

РАСЧЕТ И ПРОЕКТИРОВАНИЕ СТРОИТЕЛЬНЫХ КОНСТРУКЦИЙ ANALYSIS AND DESIGN OF BUILDING STRUCTURES

DOI: 10.22363/1815-5235-2026-22-3-253-266

EDN: LGPSQE

Research article / Научная статья

Rational Reinforcement of Continuous Three-Span Timber Beams

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Received: February 4, 2026

Revised: March 15, 2026

Accepted: March 20, 2026

Abstract. Reinforced timber structures, particularly continuous beams, offer a number of advantages that contribute to their widespread use. The economic benefits of rational reinforcement of the cross-section of multi-span reinforced timber beams in terms of material consumption, as well as the lack of similar known contemporary works, determine the relevance of the research topic. The object of the study is a three-span reinforced timber beam with equal spans, loaded with a uniformly distributed load. To achieve the goal of the study, the following tasks were solved: a review of existing methods for calculating flexural reinforced timber elements and the selection of a calculation methodology for the studied structure; an analysis of the distribution of bending moments along the beam length and the development of design solutions; and the selection of a rational option, with the relative consumption of reinforcement serving as the efficiency criterion. Design solutions with symmetrical and asymmetrical reinforcement with the placement of bars along the entire length of the beam and part of its length were considered. It was found that the difference in bending moments in the middle span and the design moments at the supports allows to have no reinforcement in a part of the middle span. Furthermore, it is possible to reduce the cross-section dimensions. The most efficient reinforcement option for a three-span beam is symmetrical reinforcement, with the reinforcement placed along part of the beam's length.

Keywords: continuous timber beams, reinforced timber structures, increased strength and rigidity, rational reinforcement

Authors' contribution: Repin V.A. — conceptualization and methodology; Martinov V.A. — text draft, review and editing; Popova M.V. — literature review and interpretation of results; Chibrikina D.A. — calculations and analysis of results. The authors read and approved the final version of the article.

Conflicts of interest. The authors declare that there is no conflict of interest.

Funding. The study was conducted under the state assignment in the field of scientific activity of the Ministry of Science and Higher Education of the Russian Federation (topic FZUN-2024-0004, state assignment of the All-Russian State University).

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Statement on the use of artificial intelligence technologies. No generative artificial intelligence technologies were used in the creation of this article.

Data availability. All data obtained during this study are included in the article being published.

For citation: Repin VA, Martinov VA, Popova MV, Chibrikin DA. Rational reinforcement of continuous three-span timber beams. *Structural Mechanics of Engineering Constructions and Buildings*. 2026;22(3):253–266. <http://doi.org/10.22363/1815-5235-2026-22-3-253-266> EDN: LGPSQE

Рациональное армирование неразрезных трехпролетных деревянных балок

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Поступила в редакцию: 4 февраля 2026 г.

Доработана: 15 марта 2026 г.

Принята к публикации: 20 марта 2026 г.

Аннотация. Армированные деревянные конструкции, в частности неразрезные балки, обладают рядом достоинств, способствующих их широкому применению. Экономический эффект от рационального армирования сечения многопролетных армированных деревянных балок с точки зрения материалоемкости, а также отсутствие аналогичных современных известных работ обуславливают актуальность тематики исследования. Объектом исследования является трехпролетная армированная деревянная балка с равными пролетами, загруженная равномерно распределенной нагрузкой. Для достижения цели исследования решены следующие задачи: обзор существующих методов расчета изгибаемых армированных деревянных элементов и выбор методики расчета исследуемой конструкции; анализ характера распределения изгибающих усилий по длине балки и формирование вариантов конструктивных решений; выбор рационального варианта, критерием эффективности при этом являлся относительный расход арматуры. Были рассмотрены конструктивные решения с симметричным и несимметричным армированием с размещением стержней по всей длине балки и на части ее длины. Установлено, что разница изгибающих усилий в среднем пролете и расчетных усилий на опорах позволяет выполнять часть среднего пролета без армирования. Кроме того, существует возможность уменьшать габариты сечения. Наиболее рациональный вариант армирования трехпролетной балки — симметричное армирование с размещением арматуры на части длины балки.

Ключевые слова: неразрезные деревянные балки, армированные деревянные конструкции, повышение прочности и жесткости, рациональное проектирование

Вклад авторов: *Репин В.А.* — концепция исследования и разработка методики расчета; *Мартынов В.А.* — подготовка, редактирование и оформление текста статьи; *Попова М.В.* — анализ литературы и интерпретация результатов; *Чибрикин Д.А.* — выполнение расчетов и анализ результатов. Авторы ознакомлены с окончательной версией статьи и одобрили ее.

Заявление о конфликте интересов. Авторы заявляют об отсутствии конфликта интересов.

Финансирование. Работа выполнена в рамках государственного задания в сфере научной деятельности Министерства науки и высшего образования Российской Федерации (тема FZUN-2024-0004, госзадание ВлГУ).

Заявление об использовании технологий искусственного интеллекта. При создании настоящей статьи технологии генеративного искусственного интеллекта не использовались.

Заявление о доступности данных. Все данные, полученные в ходе этого исследования, включены в публикуемую статью.

Для цитирования: *Репин В.А., Мартынов В.А., Попова М.В., Чибрикин Д.А.* Рациональное армирование неразрезных трехпролетных деревянных балок // *Строительная механика инженерных конструкций и сооружений*. 2026. Т. 22. № 3. С. 253–266. <http://doi.org/10.22363/1815-5235-2026-22-3-253-266> EDN: LGPSQE

1. Introduction

Load-bearing timber structures are widely used in buildings due to their unique physical and mechanical properties [1; 2].

Continuous multi-span beams have more economical cross-sections compared to single-span beams [3; 4]. In most cases, the cross-section of a multi-span beam is determined by ultimate limit states, and checking such structures against serviceability limit states is not considered relevant.

Due to their substantial dimensions, continuous structures should be divided into pre-fabricated elements in accordance with transportability requirements, and then assembled on-site into a single unit [5]. The connection between the elements can be either rigid or hinged, depending on the nature of the forces acting on it [6].

In the study by Kliger R. et al. [7], when considering the history of development of strengthening methods for timber beams, it is noted that one of the first solutions in this area was the application of steel bars glued into wood at different angles by H. Granholm in 1944. Research conducted in South Korea (Jung H.J. et al.) demonstrated the high effectiveness of using high-strength fiberglass plates in the reconstruction of damaged load-bearing timber elements [8]. In the work by Italian scientists [9], it was established that steel-reinforced timber beams require special measures to ensure flexibility in bending, because irregularities in the material cause brittle fracture of the wood in tension before the steel bars can deform. The article by Polish researchers [10] presented the results of tests conducted on solid timber beams made of new and old wood, which were subjected to bending after being reinforced with steel plates and epoxy glue. Scientists from China [11] describe an experimental testing program and theoretical analysis examining the reinforcement of glued-laminated timber beams in bending using fiber-reinforced polymer and steel materials. The focus of this study was to evaluate the influence of reinforcing materials, reinforcement ratio, and arrangement on the bending behavior.

Currently, there are numerous design solutions for beam splices using various types of joints [12]. In Russia, the development of this concept has proceeded along two lines of research. The first is the work of S.B. Turkovsky on methods and technologies for gluing inclined bars, which ensured the combined resistance of the layers of glued beam elements in building structures (TsNIISK system) [13; 14]. Currently, this system is widely used in prefabricated long-span timber structures. The technology progressed further in [15] through the development of methods for increasing the shear strength of timber structures using inclined glued connections. The second line of research, implemented in the works of V.Yu. Shchuko and S.I. Roschina [16; 17], is related to the application of longitudinal reinforcement bars in wood [18; 19] by the creation of grooves in the tension and compression zones of beams, followed by gluing of hot-rolled ribbed steel rebars into them [20; 21]. In their patent, V.I. Skribo, G.M. Shutova, and E.B. Shalkevich propose a design solution for a timber beam with reinforcement of glued laminated timber, the longitudinal grain of which is oriented perpendicular to the beam axis; this approach allows for the use of low-grade timber in load-bearing structures [22]. E.N. Aleksashkin, V.V. Veselov, and V.V. Egorov examined a repair method and a design solution for a timber beam, in which the rebars are arranged along a path corresponding to the principal normal stresses in the beam [23; 24]. Due to the placement of the reinforcement deep within the body of the timber beam, this technology is used in strengthening and fabrication of glued-laminated structures. No particular difficulties arise with the large-scale assembly of such structures.

The aim of this study is to determine the rational reinforcement arrangement for a three-span timber beam, such that the rigidity and strength of the structure are increased, while the amount of reinforcement used is minimized [25; 26]. This will ensure the most efficient utilization of the strength properties of reinforcement material [27–29].

To achieve this aim, the following objectives were addressed:

- develop design options for beams based on the arrangement of reinforcement, taking into account the distribution of bending moments along the length of the structure;
- define a criterion for evaluating the effectiveness of the considered options;
- identify the most effective options based on the bending strength condition by solving the problem in a general parametric form.

The following assumptions have been made:

- all options have the same strength;
- the cross-sectional dimensions and strength properties of timber (wood species, grade) are assumed to be the same for all options;

– reinforcement break in asymmetrically reinforced structures is assumed without accounting for the length of anchorage zones, i.e., without overlap of compression and tension bars at their break point.

The use of reinforcement in timber results in savings in the base structural material, increased strength and rigidity of the beams, and improved operational reliability. The economic benefits of rational cross-sectional reinforcement in terms of material consumption, as well as the lack of similar contemporary studies on this topic, determine the relevance of this research [30; 31].

2. Methods

The research process was conducted according to standard methodology. The first stage involved a review of existing methods for calculating flexural reinforced timber members, based on which a calculation method for the structure under study was selected. The next stage involved analyzing the distribution of bending moments along the length of the beam and developing various layout options for the structure. The third stage involved selecting the rational option.

The intensive development of experimental reinforced timber beams and their application in pilot construction projects [32–34], as well as further research into their stress-strain behavior, necessitates the refinement of design methods for such structures [35; 36].

Currently, the following analysis methods are primarily used in the design of reinforced timber elements in bending [37]:

- based on transformed geometric characteristics of the cross-sections;
- taking into account the elastic compliance of the adhesive bond between the reinforcement and the wood;
- taking into account the elastic properties of the materials and the compliance of the steel-wood adhesive bond.

The first method is based on the idealized diagram of elastic-plastic behavior of wood, assuming the hypothesis of plane sections and that the steel-wood adhesive bond ensures joint action ($\varepsilon_w = \varepsilon_r$) throughout the entire service life of the structure, up to failure.

The second method is a modification of the transformed-section analysis method. The following basic design assumptions were adopted in developing the method:

- the wood subjected to tensile and compressive forces along the grain is treated as an isotropic material;
- anisotropy and flexural rigidity of the reinforcement are not taken into account;
- the materials of the composite structure operate within elastic limit, assuming the plane-sections hypothesis;
- the compliance of the adhesive bond between the reinforcement and the wood is expressed by the difference in displacements B_r and B_w .

The third method is based on the following premises and assumptions:

- all materials in the composite structure, including the adhesive, obey Hooke's law;
- Poisson's ratio of the rebars is zero;
- flexural rigidity of the rebars is not taken into account;
- the rebars are placed strictly along the axes of the channels;
- in the base material (wood), the law of plane sections is not violated, with the exception of the zones surrounding the rebars;
- shearing angles γ_a for the adhesive layer and γ_b for the base material in the zones adjacent to the rebars in the section under consideration are assumed to be constant;
- the zones of warping of the wood cylinders must not intersect.

A common shortcoming of the considered analysis methods is the assumption that the influence of shear forces on the strength and deformability of timber structures is not taken into account. An attempt was

made to account for the combined effect of normal and shear stresses on the stress state of structures; however, the accepted theory did not account for the strength anisotropy of glued laminated timber, which affects the stress state of flexural timber elements [38; 39].

The object of this study is a three-span reinforced timber beam with equal spans, subjected to a uniformly distributed load. The model of the structure is shown in Fig. 1. The cross-section is assumed to be rectangular. The reinforcement is to be provided by steel bars of grades A400 and A500.

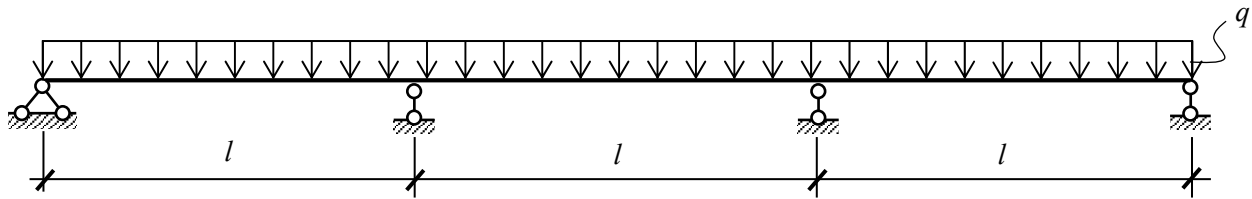


Figure 1. Model of the studied structure

Source: made by V.A. Repin.

To calculate the three-span continuous beam under study, the analysis method based on transformed geometric characteristics will be applied. The transformation factor for the reinforcement material relative to the base material is defined as the ratio of the moduli of elasticity of the reinforcement (E_r) to that of the wood (E_w). It is a constant value:

$$n = \frac{E_r}{E_w} = \frac{2 \cdot 10^6 \text{ MPa}}{10^5 \text{ MPa}} = 20. \quad (1)$$

Thus, the geometric characteristics of the transformed cross-section will take the form:

■ area:

$$F_{tr} = F_w + nF_r; \quad (2)$$

■ moment of inertia:

$$I_{tr} = I_w^2 + nI_r; \quad (3)$$

■ first moment of area:

$$S_{tr} = S_w + nS_r. \quad (4)$$

Expression (2) can be written as follows:

$$F_{tr} = F_w(1 + nF_r/F_w) = F_w(1 + n\mu), \quad (5)$$

where $\mu = \frac{F_r}{F_w}$ is the reinforcement ratio; $F_w = bh$ is the unreinforced cross-section area of the element;

$F_r = n_r \cdot \pi d_r^2 / 4$ is the cross-sectional area of the reinforcement (n_r is the number of rebars; d_r is the rebar diameter); I_w is the moment of inertia of the unreinforced cross-section; I_r is the moment of inertia of the reinforcement; S_w is the first moment of area of the unreinforced cross-section; S_r is the first moment of area of the reinforcement.

Reinforcement of timber members is typically placed in the tensile zone of the wood fibers. This approach is commonly referred to as asymmetric reinforcement (Figure 2, a). However, symmetric reinforcement of the cross-section (Figure 2, b), in which reinforcement is also placed in the compression zone, is equally effective.

The geometric characteristics of an asymmetrically reinforced cross-section (see Figure 2, *a*) are determined taking into account the fact that the neutral axis does not coincide with the axis of symmetry of the cross-section, and the position of the cross-sectional center of gravity relative to an arbitrary axis z' is calculated using the following formula:

$$y_{c.g.} = \frac{F_w y_w + nF_r y_r}{F_w + nF_r} = \frac{bh\left(-\frac{h}{2}\right) + nF_r(-h_o)}{bh(1+n\mu)} = -\frac{h + 2n\mu h_o}{2(1+n\mu)}. \quad (6)$$

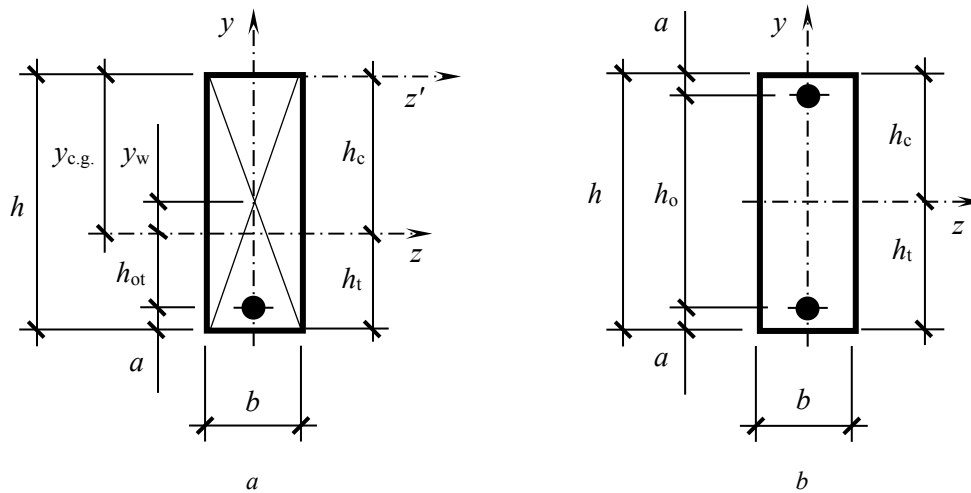


Figure 2. Cross-sections of reinforced timber beams:
a — with asymmetric reinforcement; *b* — with symmetric reinforcement
 Source: made by V.A. Repin.

From this, the dimensions of the compression and tension zones of the section, h_c and h_t respectively, can be determined:

$$h_c = |y_{c.g.}| = \frac{h + 2n\mu h_o}{2(1+n\mu)}; \quad h_t = h - h_c = \frac{h + 2n\mu a}{2(1+n\mu)}. \quad (7)$$

The coordinates of the centers of gravity of the unreinforced cross-section and the cross-section of the reinforcement:

$$y_w = \frac{h}{2} - h_p = \frac{n\mu(h_o - a)}{2(1+n\mu)}; \quad h_{ot} = h_p - a = \frac{h_o - a}{2(1+n\mu)}. \quad (8)$$

The transformed moment of inertia of the asymmetrically reinforced cross-section relative to the neutral axis:

$$I_{tr}^n = I_w' + F_w y_w^2 + nF_r h_{ot}^2 = \frac{bh^3}{12} + bh \left(\frac{n\mu(h_o - a)}{2(1+n\mu)} \right)^2 + nF_r \left(\frac{h_o - a}{2(1+n\mu)} \right)^2 = \frac{bh^3}{12} \left[1 + \frac{3n\mu}{1+n\mu} \left(\frac{2h_o}{h} - 1 \right)^2 \right], \quad (9)$$

where I_{tr} is the transformed moment of inertia of the asymmetrically reinforced cross-section relative to the neutral axis.

The transformed section modulus of the asymmetrically reinforced cross-section relative to the neutral axis:

$$W_{tr} = I_{tr} : \frac{h}{2} = \frac{bh^2}{6} \left[1 + \frac{3n\mu}{1+n\mu} \left(\frac{2h_o}{h} - 1 \right)^2 \right]. \quad (10)$$

The transformed first moment of area of the sheared part of the asymmetrically reinforced cross-section relative to the neutral axis at $h_{trunc} = h_c$: where h_{trunc} is the height of the truncated part of the section from the top edge of the element to the neutral axis.

$$S_{tr}^n = \frac{bh_c^2}{2} = \frac{b}{2} \left(\frac{h + 2n\mu h_o}{2(1+n\mu)} \right)^2 = \frac{bh^2}{8} \left(\frac{1 + 2n\mu(h_o/h)}{1+n\mu} \right)^2; \quad (11)$$

$$S_{tr}^r = nF_r h_{ot} = \frac{(h_o - a)nF_r}{2(1+n\mu)} = \frac{bh^2}{8} \left(\frac{4n\mu}{(1+n\mu)} \left(\frac{2h_o}{h} - 1 \right) \right), \quad (12)$$

where S_{tr}^n is the transformed first moment of area of the sheared part of the asymmetrically reinforced cross-section relative to the neutral axis; S_{tr}^r is the transformed first moment of area of the reinforcement of the asymmetrically reinforced cross-section relative to the neutral axis.

The symmetrically reinforced cross-section (see Fig. 2, *b*) is characterized by the fact that its center of gravity coincides with the center of gravity of the unreinforced cross-section. In this case, its geometric characteristics are as follows:

- transformed moment of inertia:

$$I_{tr} = I_w' + nF_r \left(\frac{h_o}{2} \right)^2 = \frac{bh^3}{12} \left[1 + 3n\mu \left(\frac{h_o}{h} \right)^2 \right]; \quad (13)$$

- transformed section modulus:

$$W_{tr} = I_{tr} : \frac{h}{2} = \frac{bh^2}{6} \left[1 + 3n\mu \left(\frac{h_o}{h} \right)^2 \right]; \quad (14)$$

- transformed first moment of area of the sheared part of the cross-section:

$$S_{tr} = S_w' + nF_r \frac{h_o}{4} = \frac{bh^2}{8} \left(1 + 2n\mu \frac{h_o}{h} \right). \quad (15)$$

- transformed first moment of area of the reinforcement:

$$S_{tr}^r = nF_r \frac{h_o}{4} = \frac{bh^2}{8} \left(2n\mu \frac{h_o}{h} \right), \quad (16)$$

where n is the transformation coefficient of the reinforcement material to the base material (1); h_o is the distance between the centers of the cross-sections of the rebars in the tension and compression zones; b is the width of the cross-section; μ is the reinforcement ratio; $I_w = \frac{bh^3}{12}$ is the moment of inertia of the unreinforced section; $S_w = \frac{bh^2}{8}$ is the first moment of area of the unreinforced section; F_r is the total cross-sectional area of the reinforcement.

For timber structural members, including beams, the reinforcement ratio is generally determined as follows:

- for asymmetric reinforcement $\mu = 0.012 \dots 0.025$ (1.2...2.5 %);
- for symmetric reinforcement $\mu = 0.012 \dots 0.035$ (1.2...3.5 %).

The expressions in formulas (9) and (13) enclosed in square brackets can be denoted as:

$$\beta_{\text{bear}} = 1 + \frac{3n\mu}{1+n\mu} \left(\frac{2h_o}{h} - 1 \right)^2; \quad (17)$$

$$\beta_{\text{sym}} = 1 + 3n\mu \left(\frac{h_o}{h} \right)^2. \quad (18)$$

Expressions (17) and (18) reflect the increase in rigidity and strength of the flexural element, which is achieved by the reinforcement.

In that case, the geometric characteristics of the cross-section of the reinforced timber structure can be expressed as follows:

$$I_{tr} = \beta \cdot I_w; \quad W_{tr} = \beta \cdot W_w, \quad (19)$$

where I_{tr} , I_w are the moments of inertia of the reinforced and unreinforced sections, respectively;

W_{tr} , W_w are the section moduli of the reinforced and unreinforced sections, respectively.

Based on the results of the calculations, the increase in the beam rigidity due to reinforcement will be approximately:

- for asymmetric reinforcement — by a factor of 1.3...1.9;
- for symmetric reinforcement — by a factor of 1.5...2.8.

It should be noted that, in some cases, the rigidity and strength of the reinforced beam can be up to twice that of the unreinforced beam; this holds true for reinforcement ratio $\mu = 2\%$. This fact determines the rationality of reinforcing timber structures.

Since the reinforcing bars essentially serve as the primary load-bearing fibers located in the most stressed areas of the cross-section, it is possible to use 3rd-grade timber in the construction of reinforced timber structures instead of the more expensive 1st- and 2nd-grade timber without compromising structural performance.

The analysis of such structures subjected to normal stresses in the reinforcement and wood, and shear stresses in the wood and adhesive joint, as well as the check for deformability, is performed in accordance with the requirements of SP 64.13330.2017¹ using the formulas of strength of materials.

The bending moment diagram shown in Fig. 3, *a*, demonstrates that:

- the maximum moments are observed at the intermediate supports ($M_{\text{sup}} = 0.1ql^2$);
- in the end spans, the moments are close in magnitude to the support moments ($M_{\text{end}} = 0.08ql^2$);
- in the middle span, the moment is one-fourth that of the end spans ($M_{\text{mid}} = 0.025ql^2$).

Based on this, it is necessary to identify characteristic regions within which the moments act in the same direction, which allows to assign a specific type of reinforcement to each individual region.

The boundaries of these regions are defined by points where the moments are zero. It is reasonable to design reinforcement breaks and longitudinal joints of pre-fabricated structural elements at these specific points.

The minimum bending moment (four times lower than the design value) occurs in region III, which allows to consider a scenario without reinforcement in that region.

¹ SP 64.13330.2017. *Timber structures. Revised edition of SNiP II-25-80*. Scientific Research Center “Stroitelstvo” — V.A. Kucherenko Central Scientific Research Institute for Building Structures. Moscow, 2017.

In this regard, the following beam reinforcement options have been developed:

- symmetric along the entire length (see Figure 3, *b*);
- symmetric along part of the length (see Figure 3, *c*);
- asymmetric along the entire length in tension zones (see Figure 3, *d*);
- asymmetric along part of the length (see Figure 3, *e*).

The criterion for effectiveness when selecting the most efficient option is the relative consumption of reinforcement:

$$\mu_L = \mu \cdot L_{\text{reinf}} / L_{\text{beam}}, \quad (20)$$

where $\mu = \frac{F_a}{F_\sigma}$ is the reinforcement ratio; L_{reinf} is the total length of regions with reinforcement (I, II, IV and V); $L_{\text{beam}} = 3l$ is the total beam length.

The shape and dimensions of the cross-section, as well as load q , are specified to be the same for all options.

In the first step, the required reinforcement ratio is determined based on the bending strength condition. Parameter β , which characterizes the increase in rigidity and strength, is expressed as:

$$\beta \geq \frac{M_{\max}}{W_w R_b}. \quad (21)$$

Next, by substituting expressions (17) and (18) into expression (21), the formulas for the required reinforcement ratios for asymmetrically and symmetrically reinforced cross-sections, at which they will have equal strength and rigidity, are obtained:

$$\mu_{\text{req}}^{\text{bear}} \geq \frac{\frac{M_{\max}}{W_w R_b} - 1}{3n \left(\frac{2h_o}{h} - 1 \right)^2 - n \left(\frac{M_{\max}}{W_w R_b} - 1 \right)}; \quad (22)$$

$$\mu_{\text{req}}^{\text{sym}} \geq \frac{\frac{M_{\max}}{W_w R_b} - 1}{3n \left(\frac{h_o}{h} \right)^2}, \quad (23)$$

where $M_{\max}$ is the design value of the bending moment (in this case $M_{\max} = M_{\text{sup}} = 0.1ql^2$); $W_w = \frac{bh^2}{6}$ is the section modulus of the unreinforced section; R_b is the design bending strength of timber; n is the transformation coefficient (1).

Expressions (22) and (23) can be written in more compact form:

$$\mu_{\text{req}}^{\text{bear}} \geq \frac{\Psi}{n(3\Delta h_{\text{bear}}^2 - \Psi)}; \quad (24)$$

$$\mu_{\text{req}}^{\text{sym}} \geq \frac{\Psi}{3n\Delta h_{\text{sym}}^2}, \quad (25)$$

where: $\Psi = \frac{M_{\max}}{W_w R_b} - 1$; $\Delta h_{\text{bear}} = \frac{2h_o^{\text{bear}}}{h} - 1$; $\Delta h_{\text{sym}} = \frac{h_o^{\text{sym}}}{h}$; $h_o^{\text{bear}} = h - a$; $h_o^{\text{sym}} = h - 2a$; $a = d_a / 2 - 25 \text{ mm}$.

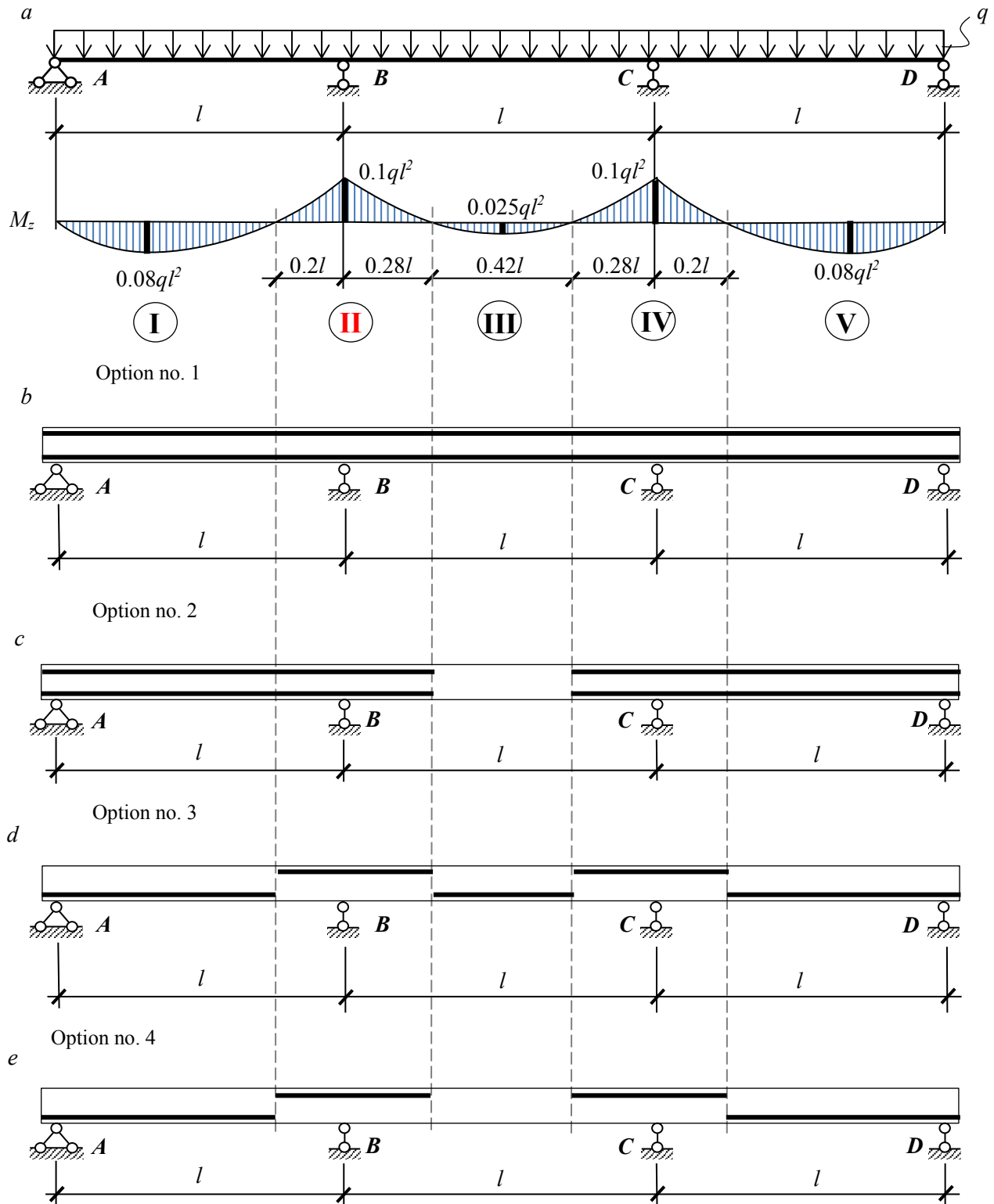


Figure 3. Calculated model and reinforcement options for a continuous three-span timber beam:

a — calculated model; b — symmetrical reinforcement of the section along the entire length;
 c — symmetrical reinforcement of the section along part of the length; d — asymmetrical reinforcement of the section along the entire length;
 e — asymmetrical reinforcement of the section along part of the length

Source: made by V.A. Repin.

3. Results and Discussion

The expression for the relative reinforcement consumption for the options with asymmetric and symmetric reinforcement along part of the length (options No. 2 and No. 4) takes the form:

$$\mu_L^{\text{bear}} = \mu_{\text{req}}^{\text{bear}} L_{\text{reinf}} / L_{\text{beam}}; \quad \mu_L^{\text{sym}} = \mu_{\text{req}}^{\text{sym}} L_{\text{reinf}} / L_{\text{beam}}. \quad (26)$$

For cases with reinforcement along the entire length of the beam (options No. 1 and No. 3), the value of μ_L will be numerically equal to the corresponding reinforcement ratios:

$$\mu_L^{\text{bear}} = \mu_{\text{req}}^{\text{bear}}; \quad \mu_L^{\text{sym}} = \mu_{\text{req}}^{\text{sym}} \quad \mu_L^{\text{сим}} = \mu_{\text{тр}}^{\text{сим}}. \quad (27)$$

It follows that the options with partial-length reinforcement (No. 2 and No. 4) are more economical, since in these cases the ratio $L_{\text{reinf}}/L_{\text{beam}}$ is less than one; more precisely: $L_{\text{reinf}}/L_{\text{beam}} = (3l - 0.42l)/(3l) = 0.86$. That is, the effect of this option is approximately 14%.

If necessary, the cross-sectional dimensions in the middle span can be reduced, as the safety factor is automatically at least 25% ($M_{\text{sup}} > M_{\text{mid}}$ by a factor of 4, and $\beta < 3$). This further enhances the efficiency of this design solution.

Since the total length of the reinforced regions L_{reinf} is the same in options No. 2 and No. 4, their effectiveness will be determined by the value of the reinforcement ratio. This fact can be established from the relationship between these values:

$$\frac{\mu_{\text{req}}^{\text{sym}}}{\mu_{\text{req}}^{\text{bear}}} = \frac{\psi n (3\Delta h_{\text{bear}}^2 - \psi)}{3n\Delta h_{\text{sym}}^2 \psi} = \frac{3\Delta h_{\text{bear}}^2 - \psi}{3\Delta h_{\text{sym}}^2}. \quad (28)$$

The expressions for Δh_{bear} and Δh_{sym} can be written as:

$$\Delta h_{\text{bear}} = \frac{2h_o^{\text{bear}}}{h} - 1 = \frac{2(h-a)}{h} - 1 = 1 - \frac{2a}{h}; \quad (29)$$

$$\Delta h_{\text{sym}} = \frac{h_o^{\text{sym}}}{h} = \frac{h-2a}{h} = 1 - \frac{2a}{h}. \quad (30)$$

It follows from expressions (29) and (30) that the values of Δh_{bear} and Δh_{sym} are equal, and a single parameter Δh can be used in their place in expressions (24) and (25):

$$\Delta h = 1 - \frac{2a}{h}. \quad (31)$$

Then, expression (28) will take the following form:

$$\frac{\mu_{\text{req}}^{\text{sym}}}{\mu_{\text{req}}^{\text{bear}}} = \frac{3\Delta h^2 - \psi}{3\Delta h^2} = 1 - \frac{\psi}{3\Delta h^2}. \quad (32)$$

By analyzing expression (32), it is clear that the result of the calculation must be non-negative; otherwise, it would be meaningless. A positive result, however, will be less than one or will approach zero:

$$0 < \frac{\mu_{\text{req}}^{\text{sym}}}{\mu_{\text{req}}^{\text{bear}}} < 1. \quad (33)$$

This means that, in any case, the value of coefficient $\mu_{\text{req}}^{\text{sym}}$ will not exceed the value of $\mu_{\text{req}}^{\text{bear}}$, i.e., the option with symmetric reinforcement will require either less rebar than the asymmetric option or virtually the same amount:

$$\mu_L^{\text{sym}} \leq \mu_L^{\text{bear}}. \quad (34)$$

It follows that the most rational option is No. 2 (see Figure 3, *c*), which features symmetric reinforcement of the cross-section — with reinforcement placed along part of the beam length, or more precisely, along its entire length except for region III. The exact value of the efficiency factor is determined by calculation, as it depends on a number of variables. However, this index does not account for the influence of shear stresses on the load-bearing capacity of the beams; therefore, when designing three-span reinforced timber beams, it is necessary to check the support sections against shear forces using formulas (21).

4. Conclusion

1. Theoretical study of continuous three-span reinforced timber beams, focusing on identifying the most rational reinforcement arrangement, was conducted. The criterion for efficiency was the relative reinforcement consumption, defined as the product of the reinforcement ratio of the section and the ratio of the total length of the reinforced regions of the beam to the total beam length: $\mu_L = \mu_{L_{\text{reinf}}} / L_{\text{beam}}$.

2. It has been determined that the difference between the bending moments in the middle span and the design moments at the supports allows part of the middle span to be constructed without reinforcement. In addition, it is possible to reduce the cross-sectional dimensions, since the minimum safety factor for the unreinforced regions is more than 25% — at the maximum reinforcement ratio.

3. The savings in steel when using partial-length reinforcement of the beams (i.e., with an unreinforced region in the middle span) amount to approximately 14% compared to structures with reinforcement along their entire length.

4. The most rational way to reinforce a three-span beam is to use symmetric reinforcement, placing the rebar along a part of the beam length.

5. The results of the study showed that the grade and species of timber do not affect the performance of the design options, as performance depends solely on the reinforcement ratio of symmetric and asymmetric cross-sections. However, more accurate performance indices for the best option can be estimated by analysing the structures using only specific parameter values.

6. A future direction for this line of research is the study of three-span reinforced timber beams subjected to various loading schemes that may occur over the service life of structure.

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