

DOI: 10.22363/1815-5235-2026-22-4-368-382

EDN: LQJMYQ

Research article / Научная статья

Nonlinear Layered Formulation: Modeling Reinforced Concrete Semi-Precast Slabs

Dara A. Mawlood[✉], Alexandr A. Koyankin^{id}

Siberian Federal University, Krasnoyarsk, Russia Federation

✉ dara.mawlood@univsul.edu.iq

Received: April 4, 2026

Revised: June 22, 2026

Accepted: July 5, 2026

Abstract. Semi-precast reinforced concrete slab systems enhance construction efficiency by substantially reducing on-site formwork. Construction involves two primary phases: (1) off-site fabrication of semi-precast components and (2) on-site casting of a concrete topping. The interfacial bond strength between the precast base and the cast-in-place overlay is a critical factor governing the composite system's performance. This study presents a comprehensive numerical investigation to evaluate the effect of surface treatment techniques on the flexural behavior of semi-precast RC slabs. A novel six-node triangular layered finite element approach is developed, incorporating material nonlinearity through an enhanced global-local plate formulation. The method employs a cross-sectional discretization strategy, decomposing the slab into distinct material layers where concrete and steel are modeled as discrete plate elements with individual constitutive laws. The computational framework utilizes a decoupled treatment of displacement fields and transverse stresses, facilitating accurate nodal stress evaluation via integrated constitutive models. Spatial discretization is achieved using a C^1 -continuous six-node triangular element, with governing equations derived from a partially mixed variation principle. Validation through benchmark cases demonstrates the method's robustness, with ultimate deflection errors below 4.68% and reaching 0.5% accuracy in optimal scenarios. The proposed triangular layered finite element method achieves superior accuracy and computational efficiency compared to conventional approaches.

Key words: semi-precast slab, strain field, stress field, layered FE discretization, and numerical results

Authors' contribution. The authors have made an equal contribution to the development of the concept, collection of data and their interpretation, and preparation of the article for publication. All authors have read and approved the final version of the manuscript.

Conflicts of interest. The authors declare that there is no conflict of interest.

Statement on the use of artificial intelligence technologies. No generative artificial intelligence technologies were used in the creation of this article.

Data availability. All data obtained during this study are included in the article being published.

For citation: Mawlood DA, Koyankin AA. Nonlinear layered formulation: Modeling reinforced concrete semi-precast slabs. *Structural Mechanics of Engineering Constructions and Buildings*. 2026;22(4):368–382. <http://doi.org/10.22363/1815-5235-2026-22-4-368-382> EDN: LQJMYQ

Dara A. Mawlood, master student, Department of Building Structures and Controlled Systems, Institute of Civil Engineering, Siberian Federal University, 79 Svobodny av., Krasnoyarsk, 660041, Russia Federation; ORCID: 0009-0003-2819-3107; e-mail: dara.mawlood@univsul.edu.iq

Alexandr A. Koyankin, Candidate of Technical Sciences, Associate Professor of the Department of Building Structures and Controlled Systems, Institute of Civil Engineering, Siberian Federal University, 79 Svobodny av., Krasnoyarsk, 660041, Russia Federation; eLIBRARY SPIN-code: 2779-8314, ORCID: 0000-0001-5271-9904; e-mail: KoyankinAA@mail.ru

© Mawlood D.A., Koyankin A.A., 2026

This work is licensed under a Creative Commons Attribution-NonCommercial 4.0 International License
<https://creativecommons.org/licenses/by-nc/4.0/legalcode>

Нелинейная послойная формулировка: моделирование сборно-монолитных железобетонных плит

Д.А. Мавлуд , А.А. Коянкин 

Сибирский федеральный университет, Красноярск, Российская Федерация

✉ dara.mawloud@univsul.edu.iq

Поступила в редакцию: 4 апреля 2026 г.

Доработана: 22 июня 2026 г.

Принята к публикации: 5 июля 2026 г.

Аннотация. Системы из сборно-монолитных железобетонных плит повышают эффективность строительства за счет существенного сокращения затрат на опалубку на строительной площадке. Строительство включает в себя два основных этапа: (1) изготовление сборных элементов за пределами площадки и (2) заливка верхнего слоя из монолитного бетона на площадке. Прочность связи между сборным основанием и монолитной облицовкой является критическим фактором, определяющим эксплуатационные характеристики композитной системы. В данном исследовании представлено комплексное численное исследование для оценки влияния методов обработки поверхности на изгиб сборных железобетонных плит. Разработан новый метод расчета с шестигрузовым треугольным слоистым конечным элементом, учитывающий нелинейность материала за счет усовершенствованной глобально-локальной формулировки пластины. В этом методе используется стратегия дискретизации поперечного сечения, при которой плита разбивается на отдельные слои материала, где бетон и сталь моделируются как отдельные элементы плиты с собственными определяющими соотношениями. В рамках данной методики расчета используется несвязанный подход к полям смещений и поперечных напряжений, что позволяет точно оценивать узловые напряжения с помощью интегрированных определяющих моделей. Пространственная дискретизация достигается с помощью C^1 -гладкого шестигрузового треугольного элемента с основными уравнениями, полученными на основе частично смешанного вариационного принципа. Проверка с помощью контрольных примеров демонстрирует надежность метода, при этом погрешности предельных отклонений не превышают 4,68 %, а в оптимальных сценариях достигают точности 0,5 %. Предложенный метод многослойных конечных элементов обеспечивает превосходную точность и вычислительную эффективность по сравнению с традиционными подходами.

Ключевые слова: сборно-монолитная плита перекрытия, поле деформаций, поле напряжений, дискретизация на многослойные конечные элементы, численные результаты

Вклад авторов. Авторы внесли равный вклад в разработку концепции, сбор данных и их интерпретацию, подготовку статьи к публикации. Все авторы ознакомлены с окончательной версией статьи и одобрили ее.

Заявление о конфликте интересов. Авторы заявляют об отсутствии конфликта интересов.

Заявление об использовании технологий искусственного интеллекта. При создании настоящей статьи технологии генеративного искусственного интеллекта не использовались.

Заявление о доступности данных. Все данные, полученные в ходе этого исследования, включены в опубликованную статью.

Для цитирования: Mawlood D.A., Koyankin A.A. Nonlinear layered formulation: Modeling reinforced concrete semi-precast slabs // Строительная механика инженерных конструкций и сооружений. 2026. Т. 22. № 4. С. 368–382. <http://doi.org/10.22363/1815-5235-2026-22-4-368-382> EDN: LQJMYQ

1. Introduction

The use of precast reinforced concrete (RC) systems has greatly enhanced construction productivity by optimizing project timelines and costs [1; 2]. Subsequently, researchers have refined semi-precast slab methods to tackle ongoing industry challenges [3; 4]. Accurate prediction of the structural performance of semi-precast reinforced concrete systems is essential for optimizing cost-efficiency and ensuring safety in design applications [5]. However, the complex composite behavior of these systems, particularly at the interface, poses significant challenges in modeling their nonlinear response, making it an active area of research in structural engineering [6–8]. The finite element (FE) method has established itself as the predominant technique for this nonlinear analysis, demonstrating greater effectiveness than alternative

Мавлуд А. Дара, магистрант кафедры строительных конструкций и управляемых систем, Инженерно-строительный институт, Сибирский федеральный университет, Российская Федерация, 660041, г. Красноярск, пр. Свободный, д. 79; ORCID: 0009-0003-2819-3107; e-mail: dara.mawloud@univsul.edu.iq

Коянкин Александр Александрович, кандидат технических наук, доцент кафедры строительных конструкций и управляемых систем, Инженерно-строительный институт, Сибирский федеральный университет, Российская Федерация, 660041, г. Красноярск, пр. Свободный, д. 79; eLIBRARY SPIN-код: 2779-8314, ORCID: 0000-0001-5271-9904; e-mail: KoyankinAA@mail.ru

approaches [9–11]. Over the past four decades, advances in FE formulations have produced increasingly sophisticated models capable of capturing the complete nonlinear load-deflection response of reinforced concrete slabs [12–14]. Among these, the layered method was developed to offer a computationally efficient alternative to conventional 3D solid elements, thereby overcoming their inherent limitations [15–17].

This challenge of accurate shear modeling contributes to a broader issue: standardized numerical modeling protocols for semi-precast slabs are still lacking [18–20]. Existing studies and formulations largely concentrate on specialized configurations, such as pre-stressed variants, fiber-reinforced composites, and hybrid structural solutions [21–23].

This research presents a novel six-node, triangular-layered finite element formulation for the materially nonlinear analysis of semi-precast reinforced concrete slabs with cast-in-place concrete overlays. The work complements prior experimental investigations on prototype systems by introducing numerical models to analyze flexural capacity as a function of systematically varied interfacial surface textures, enabling a rigorous evaluation of composite performance [3]. The proposed formulation is based on an innovative global-local plate theory. Semi-precast RC slabs are modeled by discretizing the cross-section into perfectly bonded concrete and steel layers via a superposition approach, with material nonlinearities captured by distinct constitutive models: a smeared crack formulation for concrete and an elasto-plastic framework for steel reinforcement. A key innovation is the unique incorporation of transverse stress components as nodal degrees of freedom within the six-node triangular composite plate element, which significantly enhances computational capability and accuracy.

Within this displacement-based framework, primary solution variables are displacement field parameters, while stress components are derived as secondary quantities through constitutive equations. The model's robustness is validated against benchmark cases, demonstrating superior accuracy and computational efficiency compared to conventional approaches for predicting the nonlinear load-deflection response of semi-precast slab systems.

2. Materials and Methods

The theoretical development of the proposed six-node triangular layered element is based on a plate formulation for combined membrane and bending action. It proceeds according to the following steps.

A more versatile member of the triangular element family is the six-node linear strain triangle (LST) [24–26], depicted in Figure 1.

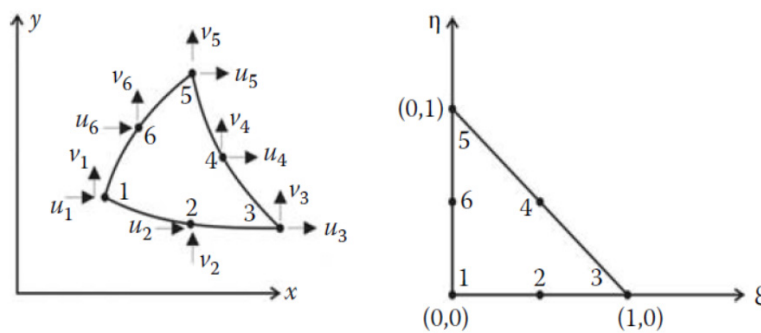


Figure 1. Linear strain triangular element
Source: made by A. Khennan [26].

The displacement field within the element is approximated by:

$$u = N_1 u_1 + N_2 u_2 + N_3 u_3 + N_4 u_4 + N_5 u_5 + N_6 u_6,$$

$$v = N_1 v_1 + N_2 v_2 + N_3 v_3 + N_4 v_4 + N_5 v_5 + N_6 v_6 \quad \text{or} \quad [U] = [N][a], \quad (1)$$

$$\text{where } \begin{bmatrix} N_1(\xi, \eta) \\ N_2(\xi, \eta) \\ N_3(\xi, \eta) \\ N_4(\xi, \eta) \\ N_5(\xi, \eta) \\ N_6(\xi, \eta) \end{bmatrix} = \begin{bmatrix} -\lambda(1-2\lambda) \\ 4\xi\lambda \\ -\xi(1-2\xi) \\ 4\xi\eta \\ -\eta(1-2\eta) \\ 4\eta\lambda \end{bmatrix} \quad \text{with } \lambda = 1 - \xi - \eta.$$

The Jacobian matrix $[J]$ of the geometric transformation is given by:

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^6 \frac{\partial N_i}{\partial \xi} x_i & \sum_{i=1}^6 \frac{\partial N_i}{\partial \xi} y_i \\ \sum_{i=1}^6 \frac{\partial N_i}{\partial \eta} x_i & \sum_{i=1}^6 \frac{\partial N_i}{\partial \eta} y_i \end{bmatrix};$$

$$[J] = \frac{1}{4} \begin{bmatrix} 1-4\lambda & 4(\lambda-\xi) & -1+4\xi & 4\eta & 0 & -4\eta \\ 1-4\lambda & -4\eta & 0 & 4\xi & -1+4\eta & 4(\lambda-\eta) \end{bmatrix} \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_5 & y_5 \\ x_6 & y_6 \end{bmatrix}, \quad (2)$$

with

$$\lambda = 1 - \xi - \eta. \quad (3)$$

Then, the membrane strain can be determined:

$$[\varepsilon_m] = [B_m][a]; \quad (4)$$

$$[B_m] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 & \frac{\partial N_4}{\partial x} & 0 & \frac{\partial N_5}{\partial x} & 0 & \frac{\partial N_6}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial y} & 0 & \frac{\partial N_4}{\partial y} & 0 & \frac{\partial N_5}{\partial y} & 0 & \frac{\partial N_6}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial y} & \frac{\partial N_4}{\partial x} & \frac{\partial N_5}{\partial y} & \frac{\partial N_5}{\partial x} & \frac{\partial N_6}{\partial y} & \frac{\partial N_6}{\partial x} \end{bmatrix}. \quad (5)$$

To evaluate the $[B]$ matrix, the partial derivatives with respect to global coordinates (x, y) must be transformed to the local (ξ, η) coordinates using the chain rule:

$$\begin{bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix} \quad \text{then} \quad \begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{bmatrix}. \quad (6)$$

The membrane stiffness matrix for the element is given by using the Gauss quadrature (often called the Hammer formula for triangular domains) [26; 27]:

$$\int_0^+ \int_0^{1-\xi} [B_m(\xi, \eta)]^T [D^i] [B_m(\xi, \eta)] \det[J(\xi, \eta)] d\eta d\xi =$$

$$= t \sum_{i=1}^{nhp} W_i [B_m(\xi_i, \eta_i)]^T [D^i] [B_m(\xi_i, \eta_i)] \det[J(\xi_i, \eta_i)], \quad (7)$$

where nhp represents the number of Hammer points.

$$D^i = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix}. \quad (8)$$

2.1. Bending Element Formulation

Modern finite element formulations for plates commonly adopt Mindlin — Reissner plate theory. Its primary advantage in FEM context is the use of independent rotational degrees of freedom, θ_x and θ_y , and transverse displacement, $w(x, y)$ [24; 26; 27].

$$w = \sum_{i=1}^6 N_i(\xi, \eta) w_i, \theta_x = \sum_{i=1}^6 N_i(\xi, \eta) \theta_{xi}, \theta_y = \sum_{i=1}^6 N_i(\xi, \eta) \theta_{yi}. \quad (9)$$

$$\text{With } [B_B] = \begin{bmatrix} 0 & \frac{\partial N_1}{\partial x} & 0 & \dots & \dots & \dots & 0 & \frac{\partial N_6}{\partial x} & 0 \\ 0 & 0 & \frac{\partial N_1}{\partial y} & \dots & \dots & \dots & 0 & 0 & \frac{\partial N_6}{\partial y} \\ 0 & \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \dots & \dots & \dots & 0 & \frac{\partial N_6}{\partial y} & \frac{\partial N_6}{\partial x} \end{bmatrix},$$

$$[B_s] = \begin{bmatrix} -\frac{\partial N_1}{\partial y} & 0 & N_1 & \dots & \dots & \dots & -\frac{\partial N_6}{\partial y} & 0 & N_6 \\ -\frac{\partial N_1}{\partial x} & N_1 & 0 & \dots & \dots & \dots & -\frac{\partial N_6}{\partial x} & N_6 & 0 \end{bmatrix}.$$

Therefore, the element stiffness matrix is composed of two distinct contributions: the bending stiffness matrix and the shear stiffness matrix.

$$[K_e]^B = [K_B] + [K_s] = \int_{A_e} [B_B]^T [D^i] [B_B] dA + \int_{A_e} [B_s]^T [D^i] [B_s] dA. \quad (10)$$

2.2. Layer Kinematic Formulation

Reddy's third-order shear deformation theory (TSDT) relaxes the restrictive kinematic assumptions of classical plate theory by explicitly accounting for transverse shear strains and their associated deformation. The displacement field within this framework is expressed as [28]:

$$u^i(x, y, z) = z\psi_x(x, y) - (4z^3)/(3h^2)(\psi_x(x, y) + (\partial w_0^i(x, y))/\partial x); \quad (11)$$

$$v^i(x, y, z) = z\psi_y(x, y) - (4z^3)/(3h^2) \left((\psi_y(x, y) + (\partial w_0^i(x, y))/\partial y) \right);$$

$$w^j(x, y) = w_0^j(x, y),$$

$$\text{where } \psi_x = -\frac{\partial w_0}{\partial x}, \psi_y = -\frac{\partial w_0}{\partial y}.$$

2.2.1. Strain Field Components

Using the kinematic strain–displacement relations, the strain state in the i -th layer of the reinforced concrete slab is formulated vector form as [19; 28].

$$\begin{aligned} \varepsilon_i^{(1)} = \begin{pmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{pmatrix} = \begin{pmatrix} \frac{\partial \psi_x}{\partial x} \\ \frac{\partial \psi_y}{\partial y} \\ \frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \end{pmatrix}, \quad \varepsilon_i^{(3)} = \begin{pmatrix} \varepsilon_{xx}^{(3)} \\ \varepsilon_{yy}^{(3)} \\ \gamma_{xy}^{(3)} \end{pmatrix} = \begin{pmatrix} -\frac{4}{3h^2} \left(\frac{\partial \psi_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2} \right) \\ -\frac{4}{3h^2} \left(\frac{\partial \psi_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2} \right) \\ -\frac{4}{3h^2} \left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} + 2 \frac{\partial^2 w_0}{\partial x \partial y} \right) \end{pmatrix}, \\ \gamma_i^{(0)} = \begin{pmatrix} \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \end{pmatrix} = \begin{pmatrix} \psi_y + \frac{\partial w_0}{\partial y} \\ \psi_x + \frac{\partial w_0}{\partial x} \end{pmatrix}, \quad \gamma_i^{(2)} = \begin{pmatrix} \gamma_{yz}^{(2)} \\ \gamma_{xz}^{(2)} \end{pmatrix} = \begin{pmatrix} -\frac{4}{h^2} \left(\psi_y + \frac{\partial w_0}{\partial y} \right) \\ -\frac{4}{h^2} \left(\psi_x + \frac{\partial w_0}{\partial x} \right) \end{pmatrix}. \end{aligned} \quad (12)$$

The strain field can be expressed as follows:

$$\varepsilon_i = z\varepsilon_i^{(1)} + z^3\varepsilon_i^{(3)}; \quad \gamma_i = \gamma_i^{(0)} + z^2\gamma_i^{(2)}. \quad (13)$$

2.2.2. Out-of-Plane Stress Formulation

The element normal stresses are derived from the strains via Hooke's law [24; 29] as follows:

$$\sigma^i = \underline{D}^i \underline{\varepsilon}^i \quad \text{or} \quad \sigma^i = \underline{D}^i \left(z\varepsilon_i^{(1)} + z^3\varepsilon_i^{(3)} \right). \quad (14)$$

Transverse shear stresses, τ_{xz} and τ_{yz} , are calculated independently for each material layer within the reinforced concrete slab. Their expressions are derived as follows:

$$\begin{bmatrix} \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \frac{E_i}{2(1+\mu)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \quad \text{or} \quad \tau_i = \underline{D}_s^i \underline{\gamma}_i, \quad \tau_i = \underline{D}_s^i \left(\gamma_i^{(0)} + z^2\gamma_i^{(2)} \right), \quad (15)$$

where parameter E_i is the elastic modulus of the i -th layer, and μ is the Poisson's ratio of the reinforced concrete. Within Reddy's Third-Order Shear Deformation Theory, the plate's constitutive law is:

$$\begin{pmatrix} \underline{M}_i^0 \\ \underline{P}_i \\ \underline{Q}_i \\ \underline{R}_i \end{pmatrix} = \begin{bmatrix} \underline{D}^i & \underline{F}^i & \underline{0} & \underline{0} \\ \underline{F}^i & \underline{H}^i & \underline{0} & \underline{0} \\ \underline{0} & \underline{0} & \underline{A}_s^i & \underline{D}_s^i \\ \underline{0} & \underline{0} & \underline{D}_s^i & \underline{F}_s^i \end{bmatrix} \begin{pmatrix} \underline{\varepsilon}_i^{(1)} \\ \underline{\varepsilon}_i^{(3)} \\ \underline{\gamma}_i^{(0)} \\ \underline{\gamma}_i^{(3)} \end{pmatrix}. \quad (16)$$

Following the standard finite element procedure, the stiffness matrix is assembled as:

$$\underline{F}^i = \begin{bmatrix} F_{11} & F_{12} & 0 \\ F_{12} & F_{22} & 0 \\ 0 & 0 & F_{66} \end{bmatrix} = \int_{z^{i-1}}^{z^i} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} z^4 dz;$$

$$\underline{H}^i = \begin{bmatrix} H_{11} & H_{12} & 0 \\ H_{12} & H_{22} & 0 \\ 0 & 0 & H_{66} \end{bmatrix} = \int_{z^{i-1}}^{z^i} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} z^6 dz;$$

$$\underline{D}^i = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} = \int_{z^{i-1}}^{z^i} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} z^2 dz;$$

$$\underline{D}_s^i = \begin{bmatrix} D_{44} & 0 \\ 0 & D_{55} \end{bmatrix} = \int_{z^{i-1}}^{z^i} \begin{bmatrix} C_{44} & 0 \\ 0 & C_{55} \end{bmatrix} z^2 dz$$

and

$$\underline{F}_s^i = \begin{bmatrix} F_{44} & 0 \\ 0 & F_{55} \end{bmatrix} = \int_{z^{i-1}}^{z^i} \begin{bmatrix} C_{44} & 0 \\ 0 & C_{55} \end{bmatrix} z^4 dz;$$

$$\underline{A}_s^i = \begin{bmatrix} A_{44} & 0 \\ 0 & A_{55} \end{bmatrix} = \int_{z^{i-1}}^{z^i} \begin{bmatrix} C_{44} & 0 \\ 0 & C_{55} \end{bmatrix} z^2 dz,$$

where

$$Q_{11} = \frac{E}{(1-\mu^2)}, \quad Q_{12} = Q_{21} = \frac{\mu E}{(1-\mu^2)}, \quad Q_{22} = \frac{E}{(1-\mu^2)}, \quad Q_{66} = G \text{ and } C_{44} = C_{55} = G.$$

3. Methods

The reinforced concrete slab geometry (Figure 2) is defined within a Cartesian coordinate system (x_1, x_2, z) , with the mid-plane at $z = 0$ serving as the reference plane. The slab is idealized as a laminated composite structure comprising perfectly bonded layers of normal concrete (NC) and steel [8]. Computationally, a smeared reinforcement approach is adopted, representing steel rebars as equivalent uniaxial layers positioned at their centroids. The thickness of an equivalent steel layer, t_{ens} , is determined from the rebar cross-sectional area A_s and spacing S as $t_{\text{ens}} = A_s/S$.

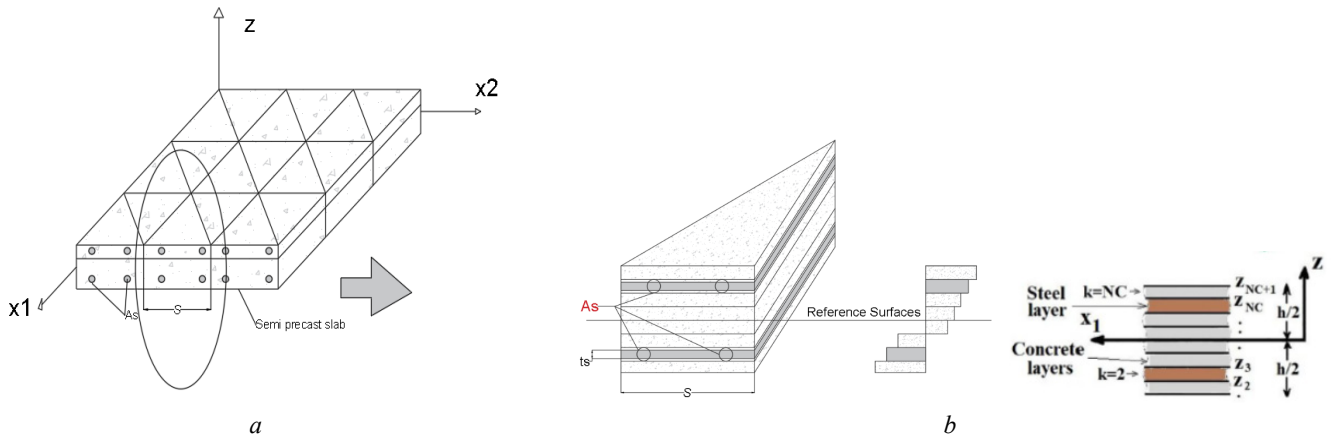


Figure 2. Configuration of the RC slab (a); Idealized laminated composite model (b)

Source: made by D.A. Mawlood, A.A. Koyankin.

3.1. Steel Reinforcement Layer

Steel reinforcement layers are modeled with an idealized elastoplastic constitutive law. For layer i , the elastic stress-strain relationship is given by [28]:

$$\sigma_{\text{steel}}^i = \underline{D}_{\text{steel}}^i \underline{\varepsilon}^i. \quad (17)$$

The sectional properties D_{steel} for steel:
$$\underline{D}_{\text{steel}}^i = \begin{bmatrix} E_s & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \underline{D}_{\text{steel}}^i = \begin{bmatrix} E_{sp} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

$E_s^{(i)}$ represents the elastic modulus of the i -th steel reinforcement layer. The parameter E_{ps} denotes the post-yield plastic modulus of the steel reinforcement layers [8].

3.2. Concrete Layer Properties

In its pre-cracked state, the i -th concrete layer exhibits isotropic, homogeneous, and linear elastic behavior, characterized by the constitutive relation:

$$\sigma_{\text{concrete}}^i = \underline{D}_{\text{concrete}}^i \underline{\varepsilon}_i, \quad \tau_{\text{concrete}}^i = \underline{D}_s^i \underline{\gamma}_i. \quad (18)$$

When the tensile strength f is exceeded in any principal direction, the isotropic constitutive relations (Eqs. 18) become invalid. This study employs the smeared crack approach [29] to characterize post-cracking concrete behavior, necessitating a transformation from isotropic to orthotropic material representation in a local coordinate system ξ, η, ζ . The post-cracking constitutive relationship for layer i in this local coordinate system is expressed as:

$$\begin{bmatrix} \sigma_{\xi} \\ \sigma_{\eta} \\ \tau_{\xi\eta} \end{bmatrix} = \frac{E_i}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \rho \left(\frac{1-\mu}{2} \right) \end{bmatrix} \begin{bmatrix} \varepsilon_{\xi} \\ \varepsilon_{\eta} \\ \gamma_{\xi\eta} \end{bmatrix}, \quad \begin{bmatrix} \tau_{\xi\zeta} \\ \tau_{\eta\zeta} \end{bmatrix} = \frac{E_i}{2(1+\mu)} \begin{bmatrix} 1 & 0 \\ 0 & \rho \end{bmatrix} \begin{bmatrix} \gamma_{\xi\zeta} \\ \gamma_{\eta\zeta} \end{bmatrix}. \quad (19)$$

Note that $\rho \in (0, 1]$ represents the shear retention factor, which is utilized to model the effects of aggregate interlock.

3.3. Mixed Stiffness Matrix

The element stiffness matrix, K_{ei} , can be additively decomposed into its membrane and bending components as follows: $[K_{ei}] = \sum_{i=1}^n [K_{e,i}^m] + [K_{e,i}^B]$, where n : number of layers.

$$[K_{e,i}^m] = \left[\int_{A_e} [B_m]^T [D] [B_m] t dA \right],$$

$$[K_{e,i}^B] = [K_B] + [K_s] = \int_{A_e} [B_B]^T [D_B] [B_B] dA + \int_{A_e} [B_s]^T [D_s] [B_s] dA.$$

Within this formulation, steel reinforcement contributes solely to the membrane stiffness, while concrete contributes to both membrane and bending stiffness components.

$$[K_{e,i}^m] = K_{\text{concrete}}^m + K_{\text{steel}}^m = K_{12,12}^m, \quad \text{then} \quad [K_{e,i}] = [K_{e,i}^m] + [K_{e,i}^B] = \begin{bmatrix} K_{12,12}^{mm} & 0 \\ 0 & K_{18,18}^{BB} \end{bmatrix}. \quad (20)$$

3.4. Application of Load

The applied load can be calculated using:

$$[K][d] = [P], \quad \text{then} \quad \begin{bmatrix} K_{12,12}^{mm} & 0 \\ 0 & K_{18,18}^{BB} \end{bmatrix} \begin{bmatrix} d^{mm} \\ d^{BB} \end{bmatrix} = \begin{bmatrix} P^{mm} \\ P^{BB} \end{bmatrix}, \quad (21)$$

where d^{mm} , d^{BB} are the membrane $(u_1, u_2, u_3, u_4, u_5, u_6, v_1, v_2, v_3, v_4, v_5, v_6)$ and bending $(w_1, \varphi_{x1}, \varphi_{y1}, w_2, \varphi_{x2}, \varphi_{y2}, w_3, \varphi_{x3}, \varphi_{y3}, w_4, \varphi_{x4}, \varphi_{y4}, w_5, \varphi_{x5}, \varphi_{y5}, w_6, \varphi_{x6}, \varphi_{y6})$ element displacements; P^{mm} , P^{BB} are the membrane and bending element loads.

The nonlinear algebraic system is solved using an incremental-iterative strategy that combines load stepping with the Newton — Raphson method [30; 31].

The algorithm for the standard (non-incremental) Newton — Raphson method is summarized as follows:

1. Find initial displacements; $\bar{d}_0 = [\bar{K}_0(E_0)]^{-1} \Delta P_1$, where $\bar{K}_0$ is found by equation (20).
2. Computing the residual (unbalanced) force vector $\bar{r}_0$: $\bar{r}_0 = [\bar{K}_1(E_1)\bar{d}_0] - \Delta P_1$.
3. A displacement correction, $\Delta \bar{d}_1$, is computed by: $\Delta \bar{d}_1 = [\bar{K}_1(E_1)]^{-1} \bar{r}_0$.
4. The total displacement is updated: $\bar{d}_1 = \bar{d}_0 + \Delta \bar{d}_1$. Thus, the iterative Newton — Raphson step can be expressed in its standard form [12]:

$$\bar{r}_j = \bar{K}_{j+1}(E_{j+1})\bar{d}_j - \bar{P}, \quad \Delta \bar{d}_{j+1} = [\bar{K}_{j+1}(E_{j+1})]^{-1} \bar{r}_j \quad \text{for } j = 1, 2, 3 \dots$$

5. The final, total displacement vector is obtained by: $\bar{d} = \bar{d}_0 + \sum_{j=0}^m \Delta \bar{d}_{j+1}$.

4. Results and Discussion

A MATLAB-based computational tool was developed to implement the proposed six-node triangular layered finite element method for nonlinear analysis. The program's performance was validated by numerically analyzing two semi-precast reinforced concrete slabs with cast-in-place toppings. These test

specimens identical in configuration to those experimentally studied by Mohamed et al. [3] were modeled using the proposed finite element approach, as illustrated in Figure 3.

Two semi-precast slab specimens were modeled using the developed six-node triangular layered FEM. The slabs had a total thickness of 125 mm (75 mm precast slab with 50 mm top slab), are illustrated in Figure 3. Complete material and structural details are provided in Tables 1 and 2.

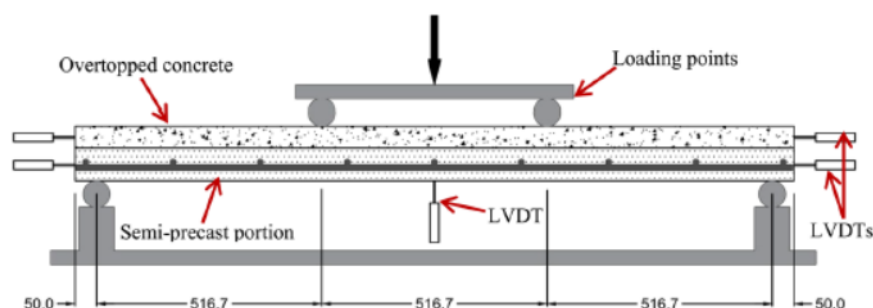


Figure 3. Schematic four-point testing arrangement of the slab

Source: made by Mohamed et al. [3].

Table 1. Description of the specimens and casted concrete strengths.

No.	Thickness, mm	Description	Semiprecast slab		Overtopping	
			Compressive strength, MPa	Splitting strength, MPa	Compressive strength, MPa	Splitting strength, MPa
S2	125	Untreated full thick slab	29.4	4.3	24.9	3.4
S3	125	Wire brushed roughened at the interface slab	30.1	4.1	23.4	3

Source: made by D.A. Mawlood, A.A. Koyankin.

Table 2. Input parameters for the numerical model

Parameters	Values
Young's moduli of concrete	17,000–24,000 MPa
Poisson's ratio of concrete	0.15
Dilation angle	31°
Eccentricity	0.1
Biaxial strength ratio	1.16
K	0.667
Peak compressive strengths of concrete	23.4–33.2 MPa
Peak tensile strengths of concrete	1.3–1.9 MPa
Young's modulus of steel	190,000 MPa
Poisson's ratio of steel	0.3
Yield tensile strength of steel	535 MPa

Source: made by D.A. Mawlood, A.A. Koyankin.

A 12×8 mesh of laminated triangular plate elements discretized the semi-precast slab, with the mesh size determined via a convergence study, illustrated in Figure 4.

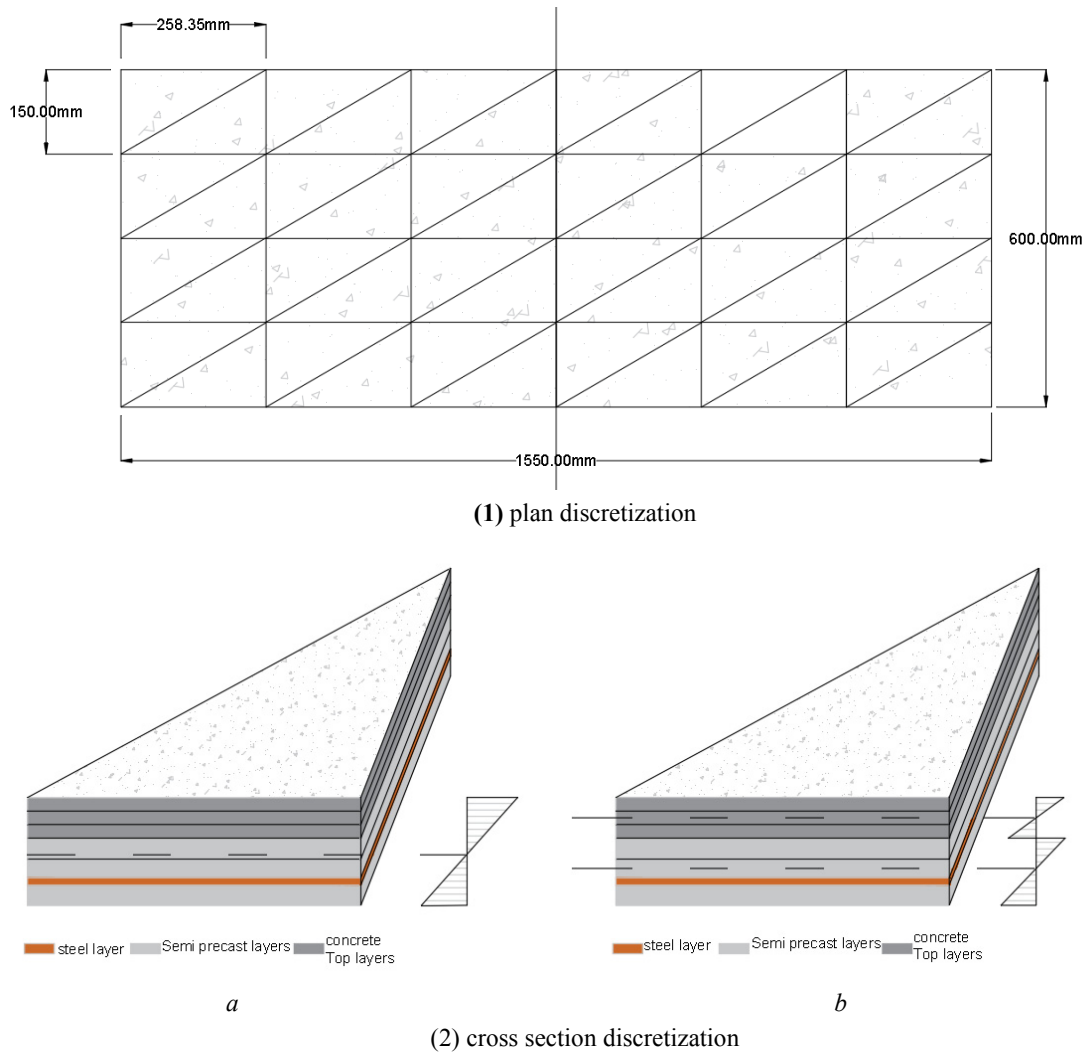


Figure 4. FE mesh of semi precast reinforced concrete slab with concrete topping:
a — neutral axes position before bonding failure; *b* — neutral axes position, after bonding failure
 Source: made by D.A. Mawlood, A.A. Koyankin.

Specimen S2 — a one-way semi-precast RC slab with an untreated, full-depth precast section and a concrete topping — was analyzed using the six-node triangular layered finite element methodology. The evaluation of mid-span deflection established a constant moment region, enabling a detailed assessment of flexural performance. Predictions from the layered FE model were compared against experimental data from Mohamed et al. [3] and results from high-fidelity 3D ABAQUS simulations in a three-way validation.

While the layered approach yielded a conservative estimate of the initial cracking load (Figures 5 and 6), excellent agreement with all reference data was observed beyond the 42 kN load threshold. The model demonstrated high fidelity in deflection prediction, with maximum values deviating by only 4.68% from experimental measurements and 7.40% from the ABAQUS simulations.

The triangular layered FE model demonstrated excellent predictive capability for Specimen S3, which featured a wire-brushed interface, accurately capturing its ultimate load capacity. As shown in Figures 7 and 8, the numerical load-deflection response exhibited remarkable agreement with both the experimental data and the ABAQUS results (Table 3). The maximum deflection prediction deviated by only 0.50% from the experimental measurement and by 12.5% from the ABAQUS simulation.

The deflection behavior and stress distributions resulting from the ABAQUS finite element analysis of the precast reinforced concrete slab are presented in Figures 9 through 14.

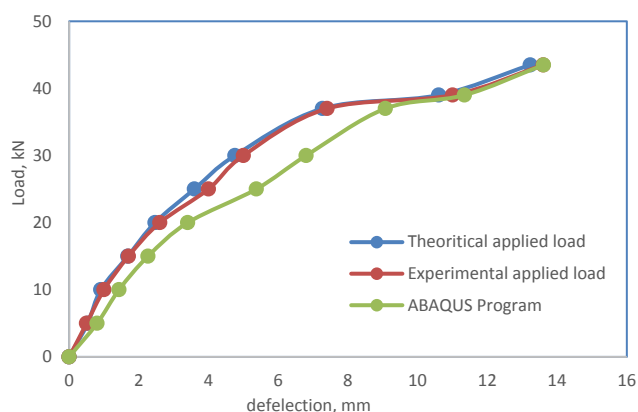


Figure 5. Load-deflection curve for sample (S2)

Source: made by D.A. Mawlood, A.A. Koyankin.

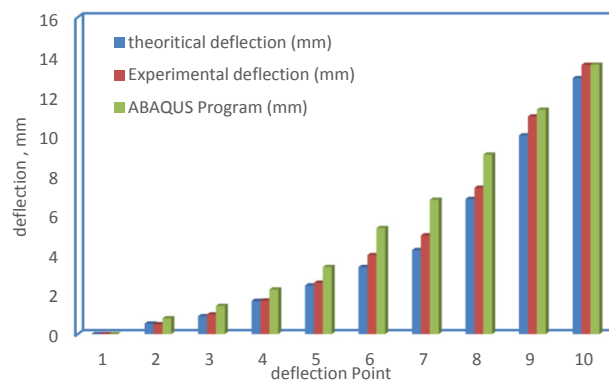


Figure 6. Deflection chart for sample (S2)

Source: made by D.A. Mawlood, A.A. Koyankin.

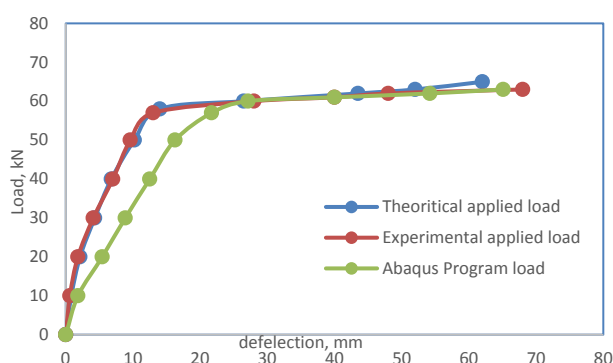


Figure 7. Load-deflection curve for sample (S3)

Source: made by D.A. Mawlood, A.A. Koyankin.

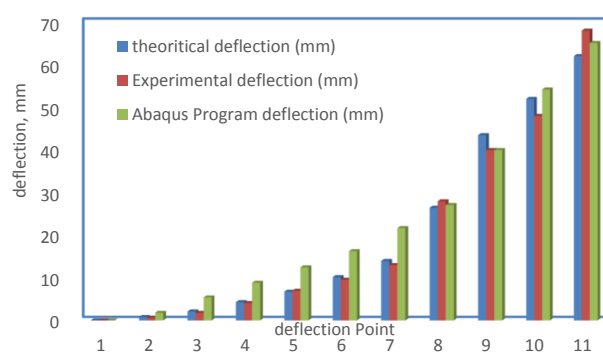


Figure 8. Deflection chart for sample (S3)

Source: made by D.A. Mawlood, A.A. Koyankin.

Table. 3. Experimental and theoretical data

Sample	Exp. Moment = $PL/4$, N. mm	Theo. Moment, N. mm	% diff.	Exp. strain = σ / E , mm/mm	Theo. Strain, mm/mm	% diff.
S2	$17.94 \cdot 10^6$	$17.02 \cdot 10^6$	-5.13	0.00287	0.00270	-5.92
S3	$25.98 \cdot 10^6$	$24.02 \cdot 10^6$	-7.54	0.003158	0.002897	-8.26

Source: made by D.A. Mawlood, A.A. Koyankin.

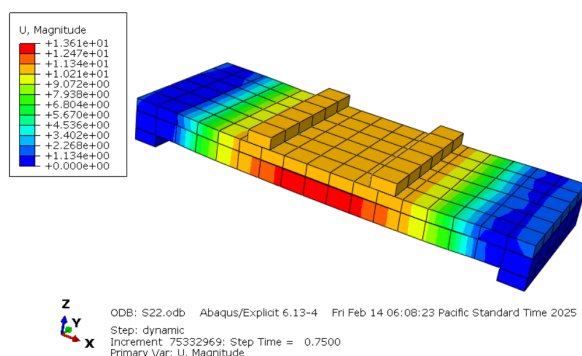


Figure 9. Plot of deflection for sample (S2)

Source: made by D.A. Mawlood, A.A. Koyankin.

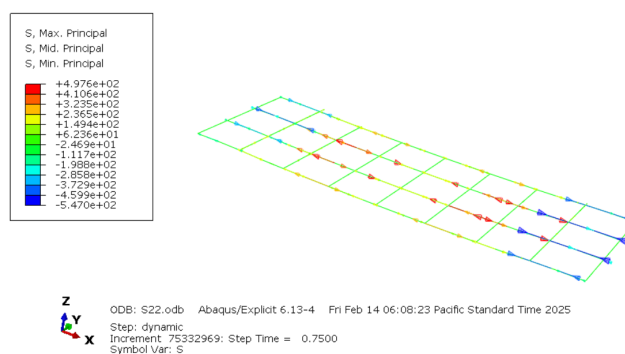


Figure 10. Plot of steel stress for sample (S2)

Source: made by D.A. Mawlood, A.A. Koyankin.

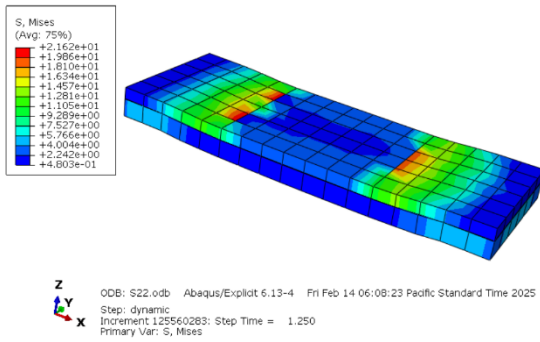


Figure 11. Plot of stress for sample (S2)
Source: made by D.A. Mawlood, A.A. Koyankin.

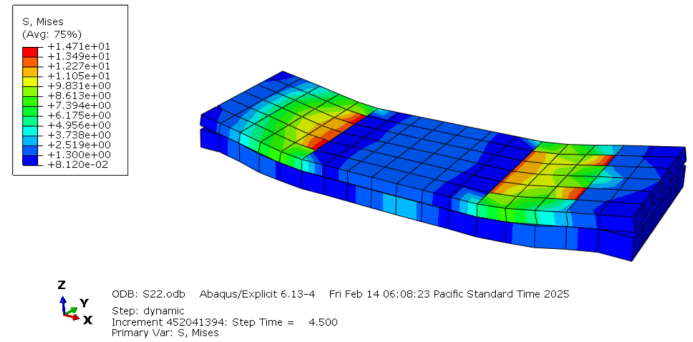


Figure 12. Plot of stress for sample (S3)
Source: made by D.A. Mawlood, A.A. Koyankin.

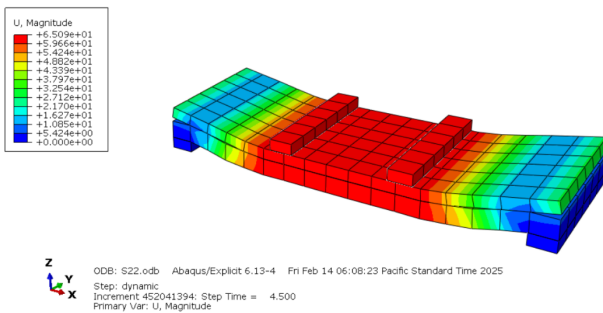


Figure 13. Plot of deflection for sample (S3)
Source: made by D.A. Mawlood, A.A. Koyankin.

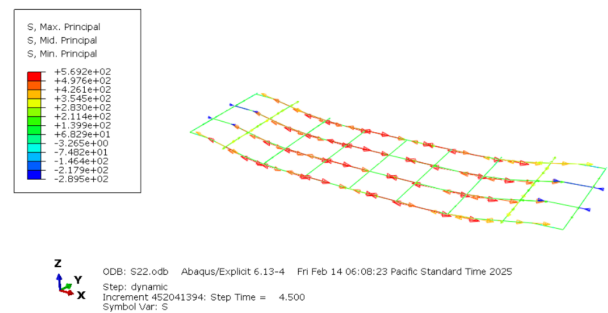


Figure 14. Plot of steel stress for sample (S3).
Source: made by D.A. Mawlood, A.A. Koyankin.

5. Conclusion

1. This study presents a six-node, triangular layered plate finite element model for the nonlinear analysis of semi-precast reinforced concrete slabs.
2. The proposed formulation employs a layer-wise discretization to accurately capture complex through-thickness material behavior.
3. The slab is discretized into multiple concrete layers, with steel reinforcement modeled as equivalent smeared layers positioned at the rebar centroids under a perfect bond assumption.
4. The method directly computes out-of-plane stress components without requiring through-thickness integration and inherently satisfies transverse shear deformation requirements, thereby eliminating the need for shear correction factors. The formulation is derived from a partial mixed variational principle, incorporating both stress and displacement fields within a six-node triangular element.
5. Numerical results demonstrate that the model successfully predicts the ultimate load capacity and failure deflections of semi-precast slabs.
6. Benchmark tests confirm the method's accuracy and computational efficiency, with maximum deflection prediction errors ranging from 0.5 to 4.68% compared to experimental and high-fidelity reference data. The model achieves an excellent balance between accuracy and computational cost.
7. Future research will extend this approach to the materially and geometrically nonlinear analysis of reinforced concrete shell structures within the presented layered framework.

References

1. Tam VWY, Fung IWH, Sing MCP. et al. Best practice of prefabrication implementation in the Hong Kong public and private sectors. *Journal of Cleaner Production*. 2015;109:216–231. <https://doi.org/10.1016/j.jclepro.2014.09.045>

2. Akmam Syed Zakaria S, Gajendran T, Rose T, et al. Contextual, structural and behavioural factors influencing the adoption of industrialised building systems: a review. *Architectural Engineering and Design Management*. 2018;141(2):3–26. <https://doi.org/10.1080/17452007.2017.1291410>
3. Mohamed MS, Thamboo JA, Jeyakaran T. Experimental and numerical assessment of the flexural behaviour of semi-precast-reinforced concrete slabs. *Advances in Structural Engineering*. 2020;23:1865–1879. <https://doi.org/10.1177/1369433220904011> EDN: GDQHBHJ
4. Saiah B, Bachene M, Guemana M, Chiker Y, et al. On the free vibration behavior of nanocomposite laminated plates contained piece-wise functionally graded graphene-reinforced composite plies. *Engineering Structures*. 2022;253:113784. <https://doi.org/10.1016/j.engstruct.2021.113784> EDN: IKFKRC
5. Ashteyat AM, Al Rjoub YS, Obaidat AT, et al. Strengthening and repair of one-way and two-way self-compacted concrete slabs using near-surface-mounted carbon-fiber-reinforced polymers. *Advances in Structural Engineering*. 2019;22(11):2435–2448. <https://doi.org/10.1177/1369433219843649>
6. Mahmood D, Rafiq S, Adbullah M. Nonlinear 3D finite element model for square composite columns under various parameters. *Iraqi Journal of Civil Engineering*. 2022;16(1):19–28. <https://doi.org/10.37650/ijce.2022.172882> EDN: NRXGDK
7. Mawlood D, Rafiq S. Nonlinear 3D finite element model for round composite columns under various eccentricity loads. *Engineering and Technology Journal*. 2022;40(11):1605–1614. <https://doi.org/10.30684/etj.2022.133106.1168> EDN: QTWLPP
8. Wang R, Fang Z, Lezgy-Nazargah M, et al. Nonlinear analysis of reinforced concrete slabs using a quasi-3D mixed finite element formulation. *Engineering Structures*. 2023;294:116781. <https://doi.org/10.1016/j.engstruct.2023.116781> EDN: HKKQXZ
9. Thamboo JA, Dhanasekar M. Characterisation of thin layer polymer cement mortared concrete masonry bond. *Construction and Building Materials*. 2015;82:71–80. <https://doi.org/10.1016/j.conbuildmat.2014.12.098>
10. Le CV, Ho VQ, Ho PLH, et al. Limit state analysis of thin plates and slabs by a numerical pseudo-lower yield design approach. *Thin-Walled Structures*. 2022;172:108852. <https://doi.org/10.1016/j.tws.2021.108852> EDN: JTJDHE
11. Van Vinh P, Avcar M, Belarbi M-O, Tounsi A, Quang Huy L. A new higher-order mixed four-node quadrilateral finite element for static bending analysis of functionally graded plates. *Structures*. 2023;47:1595–1612. <https://doi.org/10.1016/j.istruc.2022.11.113> EDN: GLXCXE
12. Cinefra M, Kumar SK, Carrera E. MITC9 Shell elements based on RMVT and CUF for the analysis of laminated composite plates and shells. *Composite Structures*. 2019;209:383–90. <https://doi.org/10.1016/j.compstruct.2018.10.039> EDN: WUDZCK
13. Amirkhani S, Lezgy Nazargah M. Nonlinear finite element analysis of reinforced concrete columns: Evaluation of different modeling approaches for considering stirrup confinement effects. *Structural Concrete*. 2022;23:2820–2836. <https://doi.org/10.1002/suco.202100532> EDN: BXJDTD
14. Yeganeh-Salman A, Lezgy-Nazargah M. Evaluating the accuracy of mass scaling method in non-linear quasi-static finite element analysis of RC structures. *Structural Engineering and Mechanics*. 2023;85:485–500. <https://doi.org/10.12989/SEM.2023.85.4.485>
15. Carrera E, Ciuffreda A. A unified formulation to assess theories of multilayered plates for various bending problems. *Composite Structures*. 2005;69(3):271–293. <https://doi.org/10.1016/j.compstruct.2004.07.003> EDN: XYMUZZ
16. Liguori FS, Corrado A, Bilotta A, Madeo A. A layer-wise plasticity-based approach for the analysis of reinforced concrete shell structures using a mixed finite element. *Engineering Structures*. 2023;285:116045. <https://doi.org/10.1016/j.engstruct.2023.116045> EDN: XEVDXS
17. Milligan GJ, Polak MA, Zurell C. Finite element analysis of punching shear behaviour of concrete slabs supported on rectangular columns. *Engineering Structures*. 2020;224:111189. <https://doi.org/10.1016/j.engstruct.2020.111189> EDN: RVLDC
18. Abeyasinghe CM, Thambiratnam DP, Perera NJ. Flexural performance of an innovative hybrid composite floor plate system comprising glass–fibre reinforced cement, polyurethane and steel laminate. *Composite Structures*. 2013;95:179–190. <https://doi.org/10.1016/j.compstruct.2012.06.019>
19. Adawi A, Youssef MA, Meshaly ME. Finite element modeling of the composite action between hollowcore slabs and the topping concrete. *Engineering Structures*. 2016;124:302–315. <https://doi.org/10.1016/j.engstruct.2016.06.016>
20. Benayoune A, Samad AAA, Trikha DN, et al. Flexural behaviour of pre-cast concrete sandwich composite panel — Experimental and theoretical investigations. *Construction and Building Materials*. 2008, Vol. 22(4):580–592. <https://doi.org/10.1016/j.conbuildmat.2006.11.023>
21. Elsanadedy HM, Almusallam TH, Alsayed SH, et al. Experimental and FE study on RC one-way slabs upgraded with FRP composites. *KSCE Journal of Civil Engineering*. 2015;19(4):1024–1040. <https://doi.org/10.1007/s12205-013-0689-y> EDN: GKSFJQ
22. Liu F, Battini J-M, Pacoste C. Finite-shell-element models for the dynamic analysis of hollow-core concrete floor. *Magazine of Concrete Research*. 2019;71(10):519–532. <https://doi.org/10.1680/jmacr.17.00480>

23. Yu Y, Wei B, Yang Y, et al. Experimental study on flexural performance of steel-reinforced concrete slim floor beams. *Advances in Structural Engineering*. 2019;22(11):2406–2417. <https://doi.org/10.1177/1369433219841917>
24. Werkle H. *Finite elements in structural analysis: Theoretical concepts and modeling procedures in statics and dynamics of structures*. Cham: Springer International Publ.; 2021. p. 191–295. <https://doi.org/10.1007/978-3-030-49840-5>
25. Oñate E. *Structural analysis with the finite element method: linear statics*. Dordrecht London: Springer; 2009. p. 158–186. ISBN 978-1-4020-8742-4
26. Khennane A. *Introduction to finite element analysis using MATLAB and abaqus*, 1st ed. Baton Rouge: Taylor & Francis Group; 2013. p. 231–379. <https://doi.org/10.1201/b15042>
27. Timošenko SP, Woinowsky-Krieger S. *Theory of plates and shells*. 2. ed., [Nachdr.]. New York: McGraw-Hill; 1996. Available from: <https://djvu.online/file/1M8cQRIBat6pU> (accessed: 27.03.2026).
28. Mittelstedt C. *Basics of elasticity theory. Theory of plates and shells*. Berlin, Heidelberg: Springer Berlin Heidelberg; 2023. p. 3–57. https://doi.org/10.1007/978-3-662-66805-4_1
29. Willam K, Pramono E, Sture S. Fundamental issues of smeared crack models. In: Shah SP, Swartz SE, editors. *Fracture of Concrete and Rock*, New York, NY: Springer New York; 1989. p. 142–157. https://doi.org/10.1007/978-1-4612-3578-1_15
30. Szilard R. *Theories and applications of plate analysis: classical, numerical, and engineering methods*. Hoboken, NJ: John Wiley; 2004. p. 662–667. <https://doi.org/10.1002/9780470172872>
31. Mawlood DA, Koyankin AA. Triangular layered finite element method for reinforced concrete slabs. *Structural Mechanics of Engineering Constructions and Buildings*. 2025;21(5), pp. 441–61. <https://doi.org/10.22363/1815-5235-2025-21-5-441-461> EDN: DEEXQA