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ANALYTICAL AND NUMERICAL METHODS OF ANALYSIS OF STRUCTURES

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Experimental and numerical investigation of thin-walled I-section beam under bending and torsion

Tesfaldet H. Gebre[1], Vera V. Galishnikova[2], Evgeny V. Lebed[2]

[1]*Peoples' Friendship University of Russia (RUDN University), Moscow, Russian Federation*
[2]*Moscow State University of Civil Engineering (National Research University), Moscow, Russian Federation*
tesfaldethg@gmail.com

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Abstract. The aim of the research – to investigate the behavior of thin-walled beam I-section loaded with bending and torsion using theoretical, numerical, and experimental approaches. In this paper, the main criteria for consideration of the different methods of analysis is the geometric characteristic of the section. The results obtained by the finite element method, the numerical method, as well as experimental data are compared. The analysis by finite element method by considering an additional degree of freedom at a node to include the restrained torsion and the dimension of the stiffness matrix is thus 14×14. The results of the calculation according to this theory are compared with the numerical solution obtained using finite element software, and with the results of the experiment. The I-beam section subject to bending with torsion is considered. The deformations, strain, and stress distributions of open thin-walled structures subjected to bending and torsion are presented using experimental methods. The comparative results for the angle of twisting, deformations, and normal stresses in the frame element subjected to combined loading are displayed graphically. To evaluate the results, a theoretical, numerical, and experimental investigation of I-beam behavior under bending and restrained torsion was carried out. As a result of the comparison, it was revealed that the results obtained according to the refined theory proposed by the authors have good convergence with experimental data and are also quite close to the values obtained using commercial software.

Keywords: experimental study, thin-walled sections, finite element method, combined loading, torsion, bending, warping torsion

Tesfaldet H. Gebre, research assistant, Department of Civil Engineering, Academy of Engineering, Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation; ORCID: 0000-0002-7168-5786, Scopus Author ID: 57212348775, eLIBRARY SPIN-code: 2587-8700; tesfaldethg@gmail.com
Vera V. Galishnikova, Dr.Sc., Professor, Vice-Rector, Moscow State University of Civil Engineering (National Research University), Professor, Department of Civil Engineering, Peoples' Friendship University of Russia (RUDN University), 26 Yaroslavskoye Shosse, Moscow, 129337, Russian Federation; ORCID: 0000-0003-2493-7255, Scopus Author ID: 55303553900, eLIBRARY SPIN-code: 2765-7069; galishni@yandex.ru
Evgeny V. Lebed, Candidate of Technical Science, Associate Professor of the Department of Metal and Wooden Structures, Moscow State University of Civil Engineering (National Research University), 26 Yaroslavskoye Shosse, Moscow, 129337, Russian Federation; ORCID: 0000-0003-3926-8701, Scopus Author ID: 57197822677, eLIBRARY SPIN-code: 5297-2700; evglebed@mail.ru

Экспериментальное и численное исследование тонкостенных балок двутаврового сечения при изгибе и кручении

Т.Х. Гебре[1], **В.В. Галишникова**[2], **Е.В. Лебедь**[2]

[1]*Российский университет дружбы народов, Москва, Российская Федерация*
[2]*Национальный исследовательский Московский государственный строительный университет, Москва, Российская Федерация*
tesfaldethg@gmail.com

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Аннотация. Цель работы – исследовать поведение тонкостенной балки I сечения, нагруженной изгибом и кручением, используя теоретические, численные и экспериментальные подходы. В данной работе основным критерием для рассмотрения различных методов анализа является геометрическая характеристика сечения. Сравниваются результаты, полученные методом конечных элементов, численным методом, а также экспериментальные данные. При анализе методом конечных элементов учитывается дополнительная степень свободы в узле для включения повторно деформированного кручения, таким образом, размерность матрицы жесткости составляет 14×14. Результаты расчета по данной теории сравниваются с численным решением, полученным с помощью программы конечных элементов, и с результатами эксперимента. Рассматривается двутавровое сечение балки, подверженной изгибу с кручением. Представлены деформации, напряжения и распределения напряжений открытых тонкостенных конструкций, подверженных изгибу и кручению, с использованием экспериментальных методов. Сравнительные результаты для угла закручивания, деформаций и нормальных напряжений в элементе рамы, подвергнутом комбинированному нагружению, отображены графически. Для оценки полученных результатов проведено теоретико-калькуляционное, численное и экспериментальное исследование поведения двутавровой балки при изгибе и ограниченном кручении. Выявлено, что результаты, полученные в соответствии с предложенной авторами уточненной теорией, имеют хорошую сходимость с экспериментальными данными и достаточно близки к значениям, полученным с помощью коммерческого программного обеспечения.

Ключевые слова: экспериментальное исследование, тонкостенные профили, метод конечных элементов, комбинированная нагрузка, кручение, изгиб, деформирующее кручение

Introduction

Thin-walled steel beams used in various engineering structures can be assembled to withstand a combined loading situation of bending and torsion. They are well-known for their high strength, high flexibility, ductility, quick construction, and effective space partitioning. Various challenges in the use of thin-walled structures emerge on a routine basis, and numerous studies are called upon to provide proper solution. Steel beams with thin-walled sections are one of the suggested possible solutions in such scenarios because they are commonly used in various fields of engineering. When a thin-walled section is subjected to a combined load, they are ineffective at resisting, and it leads to reduce the capacity of the beam. Consequently, proper experimental and numerical bearing strength analysis is required to assess it, especially for open sections such as I-profiles. Bending results in strain and shear force in common construction practice, but in thin walled sections, normal stresses

Гебре Тесфалдет Хадгембес, ассистент, департамент строительства, Инженерная академия, Российский университет дружбы народов, Российская Федерация, 117198, Москва, ул. Миклухо-Маклая, д. 6; ORCID: 0000-0002-7168-5786, Scopus Author ID: 57212348775, eLIBRARY SPIN-код: 2587-8700; tesfaldethg@gmail.com

Галишникова Вера Владимировна, доктор технических наук, профессор, проректор, Национальный исследовательский Московский государственный строительный университет, профессор департамента строительства, Российский университет дружбы народов, Российская Федерация, 129337, Москва, Ярославское шоссе, д. 26; ORCID: 0000-0003-2493-7255, Scopus Author ID: 55303553900, eLIBRARY SPIN-код: 2765-7069; galishni@yandex.ru

Лебедь Евгений Васильевич, кандидат технических наук, доцент, кафедра металлических и деревянных конструкций, Национальный исследовательский Московский государственный строительный университет, Российская Федерация, 129337, Москва, Ярославское шоссе, д. 26; ORCID: 0000-0003-3926-8701, Scopus Author ID: 57197822677, eLIBRARY SPIN-код: 5297-2700; evglebed@mail.ru

determine the bearing capacity of the beam because they are the sum of longitudinal force acting, bending moments of two planes, and bimoment [1; 2]. Considering the analysis of beams loaded for bending and torsion by considering its plastic deformations allows for a reduction in steel usage when compared the analysis with the elastic range [3]. Considering the plastic deformations in the analysis allows increasing the bearing capacity of profile [4; 5]. The significance of elastic behavior is commonly used in experimental studies to assess the behavior of thin-walled structures subjected with the combined loading.

There are different studies which accounts only the elastic behavior of the steel element [6–10]. In summary, basic structural analysis relies on the well-known Euler – Bernoulli and Vlasov beam theories, which predict displacements and stresses in the frame element [11]. Occasionally, those theories overlook the shear deformation effect for long span members [12]. Numerical and experimental investigations of thin-walled structures subjected to bending and torsion revealed that it is dependent on section properties and that non-uniform torsion is influenced by moments [13]. Various studies use section properties as a primary criterion for analyzing finite element methods of thin-walled sections with restrained torsion [14–19]. Typically, section properties were used as a benchmark for predicting the behavior of open and closed thin-walled sections in those studies [20]. Various commercial programs usually consider six degrees of freedom at each node of a member element of a 3D frame, neglecting warping effects, as a common approach [12; 21–24]. When the finite element analysis for torsion only considers Saint-Venant torsion, the analysis may overlook the torsion in the members, tends to result in an unconservative design. Numerous scholars attempted to overcome this inconsistency by developing a 14×14 stiffness matrix that does include warping due to torsion as an additional degree of freedom at each node of a thin-walled section [25–30]. In addition, there are new findings in finite element methods analysis using quadratic and linear approximations methods to develop the stiffness matrixes [31].

A theoretical, numerical, and experimental investigation of I-beam behavior under bending and restrained torsion should be carried out in this study. As various authors have studied, it is acknowledged that the load carrying capacity of an I-section tends to increase as the action of combined loading increases. This study provides an analysis of the theoretical, experimental, and numerical calculations that use the section property as the primary criterion for implementing the theoretical method. The angle of twisting, normal stresses value from bending, and warping torsion are compared using theoretical, experimental, and numerical calculations. Based on the study's findings, the parameters of test specimens and the productive solution of beam angle of twisting, normal stresses for I-sections were calculated.

Method

To incorporate the warping behavior into the study, a bar element of length L with an I-cross section beam is considered. At each node of the element, a seventh degree of freedom is added to the well-known six DOFs of the classical three-dimensional frame element to account for warping torsion. In the local coordinate system, the nodal displacement and load vector are shown below:

$$\mathbf{v}_e = \begin{bmatrix} v_{1A} & v_{2A} & v_{3A} & \beta_{1A} & \beta_{2A} & \beta_{3A} & \beta_{1,1A} & v_{1B} & v_{2B} & v_{3B} & \beta_{1B} & \beta_{2B} & \beta_{3B} & \beta_{1,1A} \end{bmatrix}^T$$

$$\mathbf{q}_m = \begin{bmatrix} n_{1A} & n_{2A} & n_{3A} & m_{1A} & m_{2A} & m_{3A} & m_{\omega A} & n_{1B} & n_{2B} & n_{3B} & m_{1B} & m_{2B} & m_{3B} & m_{\omega B} \end{bmatrix}^T$$

The stiffness matrix as it is known, the relationship between the generalized force vector q_m and the generalized displacement vector v_m is established by the stiffness matrix K_m of the element.

$$\mathbf{q}_m = \mathbf{K}_m \ \mathbf{v}_m. \tag{1}$$

The design diagram of the structure under test, which includes a cantilever thin-walled section beam under bending and torsion as shown Figure 1. The beam is subjected to a load acting in the vertical plane with eccentricity relative to the longitudinal axis, causing the beam to experience torsional and bending moments at the same time. To validate the theoretical calculation of the behavior of a thin-walled section with restrained torsion of an I-beam section with a length of 500 mm, a height of 100 mm, a flange thickness of 5.7 mm, a flange width of 55 mm, and a web thickness of 4.5 mm as shown in Figure 1. A cantilever I-profile is subjected to a combined loading which are torsional moment M_T or a point load F with lever arm e is applied at the free end as shown in Figure 1. The material properties of the prismatic cantilever steel beam are as follows: Young modulus E = 200 GPa, Poisson ration ν = 0.3, and shear modulus $G = E/(2(1+\nu))$ = 76.92 GPa. The maximum applied concentrated force is applied gradually, and its magnitude is 91.893 kg including its self-weight.

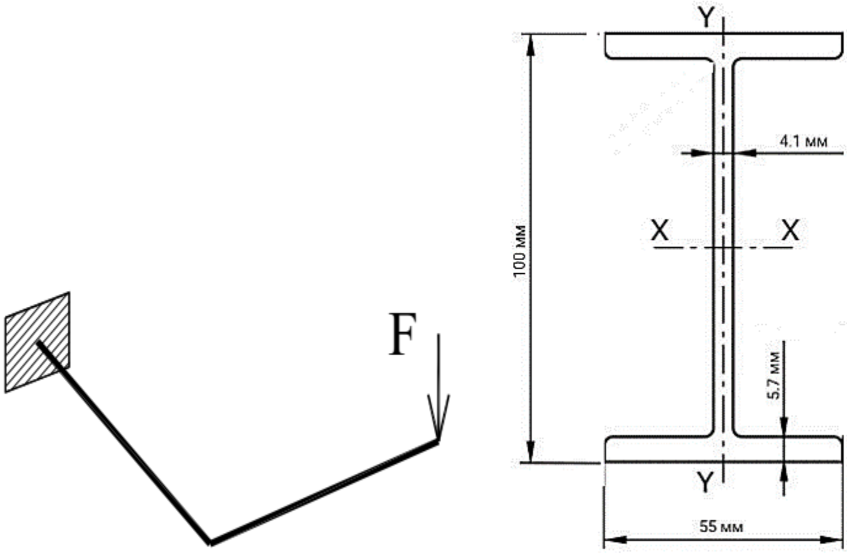

Figure 1. Geometry, boundary conditions, loading and cross-section of the beam

Experimental test set-up and instrumentation: the strain is measured using eight LVDT sensors, four of which are placed at the fixed end and four at the free end of the specimen as shown in Figure 2. At the top of the section, two LVDTs are used, and similarly, at the bottom of the section, two LVDTs are installed in both the fixed and endplates. Eight LVDTs sensers are used, of which four LVDTs are placed at fixed end and four of them are placed at the free end of the specimen used to measure the strain. Two LVDTs are used at the top of the section and similarly two LVDTs are placed at the bottom of the section in both the fixed and endplates.

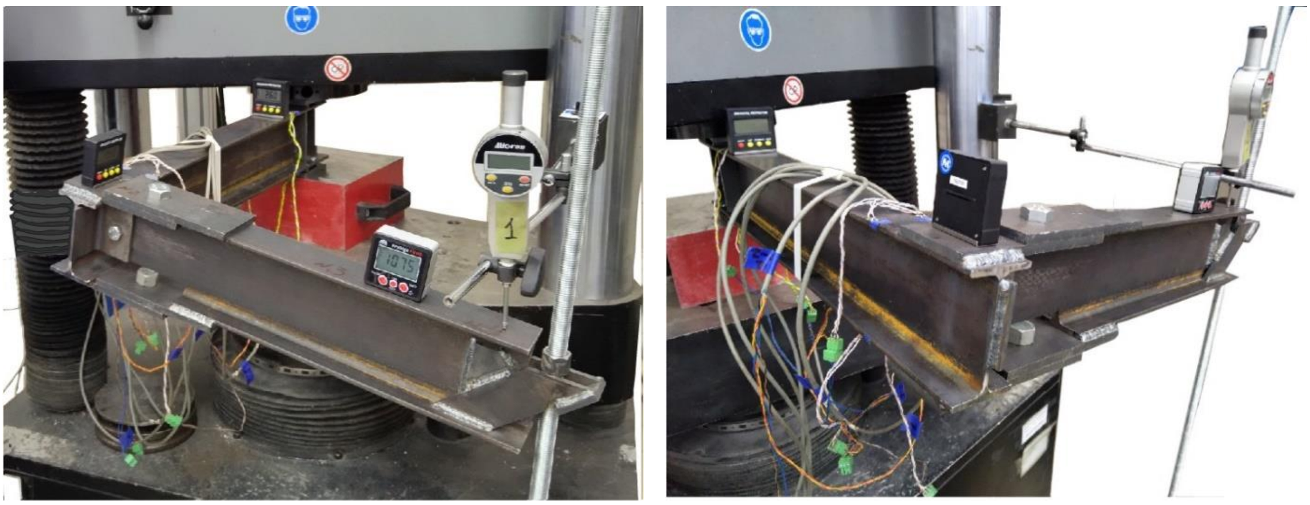

Figure 2. The specimen alignment, with measuring systems, and combined loading test stand
(photos by Tesfaldet H. Gebre)

The specimen is adjusted for creating warping restraints at one end and free at the other end as shown in Figure 2. The specimen is carefully placed on the test setup on the fixing plate so that the required loading can be applied. The test setup and instrumentation adopted is shown in Figure 2. Three angular rotation measurement devices are used to measure the angle of twisting in three different axes as shown in Figure 2.

Result and discussion

The member variables are collected in member displacement vector V_m and member load vector q_m and the matrices are arranged correspondingly in member stiffness matrix k_m.

$\mathbf{K}_m =$

k_1							k_2						
	k_3				k_4			k_6				k_4	
		k_9		k_{10}					k_{12}		k_{10}		
			k_{T1}			k_{T2}				k_{T3}			k_{T4}
		k_{10}		k_{11}					k_{13}		k_{14}		
	k_4				k_5			k_7				k_8	
			k_{T5}			k_{T6}				k_{T7}			k_{T8}
k_2							k_1						
	k_6				k_7			k_3				k_7	
		k_{12}		k_{13}					k_9		k_{13}		
			k_{T9}			k_{T10}				k_{T11}			k_{T12}
		k_{10}		k_{14}					k_{13}		k_{11}		
	k_4				k_5		k_7					k_8	
			k_{T13}			k_{T14}				k_{T15}			k_{T16}

The nonzero elements of the stiffness matrix for the 3D finite element calculation of beam element with restraint torsion are given below:

$$\mathbf{K}_1 = \frac{EA}{a}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} k_1 & k_2 \\ k_2 & k_1 \end{bmatrix} \qquad (2)$$

$$\mathbf{K}_2 = \frac{EJ_2}{a^3}\begin{bmatrix} 12 & 6a & -12 & 6a \\ 6a & 4a^2 & -6a & 2a^2 \\ -12 & -6a & 12 & -6a \\ 6a & 2a^2 & -6a & 4a^2 \end{bmatrix} = \begin{bmatrix} k_3 & k_4 & k_6 & k_4 \\ k_4 & k_5 & k_7 & k_8 \\ k_6 & k_7 & k_3 & k_7 \\ k_4 & k_8 & k_7 & k_5 \end{bmatrix} \qquad (3)$$

$$\mathbf{K}_3 = \frac{EJ_3}{a^3}\begin{bmatrix} 12 & -6a & -12 & -6a \\ -6a & 4a^2 & 6a & 2a^2 \\ -12 & 6a & 12 & 6a \\ -6a & 2a^2 & 6a & 4a^2 \end{bmatrix} = \begin{bmatrix} k_9 & k_{10} & k_{12} & k_{10} \\ k_{10} & k_{11} & k_{13} & k_{14} \\ k_{12} & k_{13} & k_9 & k_{13} \\ k_{10} & k_{14} & k_{13} & k_{11} \end{bmatrix} \qquad (4)$$

$$\mathbf{K}_T = \frac{EC_\omega}{a^3}\begin{bmatrix} k_{T1} & k_{T2} & k_{T3} & k_{T4} \\ k_{T2} & k_{T6} & k_{T7} & k_{T8} \\ k_{T3} & k_{T7} & k_{T11} & k_{T12} \\ k_{T4} & k_{T8} & k_{T12} & k_{T16} \end{bmatrix} \qquad (5)$$

$$K_{T1} = K_{T11} = S*\theta\sinh\theta,\quad K_{T6} = K_{T16} = S(\cosh\theta - \frac{\sinh\theta}{\theta})a^2;$$

$$K_{T2} = K_{T4} = S(\cosh\theta - 1)a,\quad K_{T8} = S\left(\frac{\sinh\theta}{\theta} - 1\right)a^2;$$

$$S = \left(\frac{\theta^2}{Q}\right),\quad Q = 2\left(1 - \cosh\theta\right) + \theta\sinh\theta,\qquad K_{T3} = -K_{T1},\quad K_{T7} = K_{T12} = -K_{T2}.$$

Experiments were used to obtain the deformation, angle of twisting, strain data, and stress of the corresponding pointes. The distributions of the experiment's results based on the applied loading conditions are given below. The graphs are the comparison between applied loading, angle of twisting and load application point displacement for the three experimental trials as shown in Figures 3–5.

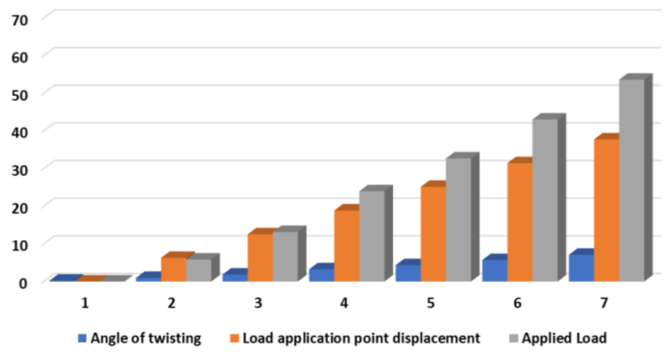

Figure 3. The distribution of the first experiment's results of applied loading, angle of twisting and load application point displacement

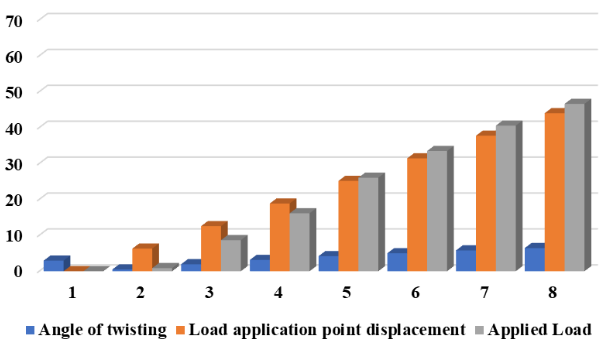

Figure 4. The distribution of the second experiment's results of applied loading, angle of twisting and load application point displacement

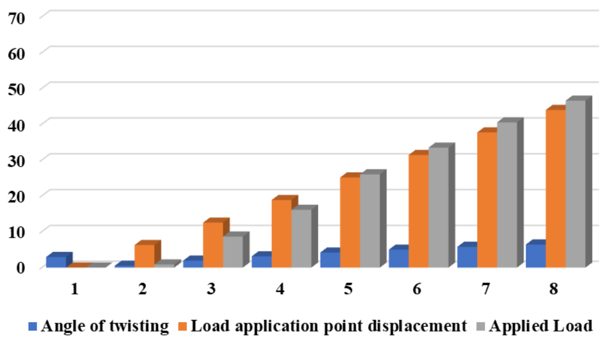

Figure 5. The distribution of the third experiment's results of applied loading, angle of twisting and load application point displacement

During the loading process, the angle of twisting, strain and deflection of beam end-span section were measured. Graphs of strains and stress vs the lever arm deformation which is measured experimentally as shown in Figure 6.

As a result of the three tests, the twist angles of the beams were measured and graphically expressed as twist angles vs load, as shown in Figure 7.

Furthermore, using the three-dimensional finite element (FE) model of the thin-walled bar with an open cross-section developed in the ANSYS environment, we compared the experimental data with both theoretical calculations and numerical simulations, as shown in Figure 8. ANSYS finite element software was also used to model the beam. Ansys simulations with beam 189 elements were performed, with an additional degree of freedom considered. A computer program for analysis is created based on the proposed finite element analysis. An illustration of the accuracy of the presented method is demonstrated by comparing using experiment and finite element software. The comparison of various results is presented graphically. A concentrated torque and point load are applied to the shear centre of the wall at the free end.

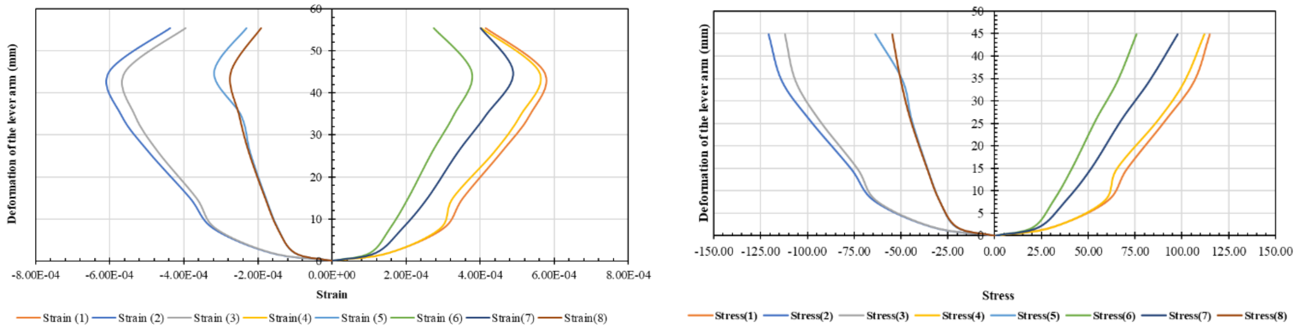

Figure 6. The strains and stress, MPa vs the lever arm deformation measured experimentally

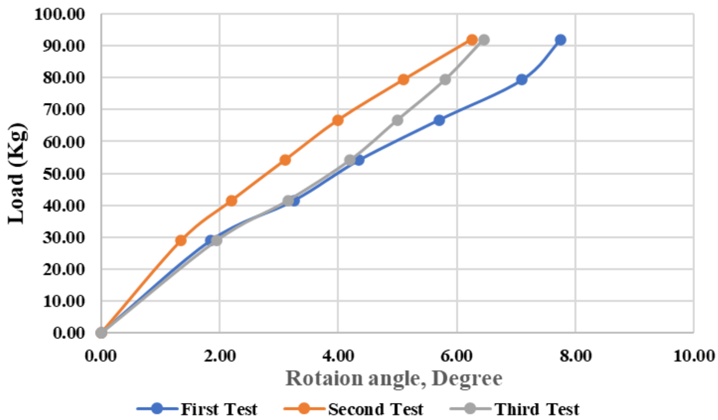

Figure 7. Applied load vs twisting angle relationships for the three tests

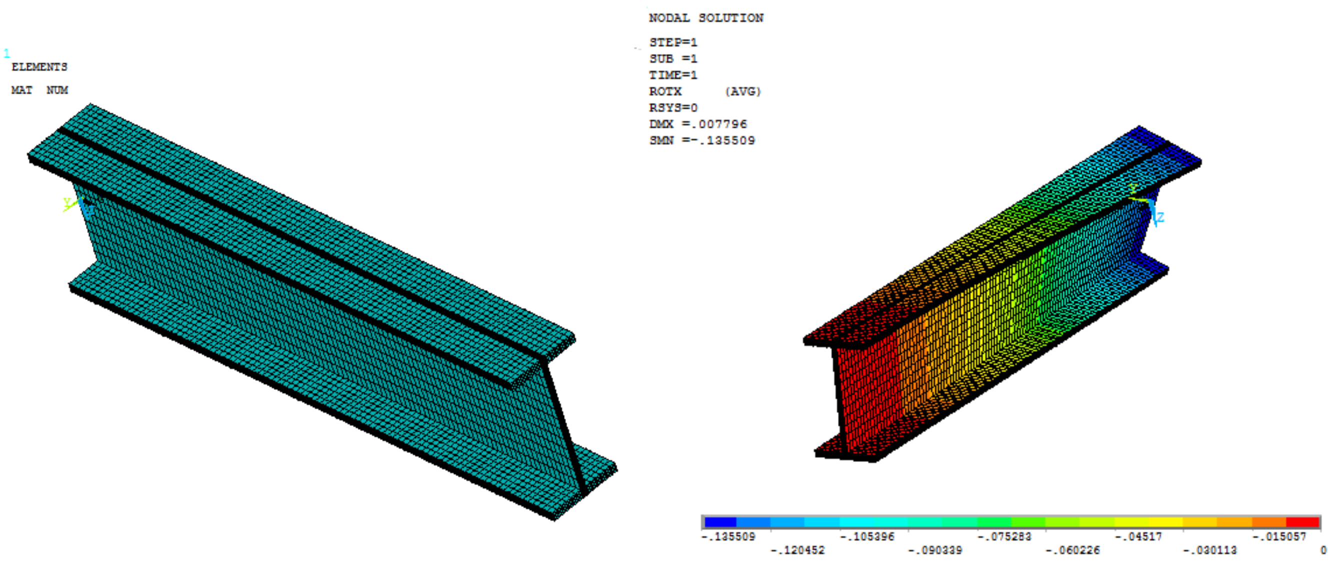

Figure 8. Open thin-walled section beam model and angle of twisting

Figure 9 depicts the percentage differences of the experimental tests in comparison to the results of the current theory and numerical results. The load-angle dependencies and their correlations for the theoretical, experimental, and numerical results are presented graphically as shown in Figure 9.

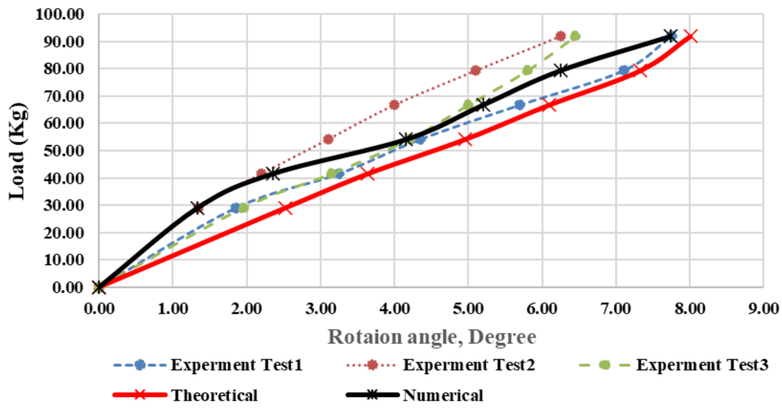

Figure 9. Load vs. angle of twisting of the three experimental tests, current theory, and numerical results

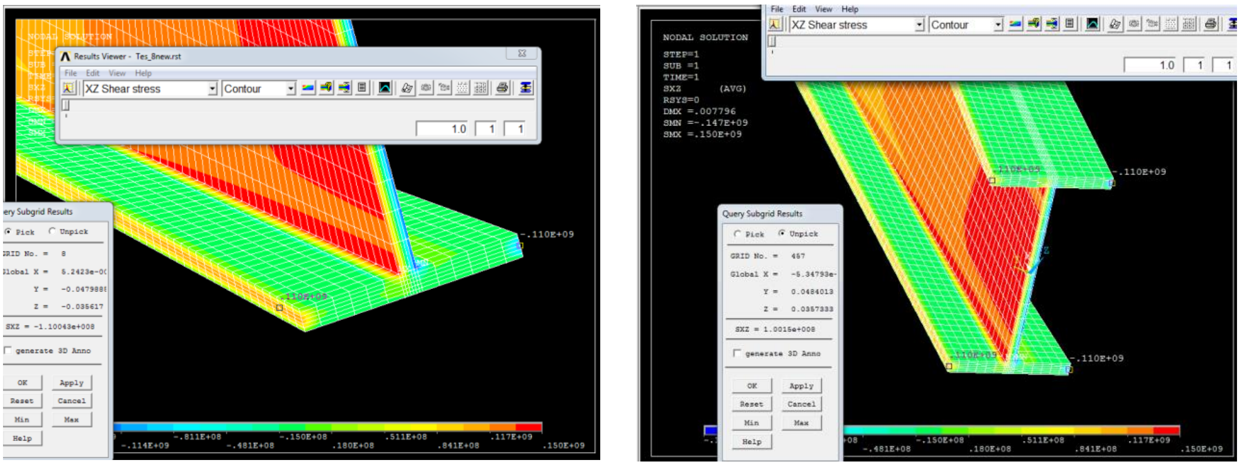

Figure 10. The section's normal stress distribution values using Ansys

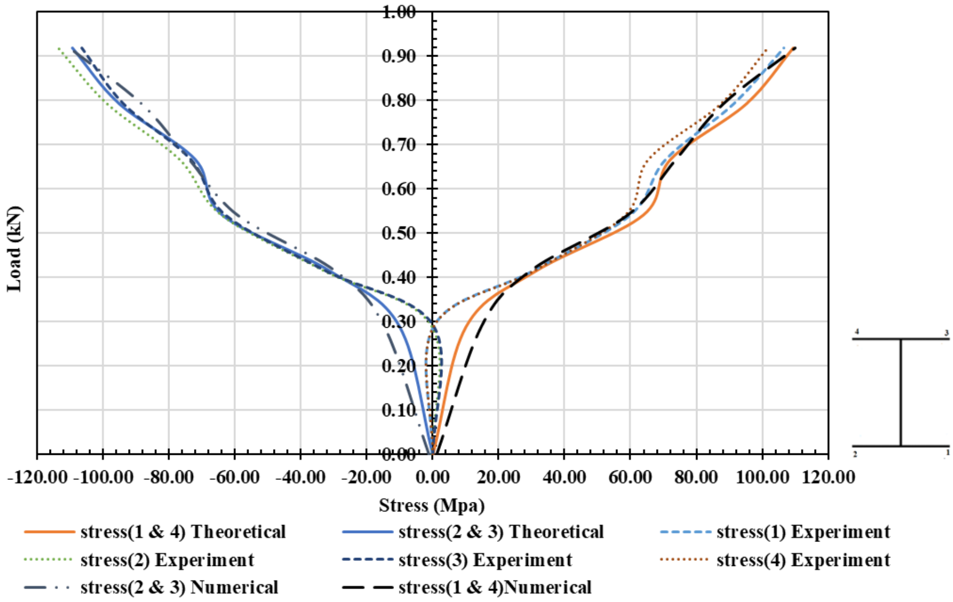

Figure 11. The stresses vs loads results by experimental, theoretical, and numerical

The obtained results are displayed for the predefined points on the cross section which is considered during the experiment. For typical beams, the stress concentrations at the sensor's locations of the I-beam thin-walled section are shown in throughout the span with the applied combined load as shown in Figure 10.

The graphical expression of the variation in stresses with increasing applied loads as determined by experimental, theoretical, and numerical calculations as shown in Figure 10.

Refereeing Figure 11 of the theoretical, numerical, and experimental normal-stress distribution graphs is practically identical to each other. The relative deviation between the three results is estimated to be between 2.5 and 3.5%

Conclusion

The following conclusions could be drawn based on the experimental results and analysis of the I-shape thin-walled steel beams under combined bending and torsion loads.

- The simple geometric properties of the section are used to generate the stiffness matrix for thin-walled beam sections with retrained torsion. By considering an additional degree of freedom at each node, the trigonometric and approximation solutions of an interpolation function are used to express the 14×14 DOFs stiffness matrix for non-uniform torsion.
- The stiffness matrix for 3D thin-walled sections subjected to combined loading is presented, making advanced structural analysis bar elements more convenient. This stiffness matrix is more applicable for open thin-walled sections because the value of characteristics number for open section is very small compared to the closed thin-walled sections.
- An investigation was carried out between the theoretical, experimental, and numerical results of an I-beam section under the combined loading conditions of bending and torsion. The behavior of experimental specimens has confirmed that beams in bending, and torsion have significant reserves of bearing capacity when compared to steel work at the elastic stage.
- As the experiment is conducted within the elastic range, the rotation angles for the three methods are nearly identical. In addition to the experiment results, numerical calculations are commonly used as a comparison in engineering design procedures because they allow for the use of a wider range of parameters engineering design procedure of beams under combined loading.
- Experimental, theoretical, and numerical investigations of I-beam profile beams section is carried out. The behavior of experimental specimens was analyzed, and it was discovered that beams in bending, and torsion have similar results to theoretical results within the elastic stage of steel work. The bearing capacity of the spaceman is determined by the distribution of normal stresses in the experiment for the I-beam with restrained torsion. When compared to each other, the rotation angles begin to increase smoothly based on the applied loading and they are similar.

References

1. Pavlenko A.D., Rybakov V.A., Pikht A.V., Mikhailov E.S. Non-uniform torsion of thin-walled open-section multi-span beams. *Magazine of Civil Engineering.* 2016;67(7):55–69. https://doi.org/10.5862/MCE.67.6

2. Mysore R., Kissinger R. Finite element analysis of thin-walled beams subjected to torsion. *In 30th Structures, Structural Dynamics and Materials Conference.* 2012. https://doi.org/10.2514/6.1989-1176

3. Tusnin A.R., Prokic M. Experimental research of I-beams under bending and torsion actions. *Magazine of Civil Engineering.* 2015;53(1). https://doi.org/10.5862/MCE.53.3

4. Vatin N.I., Sinelnikov A.S. Strength and durability of thin-walled cross-sections. In: Jármai K., Farkas J. (eds.) *Design, Fabrication and Economy of Metal Structures*. Berlin, Heidelberg: Springer; 2013. https://doi.org/10.1007/978-3-642-36691-8_25

5. Tusnin A.R., Prokic M. Selection of parameters for I-beam experimental model subjected to bending and torsion. *Procedia Engineering*. 2015;111:789–796. https://doi.org/10.1016/j.proeng.2015.07.146

6. Bischoff M., Bletzinger K.-U., Wall W.A., Ramm E. Models and finite elements for thin-walled structures. *Encyclopedia of Computational Mechanics*. 2004. https://doi.org/10.1002/0470091355.ecm026

7. Saadé K., Espion B., Warzée G. Non-uniform torsional behavior and stability of thin-walled elastic beams with arbitrary cross sections. *Thin-Walled Structures*. 2004;42(6):857–881. https://doi.org/10.1016/j.tws.2003.12.003

8. Iu C.K., Chen W.F., Chan S.L., Ma T.W. Direct second-order elastic analysis for steel frame design. *KSCE Journal of Civil Engineering*. 2008;12(6):379–389. https://doi.org/10.1007/s12205-008-0379-3

9. Jin S., Li Z., Huang F., Gan D., Cheng R., Deng G. Constrained shell finite element method for elastic buckling analysis of thin-walled members. *Thin-Walled Structures*. 2019;145:106409. https://doi.org/10.1016/j.tws.2019.106409

10. Banić D., Turkalj G., Brnić J. Finite element stress analysis of elastic beams under non-uniform torsion. *Transactions of Famena*. 2016;40(2):71–82. https://doi.org/10.21278/TOF.40206

11. Vlasov V.Z. *Thin-walled elastic beams.* Virginia: National Technical Information Service; 1984. 493 p.

12. Wu L., Mohareb M. Finite element formulation for shear deformable thin-walled beams. *Canadian Journal of Civil Engineering*. 2011;38(4):383–392. https://doi.org/10.1139/L11-007

13. Aalberg A. *An experimental study of beam-columns subjected to combined torsion, bending, and axial actions*. Trondheim; 1995.

14. Gebre T.H., Galishnikova V.V. The impact of section properties on thin walled beam sections with restrained torsion. *Journal of Physics: Conference Series.* 2020;1687(1):012020. https://doi.org/10.1088/1742-6596/1687/1/012020

15. Galishnikova V., Gebre T.H. The behaviour of thin-walled beam with restrained torsion. *Magazine of Civil Engineering*. 2022;110(2). https://doi.org/10.34910/MCE.110.9

16. Silvestre N., Camotim D. Second-order generalised beam theory for arbitrary orthotropic materials. *Thin-Walled Structures.* 2002;40(9):91–820. https://doi.org/10.1016/S0263-8231(02)00026-5

17. Gebre T., Galishnikova V., Lebed E., Tupikova E., Awadh Z. Finite element analysis of 3D thin-walled beam with restrained torsion. *Lecture Notes in Civil Engineering*. 2022;282:359–369. https://doi.org/10.1007/978-3-031-10853-2_34

18. Bernardo D. New finite element for analysis of thin-walled structures. *Journal of Structural Engineering*. 2011;137(10):1153–1167. https://doi.org/10.1061/(asce)st.1943-541x.0000372

19. Lopez R.D.E.F. *A 3D finite beam element for the modelling of composite wind turbine wings* (Master of Science Thesis). Stockholm; 2013.

20. Gebre T., Galishnikova V., Tupikova E. Warping behavior of open and closed thin-walled sections with restrained torsion. *Engineering Letters*. 2022;30(1):1–8.

21. Cambronero-Barrientos F., Díaz-del-Valle J., Martínez-Martínez J.A. Beam element for thin-walled beams with torsion, distortion, and shear lag. *Engineering Structures*. 2017;43:571–588. https://doi.org/10.1016/j.engstruct.2017.04.020

22. Nguyen P.C., Kim S.E. An advanced analysis method for three-dimensional steel frames with semi-rigid connections. *Finite Elements in Analysis and Design*. 2014;80:23–32. https://doi.org/10.1016/j.finel.2013.11.004

23. Galishnikova V.V., Gebre T.H., Tupikova E.M., Niazmand M.A. The design guide for space frames with or without warping restraint at nodes. *AIP Conference Proceedings.*2022;2559(050016). https://doi.org/10.1063/5.0099013

24. Robertson I.N., Knapp R.H. *Toward advanced analysis in steel frame design*. Hawaii; 2003.

25. Tusnin A. Finite element for calculation of structures made of thin-walled open profile rods. *Procedia Engineering.* 2016;150:1673–1679. https://doi.org/10.1016/j.proeng.2016.07.149

26. Mohri F., Eddinari A., Damil N., Potier Ferry M. A beam finite element for non-linear analyses of thin-walled elements. *Thin-Walled Structures.* 2008;46(7–9):981–990. https://doi.org/10.1016/j.tws.2008.01.028

27. Gunnlaugsson G.A., Pedersen P.T. A finite element formulation for beams with thin walled cross-sections. *Computers and Structures.* 1982;15(6):691–699. https://doi.org/10.1016/S0045-7949(82)80011-4

28. Chen H.H., Lin W.Y., Hsiao K.M. Co-rotational finite element formulation for thin-walled beams with generic open section. *Computer Methods in Applied Mechanics and Engineering*. 2006;195(19–22):2334–2370.

29. Kugler S., Fotiu P., Murín J. On the access to transverse shear stiffnesses and to stiffness quantities for non-uniform warping torsion in FGM beam structures. *Strojnicky Casopis*. 2019;69(2):27–56. https://doi.org/10.2478/scjme-2019-0016

30. Lalin V., Rybakov V., Sergey A. The finite elements for design of frame of thin-walled beams. *Applied Mechanics and Materials*. 2014;578–579:858–863. https://doi.org/10.4028/www.scientific.net/AMM.578-579.858

31. Lalin V.V., Rybakov V.A., Ivanov S.S., Azarov A.A. Mixed finite-element method in V.I. Slivker's semi-shear thin-walled bar theory. *Magazine of Civil Engineering*. 2019;89(5):79–93.