СТРОИТЕЛЬНАЯ МЕХАНИКА ИНЖЕНЕРНЫХ КОНСТРУКЦИЙ И СООРУЖЕНИЙ
STRUCTURAL MECHANICS OF ENGINEERING CONSTRUCTIONS AND BUILDINGS
2026. 22(2). 138–151
ISSN 1815-5235 (Print), 2587-8700 (Online)
HTTP://JOURNALS.RUDN.RU/STRUCTURAL-MECHANICS

РАСЧЕТ ТОНКИХ УПРУГИХ ОБОЛОЧЕК
ANALYSIS OF THIN ELASTIC SHELLS

DOI: 10.22363/1815-5235-2026-22-2-138-151
EDN: KCJJXB

Research article / Научная статья

Comparative Analysis of Calculation of a Plate of Curvilinear Trapezoidal Plan using Numerical Methods

Vyacheslav N. Ivanov, **Evgenia M. Tupikova**, **Marina I. Rynkovskaya**✉

RUDN University, *Moscow, Russian Federation*
✉ rynkovskaya-mi@rudn.ru

Received: November 30, 2025
Revised: February 12, 2026
Accepted: February 25, 2026

Abstract. Roofs in the form of plates and shells of complex curvilinear plan are common structural solutions in architecture. Such structures have a number of advantages. The mid-surface of shells and plates of curvilinear trapezoidal plan is constructed using parametric and vector equations and has a number of special aspects to consider when calculating their stress-strain state. For structures of this shape, no exact analytical solution has been obtained, but it is possible to obtain a numerical solution, for example, by the finite element method and the variational-difference method. In such a situation, for verification of calculations, comparing the results obtained using different numerical procedures is useful and relevant. A comparative analysis of the results of calculating the stress-strain state of a plate curvilinear in plan, obtained by the methods mentioned above, was conducted. In the literature, the topic of calculating plates and shells of curvilinear trapezoidal plan is insufficiently developed. The aim of the study is to obtain data on the calculation of the stress-strain state of a plate of curvilinear trapezoidal plan, as well as to assess the applicability and specifics of the two methods in calculating such structures. To accomplish the tasks, the following software was used: ANSYS APDL software for calculation by the finite element method, and the author-developed SHELLVRM program for calculation by the variational-difference method. The parameters of the stress-strain state of a plate of curvilinear trapezoidal plan have been obtained and analyzed, verification of the obtained results has been carried out, recommendations for implementing both calculation methods in the practice of structural analysis have been given, and computational difficulties and special aspects of both methods have been identified.

Keywords: thin elastic plate, node mesh, VDM, FEM, stress parameters, displacements

Authors' contribution: *Ivanov V.N.* — supervision, conceptualization, validation; *Tupikova E.M.* — software, visualization, text draft; *Rynkovskaya M.I.* — study management, data processing, text writing, review and editing. The authors read and approved the final version of the article.

Conflicts of interest. The authors declare that there is no conflict of interest.

For citation: Ivanov VN, Tupikova EM, Rynkovskaya MI. Comparative analysis of calculation of a plate of curvilinear trapezoidal plan using numerical methods. *Structural Mechanics of Engineering Constructions and Buildings.* 2026;22(2):138–151. http://doi.org/10.22363/1815-5235-2026-22-2-138-151 EDN: KCJJXB

Vyacheslav N. Ivanov, Doctor of Technical Sciences, Professor of the Department of Construction Technology and Structural Materials, Engineering Academy, RUDN University, 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation; eLIBRARY SPIN-code: 3110-9909, ORCID: 0000-0003-4023-156X; e-mail: i.v.ivn@mail.ru

Evgenia M. Tupikova, PhD, Associate Professor of the Department of Civil Engineering, Academy of Engineering, RUDN University, 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation; eLIBRARY SPIN-code: 5501-6984, ORCID: 0000-0001-8742-3521; e-mail: emelian-off@yandex.ru

Marina I. Rynkovskaya, Candidate of Technical Sciences, Associate Professor of the Department of Construction Technology and Structural Materials, Academy of Engineering, RUDN University, 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation; eLIBRARY SPIN-code: 9184-7432; ORCID: 0000-0003-2206-2563; e-mail: rynkovskaya-mi@rudn.ru

Сравнительный анализ расчета пластины на криволинейно-трапециевидном плане численными методами

В.Н. Иванов, Е.М. Тупикова, М.И. Рынковская

Российский университет дружбы народов, *Москва, Российская Федерация*
✉ rynkovskaya-mi@rudn.ru

Поступила в редакцию: 30 ноября 2025 г.
Доработана: 12 февраля 2026 г.
Принята к публикации: 25 февраля 2026 г.

Аннотация. Покрытия в виде пластин и оболочек на сложных криволинейных планах являются распространенными конструктивными решениями в архитектуре. Такие конструкции имеют ряд преимуществ. Срединные поверхности оболочек и пластин на криволинейно-трапециевидных планах строятся при помощи параметрических и векторных уравнений и имеют ряд особенностей при расчете их напряженно-деформированного состояния. Для конструкций такой формы не получено аналитического точного решения, но возможно получить численное решение, например, методом конечных элементов и вариационно-разностным методом. В такой ситуации для верификации расчетов сравнение результатов, полученных при помощи разных численных процедур, полезно и актуально. Проведен сравнительный анализ результатов расчета напряженно-деформированного состояния пластины на криволинейном плане, полученных перечисленными методами. В литературе недостаточно разработана тема расчета пластин и оболочек на криволинейно-трапециевидном плане. Цель исследования — получить данные расчета напряженно-деформированного состояния пластины на криволинейно-трапециевидном плане, а также оценить применимость и особенности двух методов при расчете подобных конструкций. Для выполнения поставленных задач применено программное обеспечение: программный пакет ANSYS APDL для расчета методом конечных элементов, авторская программа SHELLVRM для расчета вариационно-разностным. Получены и проанализированы параметры напряженно-деформированного состояния пластины на криволинейно-трапециевидном плане, произведена верификация полученных результатов, даны рекомендации по внедрению обоих способов расчета в практику анализа конструкций, выявлены вычислительные трудности и особенности обоих методов.

Ключевые слова: тонкая упругая пластина, сетка узлов, ВРМ, МКЭ, внутренние силовые факторы, перемещения

Вклад авторов: *Иванов В.Н.* — научное руководство, концепция, валидация; *Тупикова Е.М.* — программное обеспечение, графическое оформление, подготовка текста статьи; *Рынковская М.И.* — проведение исследования, обработка данных, написание текста, рецензирование и редактирование. Авторы ознакомлены с окончательной версией статьи и одобрили ее.

Заявление о конфликте интересов. Авторы заявляют об отсутствии конфликта интересов.

Для цитирования: *Иванов В.Н., Тупикова Е.М., Рынковская М.И.* Сравнительный анализ расчета пластины на криволинейно-трапециевидном плане численными методами // Строительная механика инженерных конструкций и сооружений. 2026. Т. 22. № 2. С. 138–151. http://doi.org/10.22363/1815-5235-2026-22-2-138-151 EDN: KCJJXB

1. Introduction

In architecture, shell structures in the form of various analytical surfaces are often used to cover large spans and complex ground plans [1]. The diversity of analytical surface shapes is illustrated, for example, in encyclopedia [2]. The book presents basic equations for defining surfaces, as well as a brief overview of their distinctive characteristics. S.N. Krivoshapko authored a series of seminal works on the systematization of analytical surfaces [1–3]. This topic also attracts the attention of researchers of thin-walled structures [4; 5]. Among all analytical surfaces, researchers identify certain classes that are particularly important for practical use, such as ruled and developable surfaces [6–8], umbrella surfaces [9], and a number of other surface types for application in construction [10–14]. Studies [15; 16] are devoted to the selection of optimal surface shapes of building structures in terms of strength properties. Modeling of surfaces based on two-dimensional curves is discussed in [17; 18]. Studies [19–21] are devoted to geometric modeling, while the specific area of the geometry and modeling of ellipsoidal ring surfaces has been developed by author V.N. Ivanov in [22; 23]. In most cases, such structures are analyzed using the finite element method [5; 24–27]; however, there are also studies based on other structural analysis methods, such as analytical or semi-analytical [28–31], variational difference [32–33], or a combination of methods [34–38]. Interest in the analysis of plates of ellipsoidal ring shape in plan remains strong in the studies of international authors as well. A. Merneedi et al. examined free vibrations of an elliptical plate [39], S. Çeribaşı performed static and dynamic analyses of thin plates made of functionally graded materials subjected to a uniformly distributed load [40]. However, most of the research is conducted using software based on the finite element method, such as Comsol [41].

It should be noted that no classical analytical solution has been obtained for an ellipsoidal ring plate. An analytical solution to the problem of an ellipsoidal plate in bending was derived by V.I. Pogorelov,[1] while a solution for a circular ring plate was obtained by S.P. Timoshenko [42]. Ellipsoidal ring plates and shells can be constructed in a curvilinear coordinate system to best represent their internal geometry.

To obtain an orthogonal curvilinear coordinate system, an arbitrary plane base curve of the form $\boldsymbol{r}_n(u) = x(u)\boldsymbol{i} + y(u)\boldsymbol{j}$ and a system of lines orthogonal to it are adopted [22] (Figure 1). Then, the equation of the coordinate system can then be written as:

$$\boldsymbol{r}(u,v) = \boldsymbol{r}_n(u) + v\boldsymbol{e}(u), \tag{1}$$

where $\boldsymbol{e}(u) = -\mathbf{v}$; $\mathbf{v}$ is the normal line to the base curve; v is the coordinate of the generator lines along the normal to the base curve.

```
/*AFUN,DEG
*SET,a,3
*SET,b,2
*SET,dt,1
*DO,t,0,90,dt
k,(t*100+1),a*sin(t),b*cos(t),0
k,(t*200+2),(a+2)*sin(t),(b+2)*cos(t),0
l,(t*100+1),(t*200+2)
*Enddo
/
```

Listing 1. Macro for creating guide points and lines

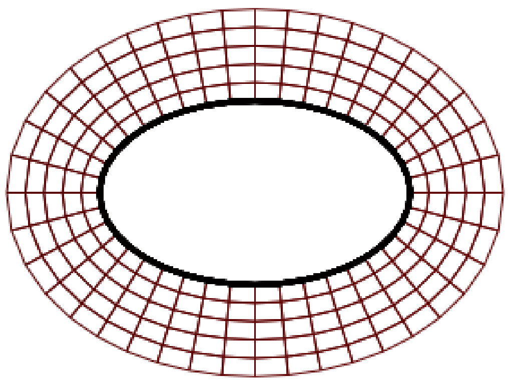

Figure 1. Ellipsoidal ring plate
S o u r c e: made by E.M. Tupikova.

The considered orthogonal curvilinear coordinate system will consist of a system of equidistant curves, i.e., curves parallel to the base curve, and a system of lines orthogonal to them, and may be referred to as pseudo-polar. If an open curve is chosen as the base curve, the result is a curvilinear trapezoidal region. If the base curve is closed, e.g., an ellipse, the result is a closed oval region.

From the perspective of coordinate system geometry, the following characteristics can be identified [22]:

$$\boldsymbol{r}_u = s'(1 - k_n v)\boldsymbol{\tau} = A\boldsymbol{\tau};$$

$$A = s'(1 - k_n v);$$

$$\boldsymbol{r}_v = \mathbf{v};$$

$$B = 1, \tag{2}$$

where s' is the coefficient of the length of the base curve; k_o is the curvature of the base curve.

[1] Pogorelov VI. *Strength and stability of thin-walled structures.* Moscow: State Publishing House of Physical and Mathematical Literature; 1963. 635 p.

If the vertical coordinate function $z(u,v)$ is defined, the equation of the surface of curvilinear trapezoidal plan can obtained in the following form [22]:

$$\rho(u,v) = \boldsymbol{r}_n(u) + v\boldsymbol{e}(u) + Z(u,v)\boldsymbol{k} . \tag{3}$$

If the vertical coordinate function $Z(u,v)$ is arbitrary, then the coordinate system of surfaces of curvilinear trapezoidal plan will not coincide with the lines of curvature of the surface, except when $Z = z(v)$, that is, the case when a constant curve moves in the normal plane of the base curve, and the surface will belong to the class of Monge surfaces [20; 33].

The coefficients of the first quadratic form of the coordinate system under consideration can be taken to be equal to the coefficients of the first quadratic form of the orthogonal coordinate system (2) [22; 23]. Then, the values of the curvatures of the surface system can be obtained from the formulas of differential geometry:

$$k_u = \frac{1}{A^3}\frac{\partial A}{\partial u}\frac{\partial Z}{\partial u} - \frac{1}{A^2}\frac{\partial^2 Z}{\partial u^2} - \frac{s' k_n}{A}\frac{\partial Z}{\partial v};$$

$$k_v = \frac{1}{A}\frac{\partial^2 Z}{\partial v^2} . \tag{4}$$

2. Materials and Methods

In the field of structural mechanics, the finite element method (FEM) is the predominant approach. It often serves as the uncontested foundation for all calculations of complex structures performed using certified software. Finite-element-based software such as SCAD, LIRA, ANSYS, SolidWorks, as well as their freely available alternatives, are universal tools for analyzing structures of virtually any shape designed by an architect. Such programs require significant computer resources and license support, and they present a number of challenges regarding usage, overcoming computational difficulties, and implementing the assumptions of the model. Most importantly, when using such software tools, the finite element mesh is critical, as the obtained results depend heavily on the generation of this mesh. Obviously, more accurate results can be obtained by using smaller finite elements (FEs), however, when dealing with complex shapes, the configuration of the finite elements (triangular or quadrilateral), the base points and lines on which the program constructs the nodes of the mesh, and the correspondence between the internal geometry of the structure or its element and the automatically generated mesh of FE nodes are also important.

Analyzing results in finite-element-based software for surfaces of complex geometry can be somewhat difficult or limited by the standard functionality available in the software for displaying displacements, strains, and stresses, for example, by the presence or absence of coordinate systems other than Cartesian, spherical, and cylindrical, the ability to use local systems, and the calculation of stresses or forces in directions characteristic of certain non-classical surfaces.

This paper presents a comparative analysis of the results obtained by two calculation methods applied to a plate of curvilinear trapezoidal plan — an ellipsoidal ring plate. The finite element analysis was performed using the ANSYS APDL software. Isoperimetric shell63 finite elements were employed.

It is assumed that the inner director ellipse of the considered ellipsoidal ring plate has dimensions a = 3 m, b = 2 m, and the width of the plate is 2 m (Figure 1). The plate is fixed along the outer contour. The plate is analysed under uniformly distributed load such as self-weight: q = 1 kN/m^2. The thickness of the plate h = 0.1 m, Young's modulus of the material $E = 3.5 \times 10^7$ kPa, Poisson's ratio $\nu = 0.15$.

To model the geometry of the structure, a macro is used that involves creating guide points and lines, followed by the generation of a surface based on the line frame (see Figure 1, Listing 1).

The model was then divided into finite elements (Figure 2). The ANSYS software supports both free automatic meshing and meshing mapped to guide points using quadrilateral elements. The latter approach was chosen, with the finite element size specified manually.

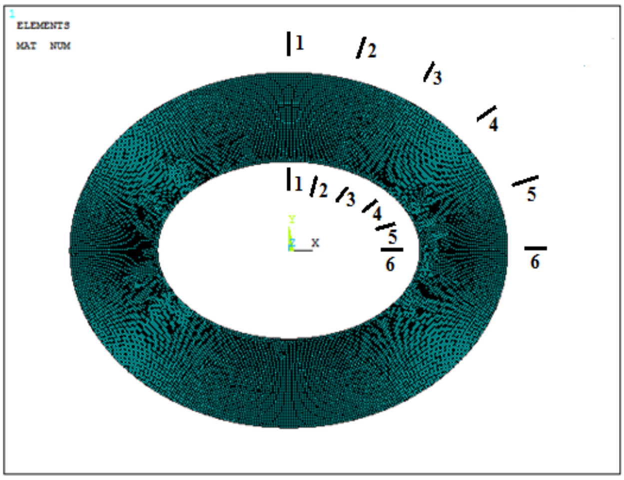

Figure 2. Model of the plate in ANSYS with characteristic cross-sections
S o u r c e: made by E.M. Tupikova.

When using automatic meshing with quadrilateral and triangular elements with element size of 0.05 m, the results show areas of questionable sharp stress jumps. However, when meshing with larger elements, up to 0.2 m, but with mapping to key points and constructed surface guide lines, the results appear more reliable.

For comparison with the finite-element solution, this study uses the solution presented in [43], obtained using the author-developed SHELLVRM program, which is based on the variational difference method (VDM). The program is compact and does not require significant computer resources, while the results exhibit accuracy comparable to that of multifunctional commercial software, as has already been demonstrated, for example, in [34–36].

3. Results and Discussion

This section of the article presents the results of the finite element analysis of the plate, as well as a comparison of these results with those obtained by the variational difference method; some of the data and figures are cited from [22]. The finite element analysis was performed in accordance with the linear Kirchhoff — Love theory, using the shell63 finite element type (four-node, quadrilateral shape) with the size of 0.2 m. The material properties are specified as linearly elastic, with Young's modulus E = 3.5×10^7 kPa and Poisson's ratio ν = 0.15. Boundary conditions: fixed outer contour; loading: self-weight q = 1 kN/m^2. Contour plots were obtained using the standard POST1 post-processor, diagrams along the specified cross-sections were constructed using the PATH function. Deflection, bending moment, and equivalent stress values were obtained. Figure 3 shows the combined deflection diagrams in the characteristic cross-sections of the plate.

The results are also presented in detail in the form of deflection graphs for characteristic sections 1–1 (Figure 4) and 6–6 (Figure 5) and contour plots of displacement (Figure 6) along the z-axis, as well as equivalent stress graphs in characteristic sections 1–1 (Figure 7) and 6–6 (Figure 8) and contour plots of equivalent stress (Figure 9). Figure 10 shows the deflections of the ellipsoidal ring plate obtained using the variational difference method of analysis in [43].

The graph of deflection in the cross-section along the major axis of the ellipse (Figure 4) has a shape similar to a parabola. The maximum deflection was 5 mm, which is in good agreement with the results obtained using VDM in [43].

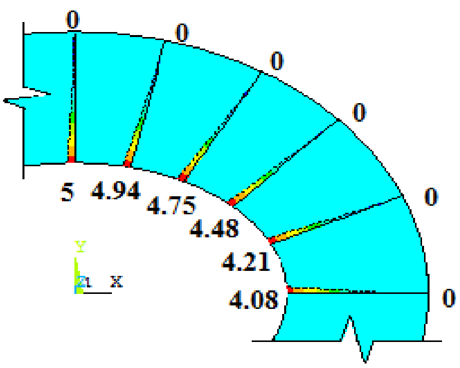

Figure 3. Combined deflection diagrams at characteristic cross-sections, mm
S o u r c e: made by E.M. Tupikova.

The graph of deflection in the cross-section along the minor axis of the ellipse (Figure 5) shows a maximum deflection of 4.2 mm, which also indicates close agreement with the calculation results from [43].

In Figure 6, which illustrates the deflection contour plot, it can be seen that, for this support arrangement (fixed outer edge), the largest deflections occur in the cross-sections passing through the minor axis of the ellipse.

The graphs of equivalent stress in characteristic sections 1–1 (Figure 7) and 6–6 (Figure 8) along the major and minor axes of the ellipse, respectively, indicate that the maximum stress values occur at the outer edge, and the minimum values are at the inner edge of the long side of the plate, and the maximum difference in equivalent stress values occurs in the cross-section along the long side of the plate. At the same time, the zone of minimum equivalent stresses in the cross-section along the short side of the plate is shifted away from the inner edge compared to the zone of minimum stresses in the cross-section along the long side of the plate.

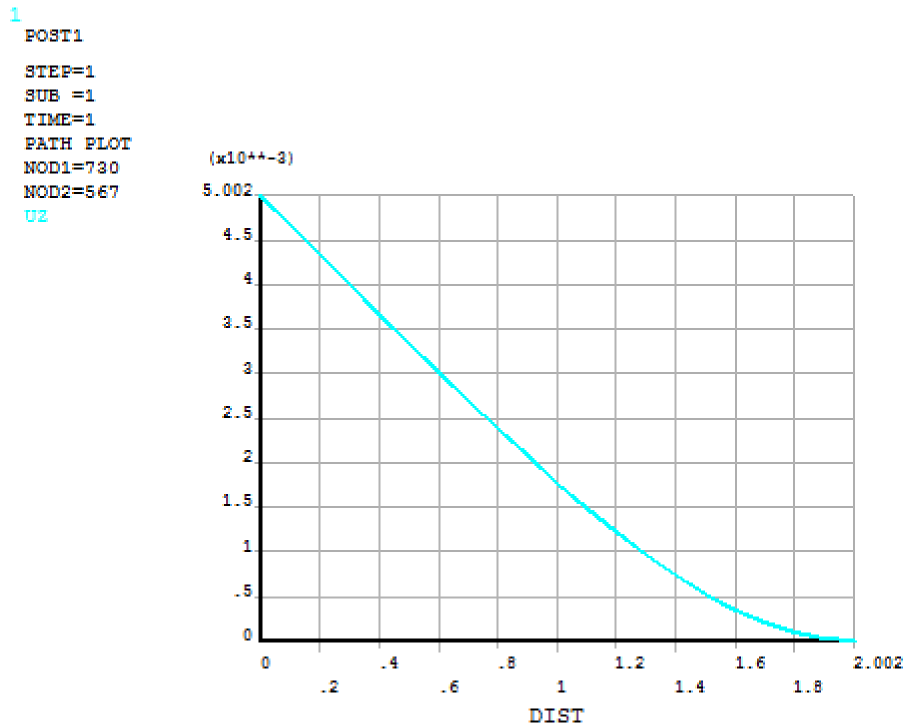

Figure 4. Graph of deflection u_z in characteristic section 1–1, m
S o u r c e: made by E.M. Tupikova.

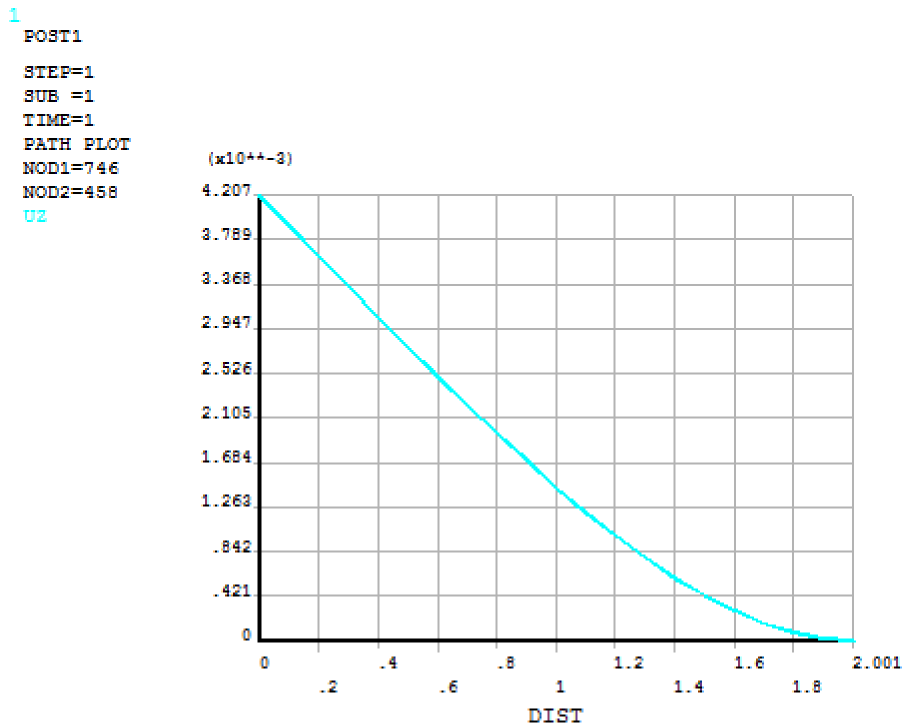

Figure 5. Graph of deflection u_z in characteristic section 6–6, m
S o u r c e: made by E.M. Tupikova.

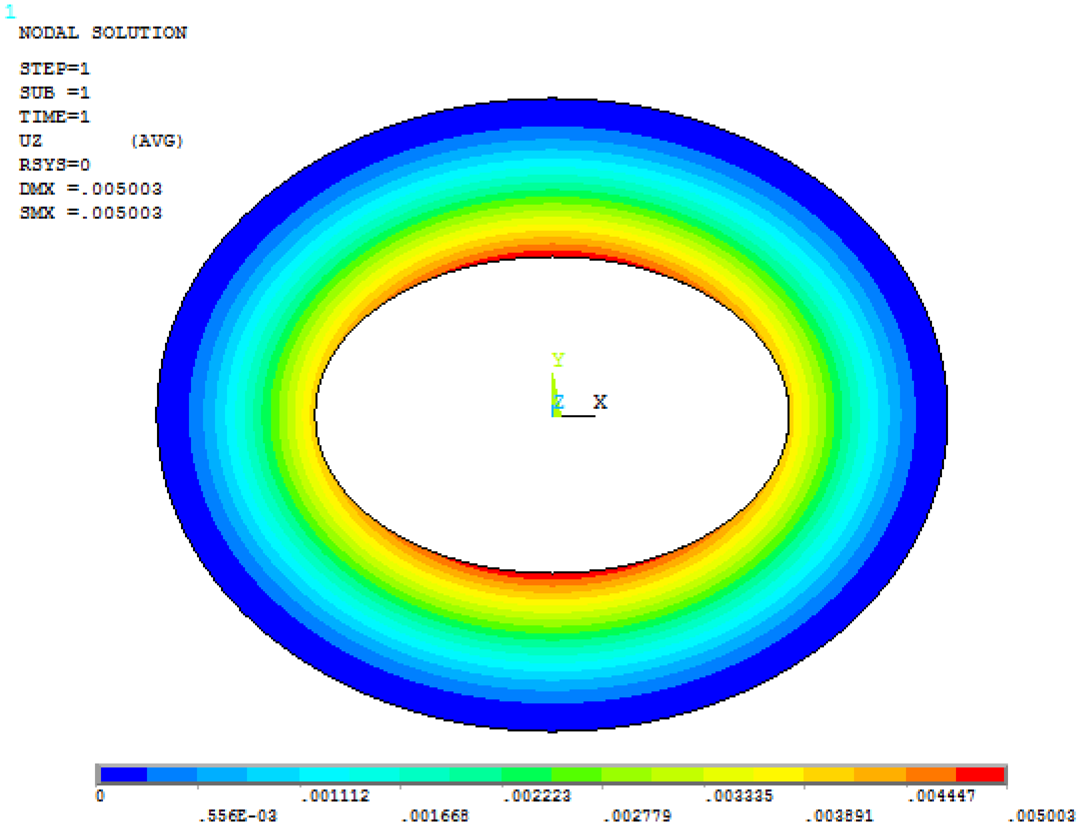

Figure 6. Contour plot of deflection u_z, m

S o u r c e: made by E.M. Tupikova.

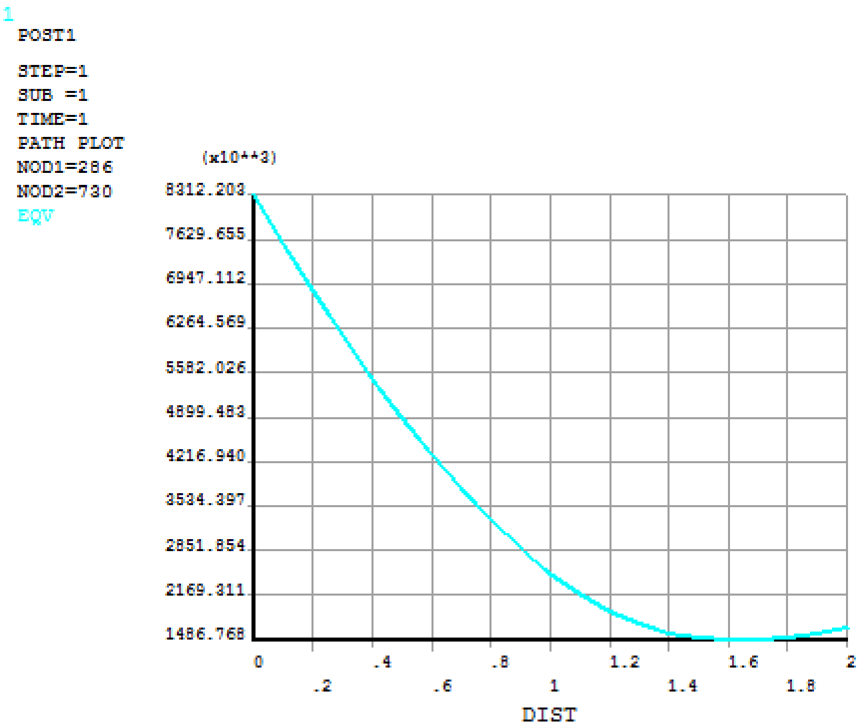

Figure 7. Graph of equivalent stress in section 1-1, N/m²

S o u r c e: made by E.M. Tupikova.

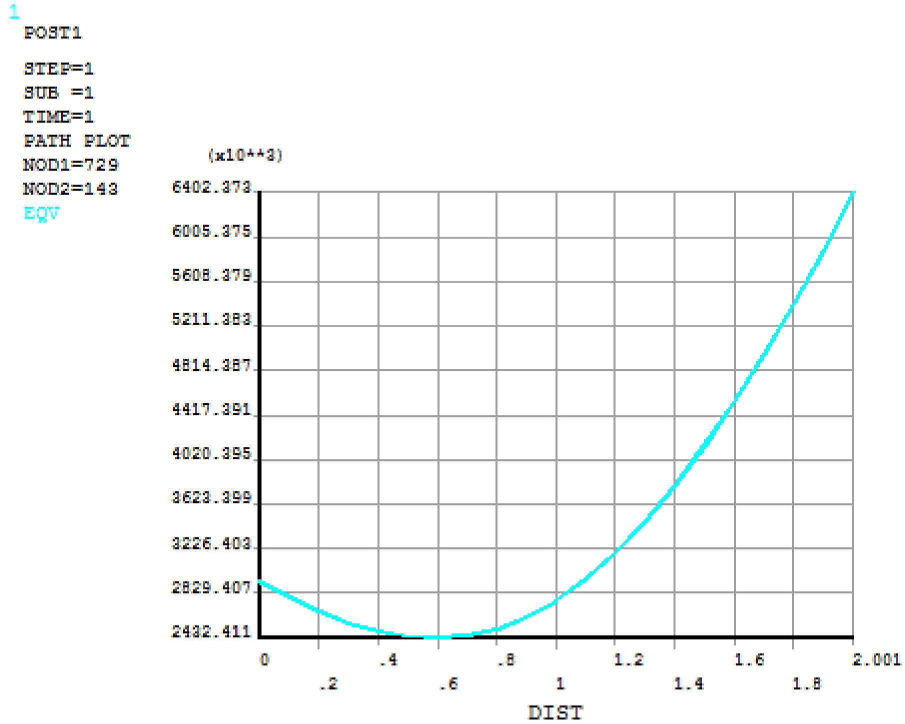

Figure 8. Graph of equivalent stress in section 6-6, N/m²

S o u r c e: made by E.M. Tupikova.

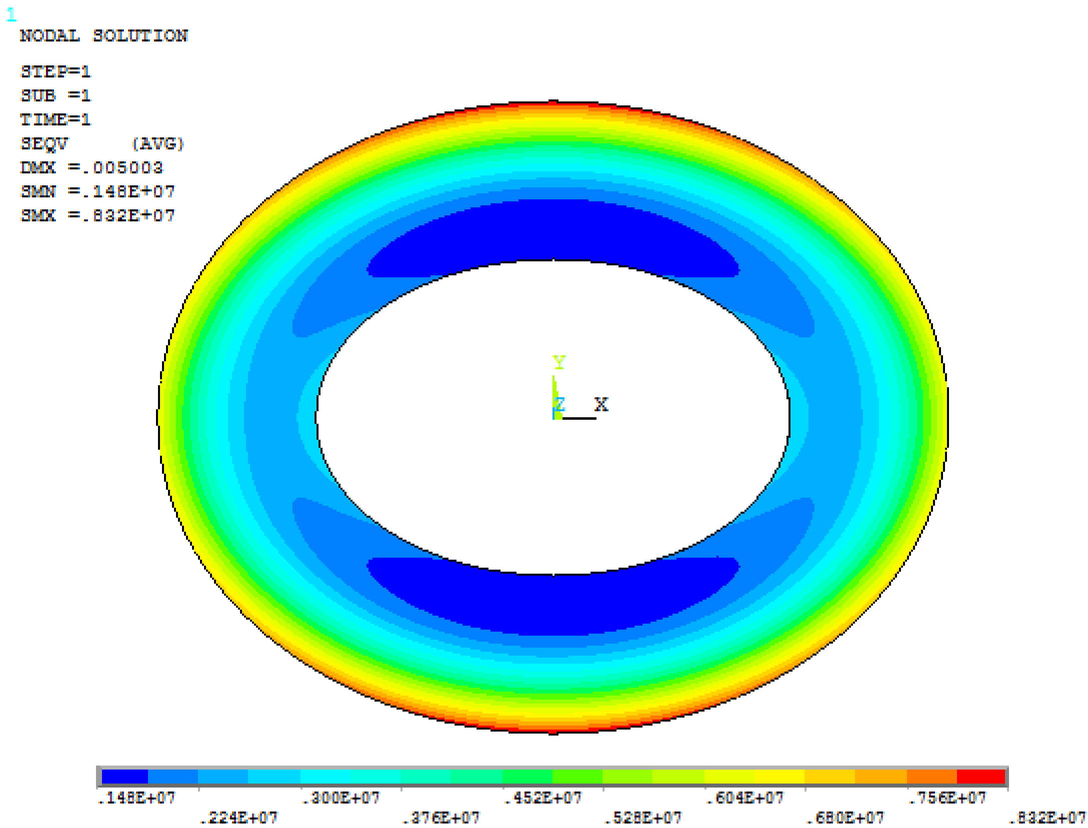

Figure 9. Contour plot of equivalent stress, N/m²

S o u r c e: made by E.M. Tupikova.

The von Mises equivalent stress (Figure 9) is calculated using standard tools of the software and characterizes the material behavior within the structure. Such data can be useful for preliminary assessment and selection of reinforcement. One can clearly see the areas of lowest stress (blue), the average background stress level (light blue and green), and the most heavily loaded, fixed outer edges (red and yellow). The model is analyzed for arbitrary reinforced concrete with linearly elastic characteristics. For a more detailed reinforcement design, these zones can serve as a rough guide.

The deflection values at characteristic cross-sections, obtained using the finite element method and the variational difference method, are presented in Table 1.

The bending moments at cross-sections 1–1 and 6–6, which are most suitable for the Cartesian coordinate system used in FEM programs, calculated using the two methods, are shown in Table 2.

Table 1. **Deflections in characteristic cross-sections according to FEM and VDM**

Section ID and analysis method	**1–1**	**2–2**	**3–3**	**4–4**	**5–5**	**6–6**
Deflection according to FEM (mm)	5.00	4.94	4.75	4.48	4.21	4.08
Deflection according to VDM (mm)	5.1	5.0	4.7	4.5	4.3	4.2

S o u r c e: made by E.M. Tupikova, M.I. Rynkovskaya.

Table 2. **Bending moments in characteristic cross-sections according to FEM and VDM**

Section ID and compared parameters	**Section 1–1**		**Section 6–6**	
Analysis method	M_x, (N·m/m)	M_y, (N·m/m)	M_x, (N·m/m)	M_y, (N·m/m)
FEM	14660	2170	11255	1168
VDM	15100	1800	11700	2300

S o u r c e: made by E.M. Tupikova, M.I. Rynkovskaya.

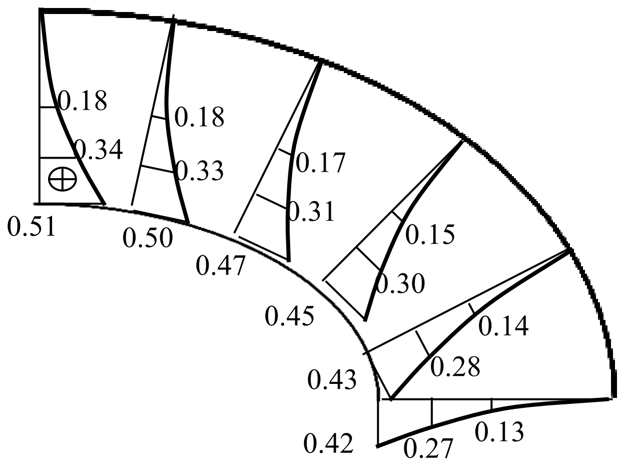

Figure 10. Deflection u_z (cm) of the ellipsoidal ring plate according to VDM

S o u r c e: made by V.N. Ivanov [43].

The bending moment diagrams for the characteristic cross-sections, obtained using the finite element method, are shown in Figures 11–14. For comparison, Figure 15 shows the corresponding results of the calculation of bending moments using the variational difference method.

The diagrams (Figures 11–14) show that bending moment *Mx* is the primary parameter determining the stress state of the plate, while the values of bending moment *My* are an order of magnitude smaller.

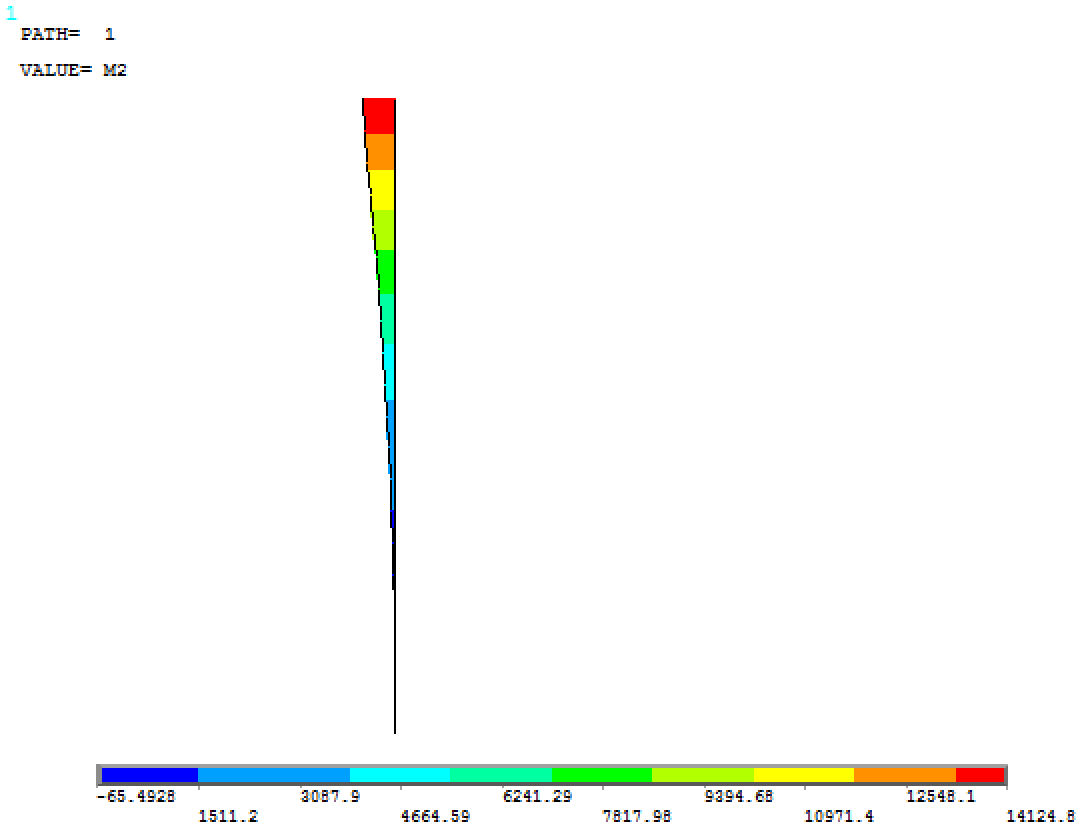

Figure 11. Diagram of bending moment M_x in characteristic section 1-1, N· m/m
S o u r c e: made by E.M. Tupikova.

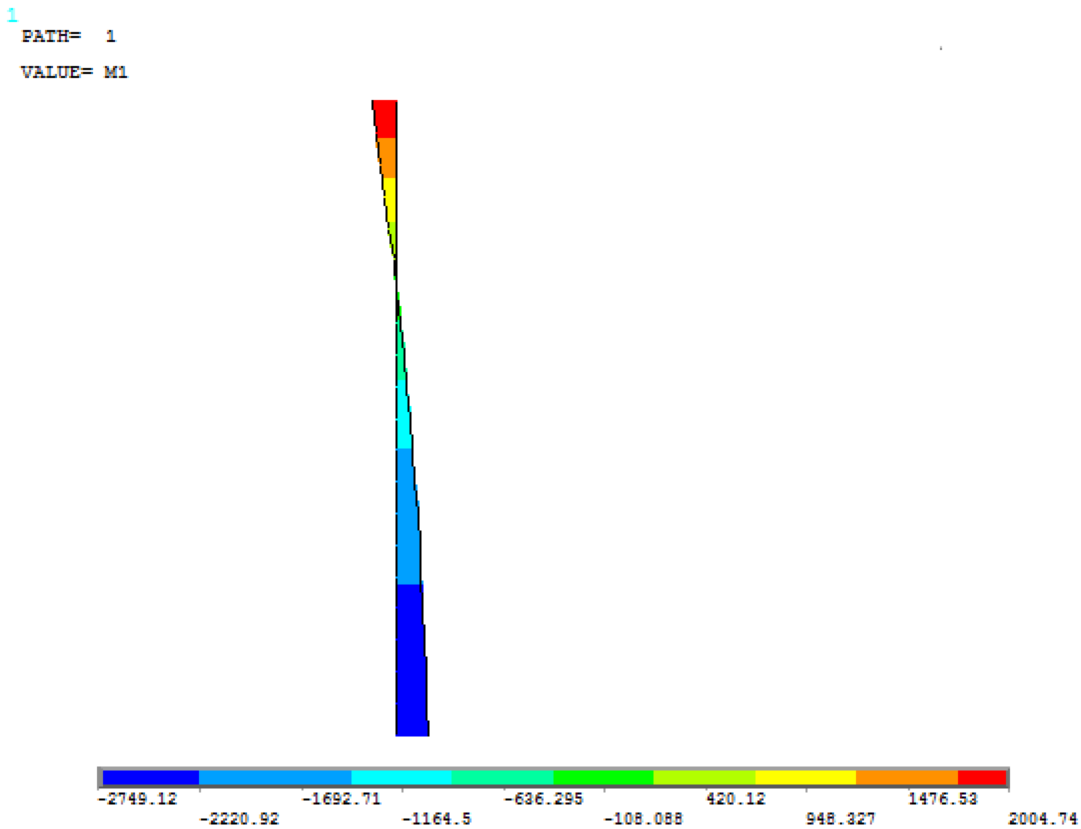

Figure 12. Diagram of bending moment M_y in characteristic section 1-1, N· m/m
S o u r c e: made by E.M. Tupikova.

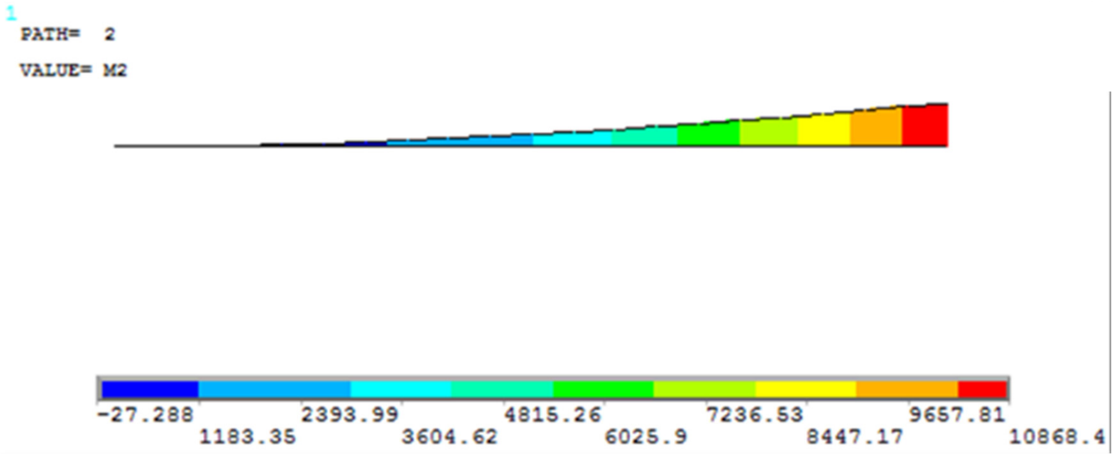

Figure 13. Diagram of bending moment M_x in characteristic section 6-6, N· m/m
Source: made by E.M. Tupikova.

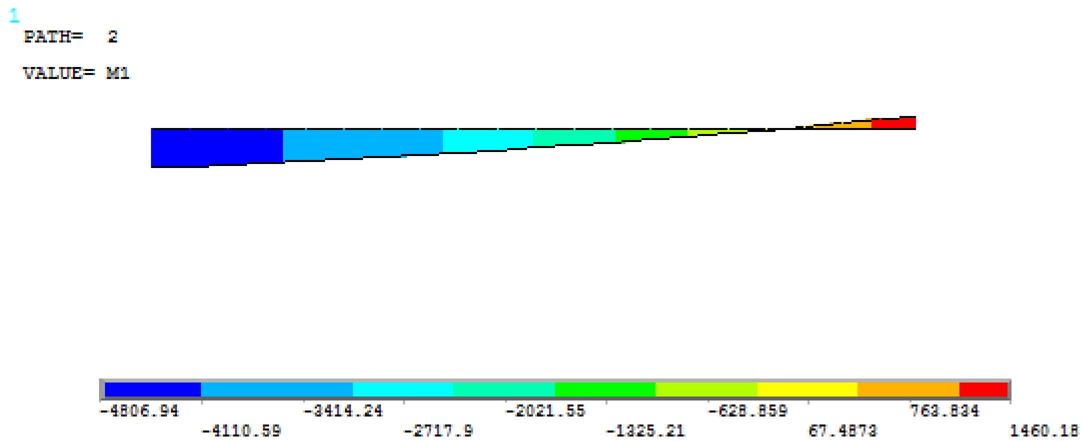

Figure 14. Diagram of bending moment M_y in characteristic section 6-6, N· m/m
Source: made by E.M. Tupikova.

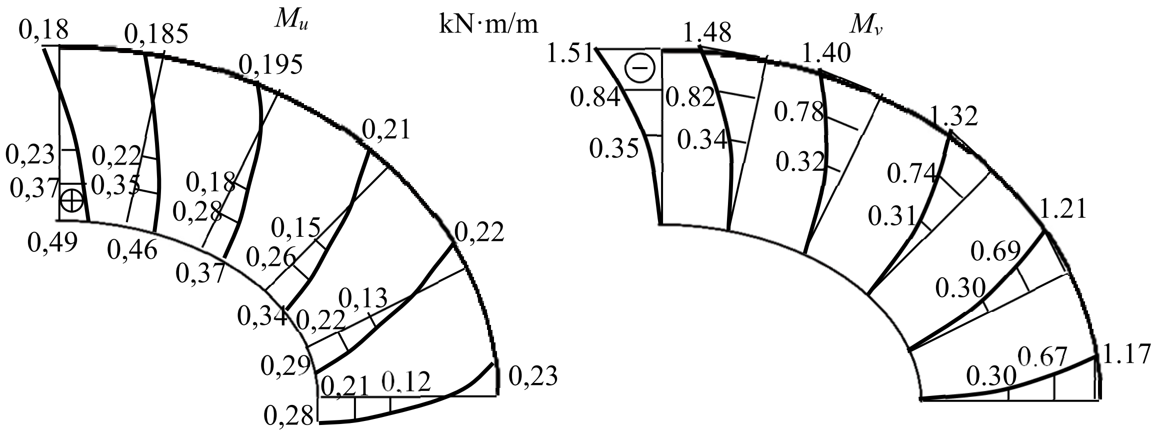

Figure 15. Diagrams of bending moments M_x and M_y (M_u and M_v , respectively,
in the curvilinear coordinate system of VDM), kN·m/m
Source: made by V.N. Ivanov [43].

The maximum moments, equal to 1.47 kN·m/m, occur in the cross-section along the minor axis of the ellipse at the fixed support.

The comparison of the results obtained using the two methods shows that they are quite similar.

4. Conclusion

This study examines the stress state of an ellipsoidal ring plate under a uniformly distributed load and compares the results obtained using the finite element method and the variational difference method. The necessary calculation procedures were performed for each method. To ensure the validity of the comparison, model parameters, such as mesh size, were set to be identical. The values of internal stresses, moments, and displacements at the nodes in identical cross-sections were obtained and analyzed. The obtained results allow the following conclusions to be drawn:

1. Analysis of the ring plate of ellipsoidal plan using the variational difference method, implemented in the author-developed program, and the finite element method in the ANSYS software yielded comparable results for deflections and bending moments in characteristic cross-sections.
2. Since the results of analyzing a relatively simple object such as a plate depend significantly on its internal geometry, it is essential to construct a finite element mesh, the key nodes of which are aligned with the director lines. Often, meshing algorithms in commercial software do not fully ensure this alignment, which negatively affects the results. When using author-developed software, this problem can be eliminated for objects with complex geometry by employing a custom node generation algorithm in case of finite element analysis.
3. When applying the variational difference method, the equation is solved by the variational method with discretization of the solution domain, which allows the problem to be solved approximately using mesh functions, and the integrals to be approximated by sums, while the derivatives are approximated by differences. When using the variational difference method, it is possible to obtain results at significant points and interpolate them.
4. Compared to the finite element method, the implementation of the variational difference method requires fewer computational resources and yields results of sufficient accuracy. Software based on the variational difference method has the potential to perform verification calculations for complex structures.

This article presents the results of the first stage of the study — plates of ellipsoidal ring plan. In the future, it is planned to perform a comparative analysis of shells of ellipsoidal ring plan. It is anticipated that differences in the FEM and VDM may have a significant impact on the analysis results of such shells.

References

1. Krivoshapko SN, Mamieva IA. *Analytical surfaces in the architecture of buildings, structures and products*. Moscow: URSS; 2025. (In Russ.) EDN: QJZEGJ

2. Krivoshapko SN, Ivanov VN. *Encyclopedia of analytical surfaces*. Moscow: URSS; 2023. (In Russ.)

3. Krivoshapko SN. Analytical ruled surfaces and their complete classification. *Structural Mechanics of Engineering Constructions and Buildings*. 2020;16(2):131–138. (In Russ.) https://doi.org/10.22363/1815-5235-2020-16-2-131-138 EDN: ILRHCX

4. Vekariya MS, Makwana EAH. A Review on thin-shell structures: Advances and trends. *International Journal of Research Publication and Reviews*. 2021;2(12):1593–1608.

5. Ganendra B, Prabowo AR, Muttaqie T, Adiputra R, Ridwan R, Fajri A, Thang Do Q, Carvalho H, Baek SJu. Thin-walled cylindrical shells in engineering designs and critical infrastructures: A systematic review based on the loading response. *Curved and Layered Structures*. 2023;10(1). https://doi.org/10.1515/cls-2022-0202 EDN: XYJUCO

6. Krivoshapko SN. Prospects and advantages of torsional surfaces in modeling mechanical engineering and building structures. *Bulletin of Civil Engineers*. 2019;1(72):20–30. https://doi.org/10.23968/1999-5571-2019-16-1-20-30 EDN: KQGSFJ

7. Chen M, Tang K. A fully geometric approach for developable cloth deformation simulation. *Visual Computer*. 2010;26(6–8):853–863. https://doi.org/10.1007/s00371-010-0467-5

8. Aleshina OO. Research on the geometry and calculation of torsional shells of the same slope. *Structural Mechanics and Analysis of Constructions*. 2019;3(284):63–70. (In Russ.)

9. Krivoshapko SN. The opportunities of umbrella-type shells. *Structural Mechanics of Engineering Constructions and Buildings*. 2020;16(4):271–278. http://doi.org/10.22363/1815-5235-2020-16-4-271-278 EDN: VCQAWD

10. Alborova L, Mamieva I. Curvilinear forms in the architecture of buildings and structures until the 21st century. *Academia. Architecture and Construction*. 2023;3:154–164. (In Russ.) https://doi.org/10.22337/2077-9038-2023-3-154-164 EDN: JEOEOY

11. Mamieva IA. Analytical surfaces for parametric architecture in contemporary buildings and structures. *Academia. Architecture and construction*. 2020;1:150–165. (In Russ.) EDN: KNYKTY

12. Gil-Oulbe M, Daou T, Mariko O. Analytical surfaces for architecture and engineering. *Structural Mechanics of Engineering Constructions and Buildings*. 2022;18(5):458–466. https://doi.org/10.22363/1815-5235-2022-18-5-458-466 EDN: EPDITE

13. Strashnov S, Rynkovskaya M. To the Question of the classification for analytical surfaces. *Geometry & Graphics*. 2022;1:36–43. (In Russ.) https://doi.org/10.12737/2308-4898-2022-10-1-36-43 EDN: YPILOJ

14. Gil-Oulbe M. Reserve of analytical surfaces for architecture and construction. *Building and Reconstruction*. 2021;6:63–72. https://doi.org/10.33979/2073-7416-2021-98-6-63-72 EDN: BCWXIS

15. Krivoshapko SN. Kinematic surfaces with congruent generatrix curves. *RUDN Journal of Engineering Research*. 2023;24(2):166–176. https://doi.org/10.22363/2312-8143-2023-24-2-166-17616:43 EDN: BNFZFA

16. Ivanov VN, Aleshina OO, Larionov EA. Determination of optimal cylindrical shells in the form of second-order surfaces. *Structural Mechanics of Engineering Constructions and Buildings*. 2025;21(1):37–47. (In Russ.) https://doi.org/10.22363/1815-5235-2025-21-1-37-47 EDN: IQCXLS

17. Ivanov V. Geometry of the normal ruled surfaces. *Structural Mechanics of Engineering Constructions and Buildings*. 2021;17(6):562–575. (In Russ.) https://doi.org/10.22363/1815-5235-2021-17-6-562–575 EDN: DBVHZU

18. Konopatskiy EV, Voronova OS, Rotkov SI, Lagunova MV, Bezditnyi AA. Modeling of the 2nd order curves and surfaces of engineering structures shells based on their basis. *Construction and industrial safety*. 2021;22(74):101–110. (In Russ.) https://doi.org/10.37279/2413-1873-2021-22-101-110 EDN: SPNBPW

19. Ivanov VN. Constructing shells and their visualization in system “MathCad” on basis of vector equations of surfaces. *IOP Conference Series: Materials Science and Engineering*. 2019;456:012018. https://doi.org/10.1088/1757-899X/456/1/012018 EDN: VVVFHR

20. Gil-Oulbé M, Ndomilep AJI. Geometry and classification of carved Monge surfaces. *Journal of Physics Conference Series*. 2021;1687:012002. https://doi.org/10.1088/1742-6596/1687/1/012002

21. Mamieva IA., Gbaguidi-Aisse GL. Influence of the geometrical researches of rare type surfaces on design of new and unique structures. *Building and Reconstruction*. 2019;5(85):23–34. https://doi.org/10.33979/2073-7416-2019-85-5-23-34 EDN: UAKPPP

22. Ivanov VN, Imomnazarov TS, Farhan IT. Orthogonal curved coordinate system and forming the surfaces on trapezium-curved plans. *RUDN Journal of Engineering Research*. 2017;18(4):518–527. (In Russ.) https://doi.org/10.22363/2312-8143-2017-18-4-518-527 EDN: YPSROF

23. Ivanov V.N. Geometric characteristics of surfaces with curved trapezoidal plan. *Structural Mechanics of Engineering Constructions and Buildings*. 2024;20(2):134–145. (In Russ.) https://doi.org/10.22363/1815-5235-2024-20-2-134-145 EDN: GPVKGU

24. Sabat L, Kundu CK. History of finite element method: a review. In: *Lecture Notes in Civil Engineering: Recent Developments in Sustainable Infrastructure*. 2021;75:395–404. https://doi.org/10.1007/978-981-15-4577-1_32

25. Wang P, Niu Q, Liu M, Li Z, Cao X, Zhang H. Numerical analysis on natural vibration of cylindrical shell with different cross-section. *MATEC Web of Conferences*. 2023;380:01015. https://doi.org/10.1051/matecconf/202338001015 EDN: DAQCOH

26. Al-Yacouby AM, Hao LJ, Liew MS, Ratnayake RMC, Samarakoon SMK. Thin-walled cylindrical shell storage tank under blast impacts: Finite element analysis. *Materials*. 2021;14:7100. https://doi.org/10.3390/ma14227100 EDN: DBXXOD

27. Trushin S, Goryachkin D. Numerical evaluation of stress-strain state of bending plates based on various models. *Procedia Engineering*. 2016;153:781–784. https://doi.org/10.1016/j.proeng.2016.08.242 EDN: YUWKVR

28. Rynkovskaya MI. Calculation and application of helical shells. *Bulletin of Peoples’ Friendship University of Russia*. Series: Engineering research. 2009;3:113–116. EDN: KVUYQX

29. Rynkovskaya MI. On the issue of strength calculation of thin ruled helical shells. *Structural Mechanics of Engineering Structures and Buildings*. 2015;6:13–15. EDN: UMQHQN

30. Rynkovskaya M. Plastic deformations occuring in shells with developable middle surfaces during bending. *IOP Conference Series: Materials Science and Engineering*. 2018;371:012054. https://doi.org/10.1088/1757-899X/371/1/012054 EDN: VBQVQK

31. Tupikova EM, Rynkovskaya MI. Analytical approach to stress-strain analysis of right and oblique helicoid structures. *Magazine of Civil Engineering*. 2021;106(6):10609. https://doi.org/10.34910/MCE.106.9 EDN: MHSSFD

32. Maksimyuk VA, Storozhuk EA, Chernyshenko IS. Variational finite-difference methods in linear and nonlinear problems of the deformation of metallic and composite shells. *International Applied Mechanics*. 2012;48(6):613–87. https://doi.org/10.1007/s10778-012-0544-8 EDN: XMGQZR

33. Ivanov V, Rynkovskaya M. Analysis of thin walled wavy shell of monge type surface with parabola and sinusoid curves by variational-difference method. *MATEC Web of Conferences*. 2017;95:12007. https://doi.org/10.1051/matecconf/20179512007 EDN: YVFNSX

34. Ivanov VN, Alyoshina OO. Comparative analysis of the results of determining the parameters of the stress-strain state of equal slope shell. *Structural Mechanics of Engineering Constructions and Buildings*. 2019;15(5):374–83. https://doi.org/10.22363/1815-5235-2019-15-5-374-383 EDN: LZSVVI

35. Ivanov VN, Aleshina OO. Comparative analysis of the parameters of the stress-strain state of a torso with a guiding ellipse using three calculation methods. *Structural Mechanics and Analysis of Structures*. 2020;3(290):37–46. https://doi.org/10.37538/0039-2383.2020.3.37.46 EDN: STKBFX

36. Aleshina OO, Ivanov VN, Grinko EA. Investigation of the equal slope shell stress state by analytical and two numerical methods. *Structural Mechanics and Analysis of Constructions*. 2020;6:2–13. https://doi.org/10.37538/0039-2383.2020.6.2.13 EDN: YXWWNT

37. Govind PL. Complicated features and their solution in analysis of thin shell and plate structures. *Structural Mechanics of Engineering Constructions and Buildings*. 2018;14(6):509–515. https://doi.org/10.22363/1815-5235-2018-14-6-509-515 EDN: YUZVSP

38. Aleshina OO, Ivanov VN, Cajamarca-Zuniga D. Stress state analysis of an equal slope shell under uniformly distributed tangential load by different methods. *Structural Mechanics of Engineering Constructions and Buildings*. 2021; 17(1):51–62. https://doi.org/10.22363/1815-5235-2021-17-1-51-62 EDN: TSDXQW

39. Merneedi A, Nalluri MR, Vissakodeti VSR. Free vibration analysis of an elliptical plate with cut-out. *J Vibroeng*. 2017;19(4):2341–2353. https://doi.org/10.21595/jve.2016.17575

40. Çeribaşı S. Static and Dynamic Analyses of Thin Uniformly Loaded Super Elliptical FGM Plates. *Mechanics of Advanced Materials and Structures*. 2012;19(5):323–335. https://doi.org/10.1080/15376494.2010.528160

41. Sharma P, Khinchi A, Singh R. Modal Study on FGM Elliptical Plate Under Thermal Environment. In: Maiti DK, et al., editors. *Recent Advances in Computational and Experimental Mechanics, Vol II*. Lecture Notes in Mechanical Engineering. Singapore: Springer; 2022. https://doi.org/10.1007/978-981-16-6490-8_8

42. Timoshenko S, Voinovsky-Krieger S. *Theory of plates and shells*. 2nd ed. New York Toronto London: McGraw-Hill Book Company INC, 1959. ISBN 978-0-07-085820-6

43. Ivanov VN. Design and calculation of plates and shallow shells on curvilinear-trapezoidal plans. *Structural Mechanics and Analysis of Constructions.* 2025;(5):72–82. https://doi.org/0.37538/0039-2383.2025.5.72.78 EDN: EIVYNT