3

The values of the tangential force S in sections 1-1 and 11-11 are equal to zero by all calculation methods. The maximum deviations of the results of the tangential force S by MLT from the results by FEM and VDM in the nodes with coordinates u = 1.4 m, u = 1.6 m and u = 1.8 m are 112.4% in section 2-2, 62.9% – in section 3-3 (Figure 6), 23.4% – in section 4-4. In the remaining nodes of sections 2-2, 3-3 and 4-4, the deviations do not exceed 9.0%. The maximum deviations in section 5-5 are 3.2%, 7.4% – in section 6-6, 9.1% – in section 7-7, 7.7% – in section 8-8, 5.6% – in section 9-9 (Figure 7) and in section 10-10 the maximum deviations are 4.1%.

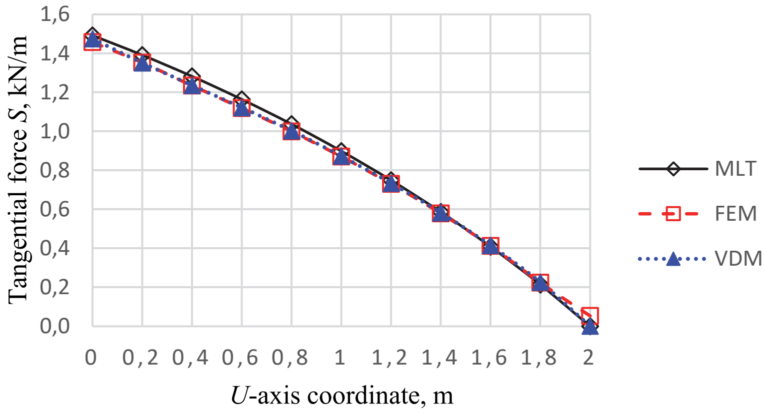

Figure 7. Comparison of numerical results for tangential force S in section 9-9

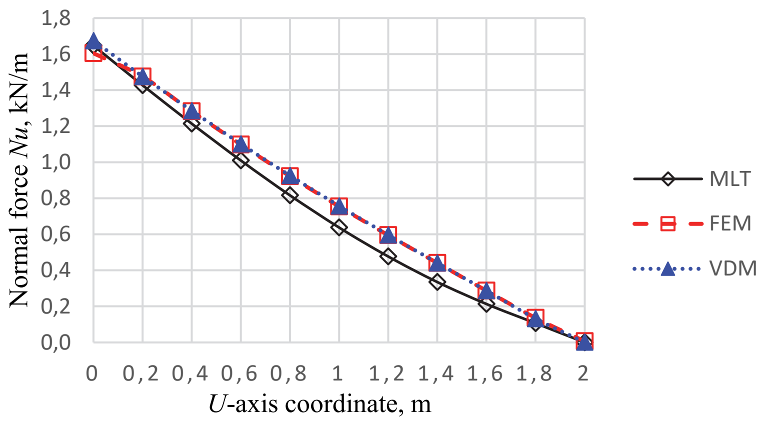

Figure 8. Comparison of numerical results for normal force N_u in section 5-5

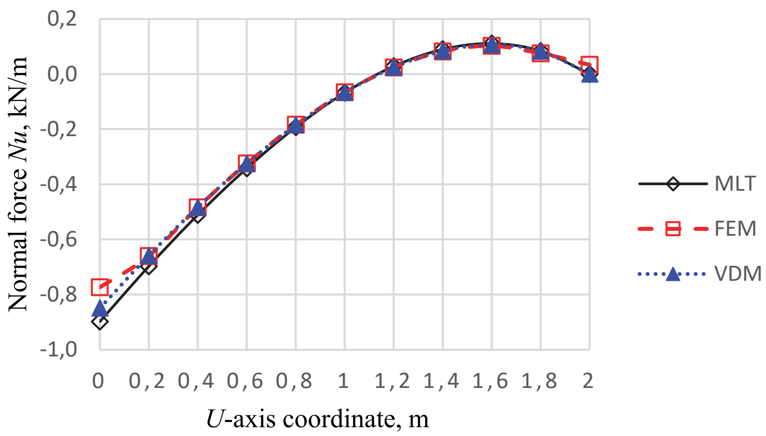

Figure 9. Comparison of numerical results for normal force N_u in section 11-11

The maximum deviations of the results of the normal force N_u by MLT from the results by FEM and VDM are 59.0% in section 1-1, 27.1% – in section 2-2, 23.1% – in section 3-3, 32.7% – in section 4-4, 25.8% – in section 5-5 (Figure 8), 10.8% – in section 6-6, 11.1% – in section 7-7, 37.6% – in section 8-8, 33.1% – in section 9-9, 27.1% – in section 10-10, and 18.6% – in section 11-11 (Figure 9). The concentration of the greatest deviations is in the upper zone of the shell in sections 1-1 – 5-5 at the nodes with coordinates u = 1.40 m, u = 1.60 m and u = 1.80 m. Also, in sections 8-8 – 11-11, the maximum deviations of the results appear in the regions of the transition from the stretched into the compressed zone of the shell.

The general stress state of the torse shell under the action of internal pressure is shown in Figures 10–12. All the contour graphs are obtained in the SCAD Office software.

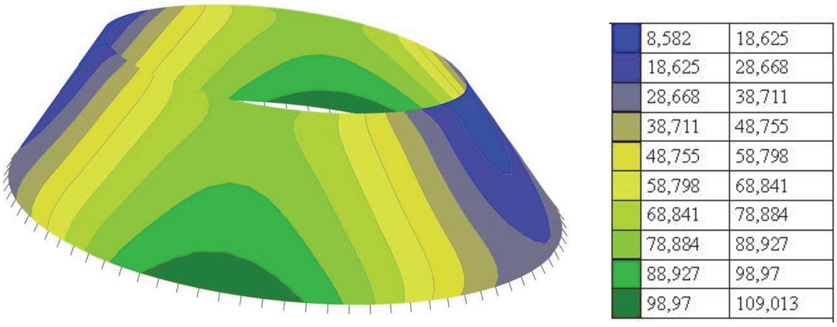

Figure 10. Normal stress σ(N_v) by FEM, kN/m²

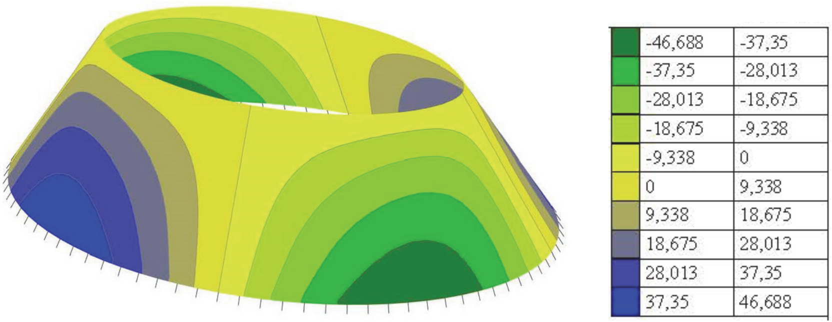

Figure 11. Tangential stress τ(S) by FEM, kN/m²

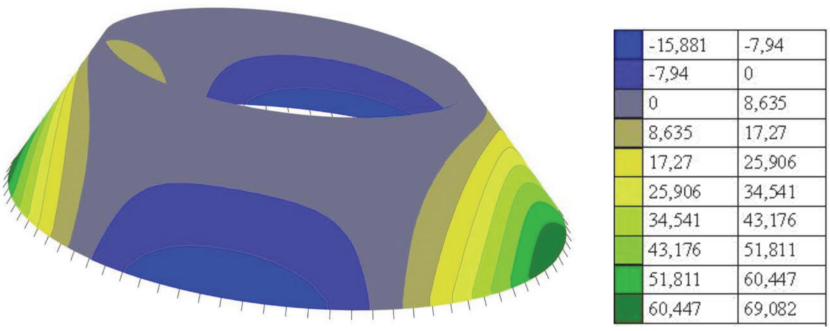

Figure 12. Normal stress σ(N_u) by FEM, kN/m²

Comparison of the obtained forces by different analysis methods shows good convergence. The largest deviations of the values of the normal forces N_v, N_u and tangential force S are localized at the nodes with coordinates u = 0.00 m, u = 1.80 m and u = 2.00 m. Deviations of the results at the nodes with coordinates u = 0 m can be explained by the fact that in the momentless theory only the boundary condition at the upper edge of the torse (u = 2 m) is considered.

Analytical results of the forces N_u and S under the action of uniform internal pressure at the nodes of all sections with coordinate u = 2.00 m should be $N_u = 0$ and $S = 0$, which is confirmed by the rules of strength of materials. However, the values of forces N_u and S of FEM and VDM are different from zero, at the same time, the results of VDM are more accurate as compared to FEM.

The greatest deviations of the values of normal forces N_v along curvilinear directrixes and N_u along straight generatrixes are concentrated in the upper free edge of the torse in sections from 1-1 to 6-6. Similar results have been obtained in [2–4]. When analyzing the geometry of the middle surface of the torse, we see that this region of the shell has the greatest change in the radius of curvature along the curvilinear directrixes. As is known, a smooth change in the geometry of the middle surface is one of the conditions for the application of the momentless theory. The momentless theory allows us to take into account only normal and tangential forces, and transverse forces and moments also affect the overall picture of the stress-strain state. Moreover, the correct choice of the finite element size (mesh) affects the accuracy of analysis using FEM and VDM [15]. It is noted that comparison of the results of VDM and FEM analysis for identical meshes shows similar accuracy, and in some cases VDM gives even more accurate results [14]. Thus, the influence of middle surface geometry and choice of finite element dimensions (mesh) are topics for further investigation of the stress-strain state of a class of torse shells of equal slope.

Conclusion

The application of the analytical method to solve the problem of determining the internal forces in the torse under the action of internal pressure turned out to be a labor-intensive task. Comparison of the results of the analytical method with two numerical methods (FEM and VDM) shows good convergence, indicating that the derived differential equations of equilibrium and expressions for determining the numerical values of forces N_u, N_v and S are correct. The SHELLVRM and SCAD programs simplify this task. However, the SHELLVRM program is not distributed, and it is difficult to implement the momentless condition of shell in SCAD. In this paper, when choosing a method for solving the problem, the preference is the SCAD program, which is most universal for solving a research problem.

The values of normal forces N_u and N_v indicate that this torse shell of equal slope with ellipse at the base works mainly in stretching. Taking into account the results of the stress state of torse and the properties of this class of surfaces to be unfolded on the plane without folds and breaks, it can help distribute this class of shells for the design of various buildings and structures among architects and designers.

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