

РАСЧЕТ И ПРОЕКТИРОВАНИЕ СТРОИТЕЛЬНЫХ КОНСТРУКЦИЙ ANALYSIS AND DESIGN OF BUILDING STRUCTURES

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Rational Outline of Timber Beams

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Abstract. Timber structures, in particular glued laminated beams, have a number of advantages that contribute to their wide use in industrial and civil construction. The development and research of design solutions that utilize the strength properties of structural timber more efficiently is a key area for improving the performance of timber structures. The object of this study is a timber beam of rectangular cross-section loaded with a uniformly distributed load, the outline of which is based on the uniform-strength trajectories of beams in bending and horizontal shear. The construction of the outline of rational timber beams (RTB) was based on determining the coordinates of nodal points by constructing straight lines tangent to the contour of a beam of uniform strength. The RTB is a beam with a zone of constant rigidity in the middle of the span and undercuts in the support zones. The purpose of this approach was to create a resource-efficient beam structure made of glued timber, characterized by lower material consumption compared to beams of constant cross-section height along the entire span, as well as relative ease of manufacturing. The results of the study show that the relative values of the RTB parameters, such as: the height of the support section relative to the maximum height of the section h_{sup}/h , the length of the undercut relative to the span l_{cut}/L and the angle of the undercut α_{cut} , depend only on the ratio of the maximum height of the section to the span h/L . The change in the width of the section b and the value of the external load q have no effect on them. The theoretical savings of structural timber of grades 1 and 2 are 6.2%...18.3%.

Keywords: timber beams of variable rigidity, timber beams of uniform strength, timber beams of rational outline, resource-efficient timber beams, structural timber saving

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Рациональное очертание деревянных балок

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Аннотация. Деревянные конструкции, в частности дощатоклееные балки, обладают рядом достоинств, способствующих их широкому применению в промышленном и гражданском строительстве. Разработка и исследование конструктивных решений, позволяющих более рационально использовать прочностные характеристики конструкционной древесины, являются одним из актуальных направлений повышения эксплуатационных качеств деревянных конструкций. Объект исследования — деревянные балки прямоугольного сечения, нагруженные равномерно распределенной нагрузкой, очертание которых построено на основе траекторий равнопрочности балок при изгибе и скалывании. Построение очертания рациональных деревянных балок (РДБ) основывалось на определении координат узловых точек путем построения прямых, касательных к контуру равнопрочной балки. РДБ представляет собой балку с участком постоянной жесткости в середине пролета и подрезками в опорных зонах. Целью такого подхода являлось создание ресурсоэффективной балочной конструкции из клееной древесины, отличающейся меньшей материалоемкостью по сравнению с балками постоянной высоты сечения по всей длине пролета, а также относительной простотой исполнения. Результаты исследования показывают, что относительные значения параметров РДБ, такие как высота опорного сечения относительно максимальной высоты сечения h_{sup}/h , длина подрезки относительно пролета l_{cut}/L и угол подрезки α_{cut} , зависят только от соотношения максимальной высоты сечения к пролету h/L . Изменение же ширины сечения b и значения внешней нагрузки q никакого влияния на них не оказывают. Теоретическая экономия конструкционной древесины 1 и 2-го сортов составляет 6.2 %...18.3 %.

Ключевые слова: деревянные балки переменной жесткости, равнопрочные деревянные балки, деревянные балки рационального очертания, ресурсоэффективные деревянные балки, экономия конструкционной древесины

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1. Introduction

Load-bearing timber structures (TS) have found widespread application in the construction of buildings and structures due to their physical and mechanical properties, as well as their renewability, carbon neutrality, and economic efficiency [1–5]. Study [6] is devoted to comparing TS with other types of building structures. One way to improve their performance is to refine structural solutions by seeking improved cross-sectional shapes, rational axis outlines of elements, and through the use of variable rigidity, as discussed in [7]. Adopting a structurally justified solution at the design phase allows to reduce the cost of the structure and cut construction and operational expenses, as demonstrated by the design of Olympic facilities made of timber in the Russian Federation. Therefore, improving structural solutions for these load-bearing elements remains a relevant task for the construction industry.

Adopting a technically justified solution at the design phase helps reduce the overall cost of the structure and lower the construction and operating costs of the entire building. This process involves modeling and complex mathematical calculations. Existing approaches include numerous variational methods, as well as the well-developed theory of uniform strength [8], and the numerical finite element methods described in [9; 10]; however, they require significant computational resources and specialized software.

Variability of the properties of wood, as a natural material, poses challenges for accurately evaluating the mechanical characteristics of glued laminated beams. Therefore, it is crucial to develop and validate models for predicting the mechanical characteristics in order to fully exploit the potential of this material [11]. A stochastic model, which enables the establishment of relationships between safety factors and individual mechanical characteristics by combining structural analysis tools with probabilistic descriptions of timber planks, has become an important step in the design of timber structures, including for the purpose of enhancing the competitiveness of timber compared to other building materials [12]. The transversely isotropic model proved to be the most appropriate for describing glued structures, due to the averaging of anisotropic properties in a multilayer solid [13]. Structural optimization is closely linked to the grain distribution along the cross-section. The grain angle should depend on the magnitude of the applied load, which led to development of a nomogram for the rational lamination, in which the proportion of low-quality timber should not exceed 50%. Experimental and numerical analyses [14] indicate the advantages of using hardwood for manufacturing glued laminated beams, particularly in a combined configuration where the layers in the middle of the cross-section are made of low-grade timber, which opens up broad opportunities for resource optimization. Study [15] also demonstrates the effective use of resources in glued laminated beams by inserting lower-quality lamellas into less-loaded zones, typically in the center of the beam.

The influence of geometric parameters on the stress state was numerically determined in [16]. The relationship between the section height and stress is parabolic, while the relationship between the wall thickness and modulus of elasticity is linear. The authors confirm the theory that, in the beam outline, the height axis is the most critical parameter for the structural shape. Timber lamellas in glued laminated structures contain knots formed as a result of the natural growth process of trees, which significantly affect the mechanical properties [17]. Accounting for the location of knots and finger joints in glued laminated beams, that can cause significant stress and strain concentrations, is important, as demonstrated in [18]. Special stresses are concentrated in the support zones of deep beams, during the study of which an engineering calculation of maximum stress was proposed, taking into account the rheological properties of wood, and a method of reinforcement with polymers based on glass fabric with carbon nanotubes was developed, increasing the horizontal shear strength by 10–20% [19]. The developed reinforcement method demonstrated the possibility of conserving wood resources in glued laminated beams while maintaining the design strength [20]. Study [21] demonstrates that replacing up to 34% of the lamellas with lower-quality timber (after fire exposure resulting from a forest fire) reduces the load-bearing capacity by only 2.5%.

Approaches to improving beam geometry have evolved from individual formulas to comprehensive analysis and design methods [22]. In [23], a solution for the skew bending of rectangular beams using the Lagrange method was proposed, which minimizes the cross-sectional area subject to strength constraints. In [24], a variational analysis method was developed and a system of integral equations for calculating forces in all components of the structure of a multi-layer beam was obtained. A simple optimization method for “scissor” trusses, developed in [25], revealed that the slope of the upper chord minimizes forces due to scaling of the load diagrams.

Finite element methods (FEM) have become the standard for analyzing complex structures. Numerical modeling of glued laminated timber beams is a complex problem; for example, the quasi-brittle nature of wood requires modeling of progressive failure mechanisms and nonlinear behavior under load [26]. Simulation in MATLAB can predict flexural rigidity with high accuracy and provide sufficiently accurate predictions of the flexural strength of glued laminated beams [27]. Developing a model in the ANSYS software allows to simulate stress states of any complexity. In the practice of large-scale design, modelling of a lattice timber dome with a diameter of 100 m allowed to reduce wood consumption by 10–15% [28].

The method for buckling analysis of glued laminated beams with variable cross-sections, proposed by the authors [29], revealed an underestimation of the design coefficients when using the Bubnov — Galerkin method. Compound timber elements connected by dowels were studied in [30], showing compliance coefficients in the range of 0.85–0.93. There is a trend toward increased use of glued laminated timber in increasingly slender structures, which requires special attention to stability issues, such as lateral buckling [31]. Lateral buckling of narrow beams with bracing, investigated in [32], was refined in an orthotropic formulation with an equation that revealed overestimation of standard values for a single bracing. Study [33] is devoted to the investigation of out-of-plane bending of glued laminated structures, presenting a comparison of the bending and shear characteristics of CLT and GLT beams.

The development of the strength criteria proceeded in parallel with the refinement of standards, as shown in [34]. The authors of the study demonstrated the inadequacy of the Navier — Zhuravsky calculations for glued laminated structures and proposed Ashkenazi tensor criterion with checks for radial stresses and splitting across the grain in the near-support zones. In [35], a critical reassessment of building codes for timber structures was performed, and errors in the coefficients for lateral buckling were identified. The authors prepared tables for all types of restraints, refining the formulas and force diagram shapes. Field test results confirmed the theoretical hypotheses with a discrepancy of no more than 12%. The results demonstrate the need to refine the regulatory framework for structurally justified timber structure outlines and the validity of applying the principle of uniform strength in practical calculations.

Practical implementation confirms the justification of the improvements to the TS shapes. Technically justified structures are being actively incorporated into the designs of real projects. Compound composite timber beams with mechanical connections, developed in [36], have increased the strength by a factor of 1.5–1.8 and stiffness by a factor of 2.5–3. Software [37] allows to automate the calculation of prestressed compound beams with uniform stress distribution.

Having conducted the analysis of the use of timber structures in construction, including ways to improve the shape and configuration of cross-sections, it can be concluded that the study of load-bearing TS remains relevant. Contemporary researchers are exploring structural solutions aimed at conserving timber resources without compromising the overall strength. Most of the research is related to the use of composite materials, compound structures, or refinement of design codes. However, the use of technically justified structural shapes or cross-sectional configurations is not widely covered in the literature. These factors allow the authors to formulate the objective of this study, which is to determine the cross-sectional shape of a timber beam that will make the most complete use of the strength characteristics of structural timber, and to evaluate the effectiveness of the resulting designs.

The object of this study is a simply supported glued laminated timber floor or roof beam with a rectangular cross-section and subjected to a uniformly distributed load.

The ratio of the wood consumption of beams with a rational outline to that of beams with constant rigidity along their entire length is used as a criterion for evaluating the effectiveness of this design.

The outline of a rational timber beam (RTB) is determined by constructing tangents to the function of the height of the cross-section of a beam of uniform-strength with a constant cross-sectional width. This method ensures a linear variation in rigidity in the near-support zones of the structure, while the cross-sectional height remains constant in the middle of the span.

2. Methods

In the first stage of the study, the contour of a beam of uniform strength was constructed using diagrams of the relationship between the section height and the distribution of bending moments and shear forces $y = h_Q(x)$ along the axis of the member (Figure 1).

The height of the cross-section of the beam of uniform strength was determined based on the strength conditions for timber beams with respect to normal and shear stresses:

$$\sigma = \frac{M(x)}{W(x)} \leq R_b, \quad (1)$$

$$\tau = \frac{Q(x)S(x)}{bJ(x)} \leq R_{hs}, \quad (2)$$

where $M(x) = qx(L - x)/2$ is the function of the bending moment distribution along the beam axis; $Q(x) = q(L/2 - x)$ is the function of the shear force distribution along the beam axis; $W(x) = bh(x)^2/6$ is the section modulus of the beam of variable rigidity; $S(x) = bh(x)^2/8$ is the statical moment of the beam of variable rigidity; $J(x) = bh(x)^3/12$ is the moment of inertia of the beam of variable rigidity; b is the width of the beam section; R_b , R_{hs} are the design strengths of timber in bending and horizontal shear, respectively.

The expressions for $M(x)$ and $W(x)$ were substituting into formula (1), the expressions for $Q(x)$, $S(x)$ and $J(x)$ were substituted into formula (2), and the variable section height $h(x)$ was then expressed:

$$\sigma = \frac{6qx(L - x)}{2bh(x)^2} = R_b \Rightarrow h_M(x) = \sqrt{\frac{3qx(L - x)}{bR_b}}. \quad (3)$$

$$\tau = \frac{12qx(L/2 - x) \cdot bh(x)^2}{8b \cdot bh(x)^3} = \frac{1.5qx(L/2 - x)}{bh(x)} = R_{hs} \Rightarrow h_Q(x) = \frac{1.5q(L/2 - x)}{bR_{hs}}. \quad (4)$$

Next, three tangents to the curve of function $h_M(x)$ were constructed. One of them ran horizontally through its extremum (point D), the vertical coordinate of which corresponds to the maximum height of the cross-section at the midpoint of the span h_{max} . The other two tangents were drawn from points A and G , the vertical coordinates of which correspond to the heights of the support cross-sections h_{sup} . These tangents were, in turn, the extrema of function $h_Q(x)$ and touched the graph of function $h_M(x)$ at points B and E . Solving the problem of determining the rational outline of the timber beam reduced to determining the coordinates of the nodal points of the contour A, B, C, D, E, F, G (see Figure 1).

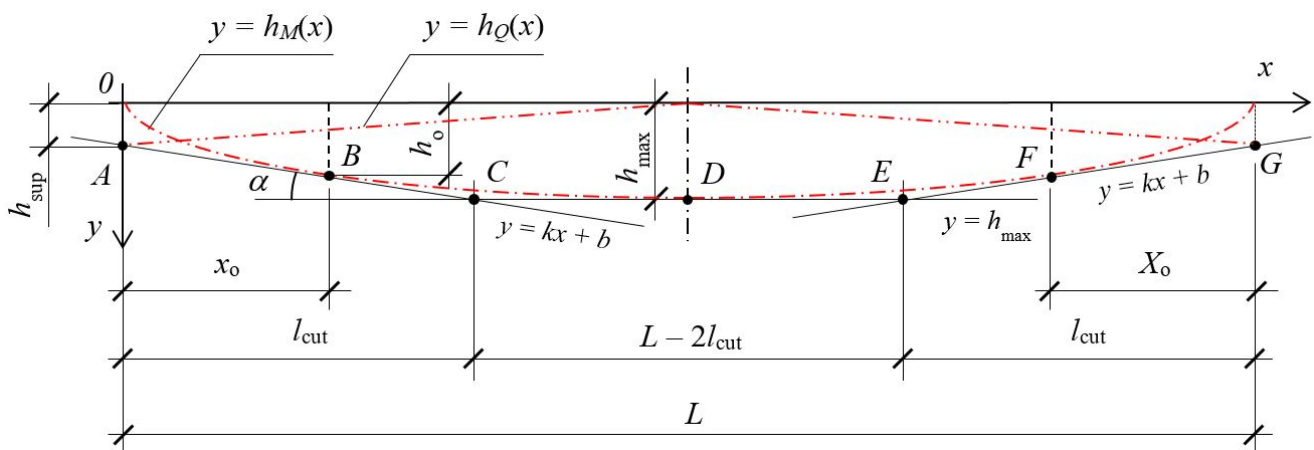


Figure 1. Diagram for determining the rational outline of a timber beam

Source: made by A.V. Repin.

In the second stage, half of the beam span from the origin was considered and the coordinates of points A, D , and G were determined. A coordinate system with its origin at point O was defined, which coincided with the starting point of diagram $h_M(x)$, and the y -axis was oriented vertically downward. The horizontal

coordinates of points A , D , and G corresponded to the beginning, middle, and end of the beam span: $x_A = 0$; $x_D = L/2$; $x_G = L$. The vertical coordinates of points A , D , and G corresponded to the heights of the support sections and the middle of the span (maximum height): $y_A = y_G = h_{\text{sup}}$; $y_D = h_{\text{max}}$. The final coordinates of the points were as follows: $A(0; h_{\text{sup}})$, $D(L/2; h_{\text{max}})$, $G(L; h_{\text{sup}})$.

Thus, based on the above, the following equations were obtained:

$$y_D = h_{\text{max}} = h_M(L/2) = \sqrt{\frac{3qL(L-L/2)/2}{bR_b}} = L\sqrt{\frac{0.75q}{bR_b}}, \quad (5)$$

$$y_A = h_{\text{sup}} = h_Q(0) = \frac{1.5q(L/2-0)}{bR_{\text{hs}}} = \frac{0.75qL}{bR_{\text{hs}}}. \quad (6)$$

In the next stage, the equations for the tangent line to curve $h_M(x)$ were composed:

$$y = h_M(x_o) + h_M'(x_o)(x - x_o) = \sqrt{\frac{3qx_o(L-x_o)}{bR_b}} + \left(\sqrt{\frac{3qx_o(L-x_o)}{bR_b}} \right)' (x - x_o) \Rightarrow$$

$$y = \sqrt{\frac{0.75q}{bR_b x_o(L-x_o)}} (Lx_o + x(L-2x_o)). \quad (7)$$

$$\text{where } h_M'(x_o) = \left(\sqrt{\frac{3qx_o(L-x_o)}{bR_b}} \right)' = \frac{(3qx_o(L-x_o)/(bR_b))'}{2\sqrt{3qx_o(L-x_o)/(bR_b)}} = (L-2x_o) \sqrt{\frac{0.75q}{bR_b x_o(L-x_o)}}.$$

Next, the position of point B was determined using the equation of the tangent line (7). For this, it was specified in the equation that $x_B = x_o$ and the coordinates of point A were substituted for x and y :

$$\frac{0.75qL}{bR_{\text{hs}}} = \sqrt{\frac{0.75q}{bR_b x_B(L-x_B)}} (Lx_B + 0(L-2x_B)).$$

Then, using the equation given above, x_B was expressed:

$$x_B = \frac{0.75qR_b}{bR_{\text{hs}}^2 + 0.75qR_b} L. \quad (8)$$

The horizontal coordinate of point B is numerically equal to the distance from the support axis to the critical section in the near-support zones of variable rigidity.

The vertical coordinate of point B was determined by substituting $x = x_o = x_B$:

$$y = h_M(x_o) + h_M'(x_o)(x_o - x_o) = h_M(x_o) = \sqrt{\frac{3qx_o(L-x_o)}{bR_b}} \Rightarrow$$

$$y_B = \frac{1.5qR_{\text{hs}}}{bR_{\text{hs}}^2 + 0.75qR_b} L. \quad (9)$$

The vertical coordinates of points B and F are numerically equal to the height of the critical section in the near-support zones of variable rigidity: $h_0 = y_B$.

To determine the coordinates of the points where tangents C and E intersect, y in the equation of the tangent (7) was substituted with expression (5) for h_{\max} :

$$\sqrt{\frac{0.75qL^2}{bR_b}} = \sqrt{\frac{0.75q}{bR_b x_o(L-x_o)}} (Lx_o + x(L-2x_o)).$$

Next, the value of x_C from the equation given above was derived:

$$x_C = \frac{\sqrt{x_o(L-x_o)} - x_o}{L-2x_o}. \quad (10)$$

Substituting x_0 in equation (10) with the expression for x_B , the following is obtained:

$$x_C = \frac{L}{1 + R_{hs} \sqrt{b / (0.75qR_b)}}. \quad (11)$$

The horizontal coordinate of point C is numerically equal to the length of the undercut in the support zones of the beam: $l_{\text{cut}} = x_C$.

In the next stage of the study, based on the expressions given above, calculations were performed to determine the rational outline of the timber beam, and the results of the study were analyzed.

During the process of constructing the trajectory of the beam of uniform strength, cross-section heights were obtained for which the ratio of internal forces to the ultimate strength was equal to one; therefore, the structural capacity was virtually exhausted. It was decided that when determining the outline of a rational beam, a safety factor of, for example, 8% is applied. The values of the design strengths will be multiplied by safety factor $\eta = 1.08$.

Design parameters:

- of 1st-grade pine timber:
 - bending and tensile strength $R_b^A = 21.0$ MPa;
 - shear strength along the grain of glued members in bending $R_{hs}^A = 2.4$ MPa.
- for 2nd-grade pine timber:
 - bending and tensile strength $R_b^A = 19.5$ MPa;
 - shear strength along the grain of glued members in bending $R_{hs}^A = 2.25$ MPa.

Values of service condition factors:

– long-term strength factor $m_{lt} = 0.66$ is adopted for service conditions involving the combined action of dead, long-term temporary and occupancy loads on the floors of residential and public buildings, or the combined action of dead and short-term snow loads;

– coefficient m_b depends on the height of the rectangular cross-section of glued laminated elements in bending. In this study, the calculation was performed for beams with different cross-sectional heights; therefore, in each case, the values of m_b , and the design strength values of timber in bending R_b^D and horizontal shear R_{hs}^D will differ (see Table 1);

– the values of the remaining service condition factors such as m_s, m_h, m_t, m_l are taken as equal to one, and $m_o, m_a, m_c, m_{s,l}, m_{cr}, m_{ptl}$ are not used.

Rational timber beams (RTB) with spans of 6.0, 9.0, 12.0, and 15.0 m were examined, with cross-sectional dimensions relative to the span as follows: height $h = (L/10; L/12; L/15)$; width $b = (h/4; h/5; h/6)$.

The analysis was performed in two directions:

- at each step, relative cross-sectional dimensions h/L and b/h were assigned, based on which the absolute values of h and b , the geometric characteristics of the cross-section, etc., were calculated;
- by varying the value of the design load q , a perfect match was achieved between the values of the extrema of the trajectory of a beam of uniform strength and the absolute value of h .

Table 1. Values of design strength of timber taking into account service condition factors m_i and the specified safety factor

Design characteristics	Section height, cm					
	50 and lower	60	70	80	100	120 and higher
Coefficient m_b	1	0.96	0.93	0.90	0.85	0.8
$R_b^D = R_b^A \cdot m_{lt} \cdot \prod m_i / \eta$, MPa	13.86	13.31	12.89	12.47	11.78	11.09
$R_{hs}^D = R_{hs}^A \cdot m_{lt} \cdot \prod m_i / \eta$, MPa	1.584	1.521	1.473	1.426	1.346	1.267

Source: made by M.S. Lisyatnikov.

Based on the data provided above, the coordinates of the key nodal points governing the rational outline of the timber beam were determined. Next, the geometric parameters of the RTB were calculated: the height of the support section h_{sup} , the height of the critical section in the zones of variable beam rigidity h_0 , the length of the undercut in the near-support zones l_{cut} , and the undercut angle α_{cut} . The economic efficiency of the new structural design was evaluated by comparing the difference in wood consumption between the RTB and beams of constant rigidity. For the same cross-sectional width, the wood consumption is numerically equal to the ratio of the lateral surface areas of the compared structures:

$$\omega = \frac{V_{RB}}{V_{CB}} = \frac{S_{RB}}{S_{CB}} = \frac{h(L - 2l_{cut}) + l_{cut}(h_{sup} + h)}{h \times L}, \quad (12)$$

where V_{RB} , V_{CB} , S_{RB} , S_{CB} are the volumes and surface areas of the rational beam and the constant (rigidity) beam; h , h_{sup} are the cross-sectional heights at the midspan and at the supports, respectively; L is the span of the beam; l_{cut} is the length of the undercut in the near-support zones of the beam.

The calculation process and verification of the results were performed using the *MS Excel* spreadsheet program.

3. Results and Discussion

Figure 2 shows the diagrams of RTB with a span of 9.0 m for $L/h = 10$, 12, and 15, generated using the Chart Wizard in *MS Excel*.

The results of the analysis are presented in Table 2. They show that the relative parameters of the rational beam outline depend on the value of the relative section height h at the midspan. Other characteristics — span L , cross-sectional width b , etc. — have virtually no effect. It has been established that as the section height h increases, the values of such relative values as the length of the undercut l_{cut} / L , the height of the support section h_{sup} , and the coordinate of the critical section in zones of variable rigidity also increase.

By performing calculations for intermediate values of the relative section height, taking into account the strength characteristics of structural timber of grades 1 and 2, the relative geometric characteristics of the RTB as a function of various L/h ratios are obtained. The calculation results are presented in Tables 3 and 4 and in Figures 3 and 4.

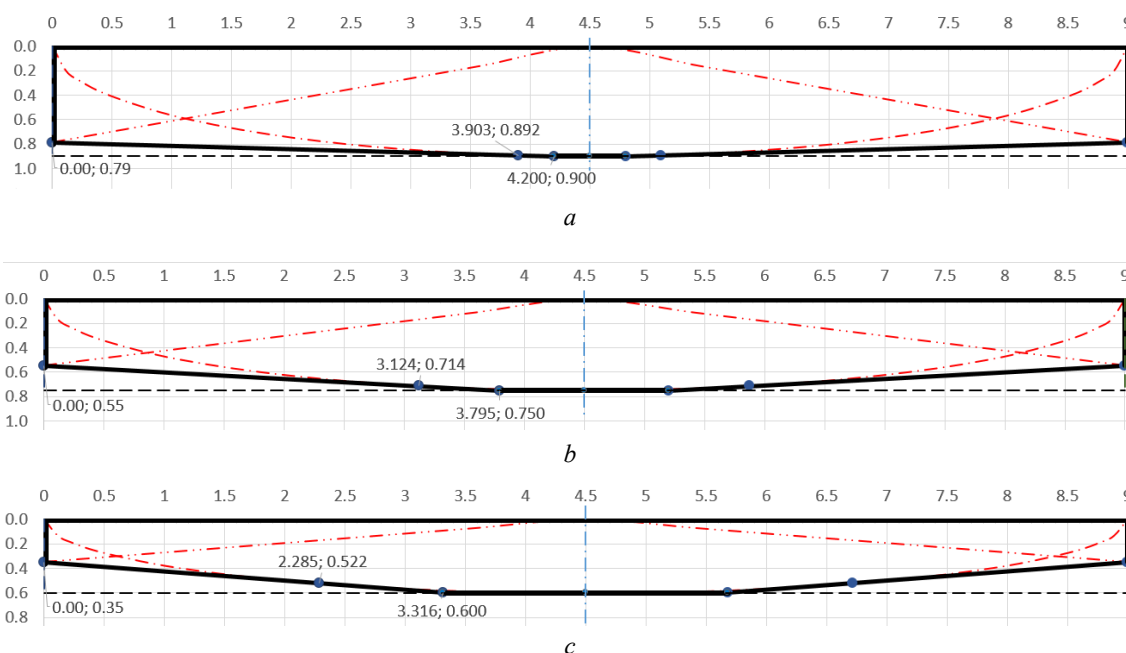


Figure 2. General view of rational beam outlines using the example of structures with a span of $L = 9.0$ m for:

a — $L/h = 10$; b — $L/h = 12$; c — $L/h = 15$

Source: made by A.V. Repin.

Table 2. Main results of the calculation for determining the parameters of the rational outline of timber beams made of grade 1 timber

L, m	Parameters and indices	$h = L/10$			$h = L/12$			$h = L/15$		
		$b = h/4$	$b = h/5$	$b = h/6$	$b = h/4$	$b = h/5$	$b = h/6$	$b = h/4$	$b = h/5$	$b = h/6$
6.0	$q, kN/m$	26.61	21.29	17.74	16.04	12.83	10.69	8.21	6.57	5.48
	h, m	0.600			0.500			0.400		
	h_{sup}, m	0.525			0.365			0.233		
	b, m	0.150	0.120	0.100	0.120	0.096	0.080	0.100	0.080	0.067
	X_0, m	2.602			2.083			1.523		
	l_{cut}, m	2.800			2.530			2.211		
	$\alpha_{cut}, ^\circ$	1.534			3.064			4.312		
	h_{sup} / h	0.875			0.729			0.583		
	X_0 / L	0.434			0.347			0.254		
	l_{cut} / L	0.467			0.422			0.368		
$\omega, \%$	6.2			12.9			18.1			
9.0	$q, kN/m$	35.34	28.28	23.56	21.66	17.33	14.44	11.83	9.46	7.88
	h, m	0.90			0.75			0.60		
	h_{sup}, m	0.79			0.55			0.35		
	b, m	0.23	0.18	0.15	0.19	0.15	0.13	0.15	0.12	0.10
	X_0, m	3.903			3.124			2.285		
	l_{cut}, m	4.200			3.795			3.316		
	$\alpha_{cut}, ^\circ$	1.535			3.064			4.312		
	h_{sup} / h	0.875			0.729			0.583		
	X_0 / L	0.434			0.347			0.254		
	l_{cut} / L	0.467			0.422			0.368		
$\omega, \%$	6.2			12.9			18.1			

Ending of the Table 2

L, m	Parameters and indices	$h = L/10$			$h = L/12$			$h = L/15$		
		$b = h/4$	$b = h/5$	$b = h/6$	$b = h/4$	$b = h/5$	$b = h/6$	$b = h/4$	$b = h/5$	$b = h/6$
12.0	$q, kN/m$	44.35	35.48	29.57	27.27	21.82	18.18	14.78	11.83	9.86
	h, m	1.20			1.00			0.80		
	h_{sup}, m	1.05			0.73			0.47		
	b, m	0.30	0.24	0.20	0.25	0.20	0.17	0.20	0.16	0.13
	X_0, m	5.203			4.165			3.047		
	l_{cut}, m	5.60			5.06			4.42		
	$\alpha_{cut}, ^\circ$	1.535			3.064			4.312		
	h_{sup} / h	0.875			0.729			0.583		
	X_0 / L	0.434			0.347			0.254		
	l_{cut} / L	0.467			0.422			0.368		
$\omega, \%$	6.2			12.9			18.1			
15.0	$q, kN/m$	55.43	44.35	36.96	32.08	25.67	21.39	17.45	13.96	11.64
	h, m	1.50			1.25			1.00		
	h_{sup}, m	1.31			0.91			0.58		
	b, m	0.38	0.30	0.25	0.31	0.25	0.21	0.25	0.20	0.17
	X_0, m	6.50			5.21			3.81		
	l_{cut}, m	7.00			6.33			5.53		
	$\alpha_{cut}, ^\circ$	1.536			3.065			4.312		
	h_{sup} / h	0.875			0.729			0.583		
	X_0 / L	0.434			0.347			0.254		
	l_{cut} / L	0.467			0.422			0.368		
$\omega, \%$	6.2			12.9			18.1			

Source: made by M.S. Lisyatnikov.

Table 3. Summary table of calculation results for beams made of grade 1 timber

Parameters and indices	Relative height of the section h										
	$L/10$	$L/10.5$	$L/11$	$L/11.5$	$L/12$	$L/12.5$	$L/13$	$L/13.5$	$L/14$	$L/14.5$	$L/15$
$\alpha_{cut}, ^\circ$	1.535	2.000	2.404	2.755	3.064	3.337	3.577	3.792	3.984	4.156	4.312
h_{sup} / h	0.875	0.833	0.795	0.761	0.729	0.700	0.673	0.648	0.625	0.603	0.583
X_0 / L	0.434	0.410	0.388	0.367	0.347	0.329	0.312	0.296	0.281	0.267	0.254
l_{cut} / L	0.467	0.455	0.443	0.432	0.422	0.412	0.402	0.393	0.385	0.376	0.368
$\omega, \%$	6.2	8.2	10.0	11.5	12.9	14.1	15.1	16.1	16.9	17.5	18.1

Source: made by M.S. Lisyatnikov.

Table 4. Summary table of calculation results for beams made of 2nd grade timber

Parameters and indices	Relative height of the section h										
	$L/10$	$L/10.5$	$L/11$	$L/11.5$	$L/12$	$L/12.5$	$L/13$	$L/13.5$	$L/14$	$L/14.5$	$L/15$
$\alpha_{cut}, ^\circ$	1.645	2.107	2.505	2.855	3.159	3.429	3.669	3.881	4.071	4.242	4.395
h_{sup} / h	0.867	0.825	0.788	0.754	0.722	0.693	0.667	0.642	0.619	0.598	0.578
X_0 / L	0.429	0.405	0.383	0.362	0.343	0.325	0.308	0.292	0.277	0.263	0.250
l_{cut} / L	0.464	0.452	0.441	0.430	0.419	0.409	0.400	0.391	0.382	0.374	0.366
$\omega, \%$	6.6	8.6	10.3	11.8	13.2	14.4	15.4	16.3	17.0	17.7	18.3

Source: made by M.S. Lisyatnikov.

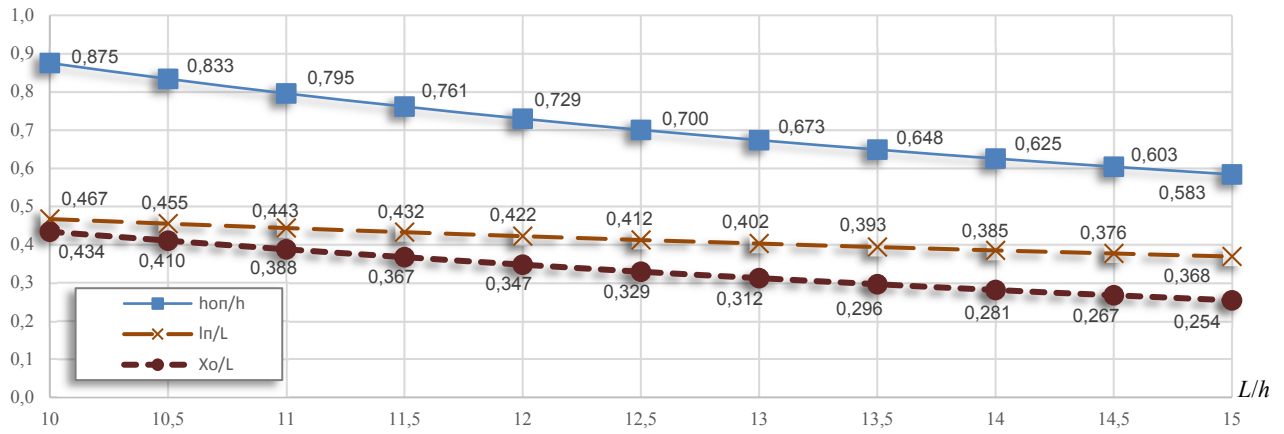


Figure 3. Diagram of the relationship between the relative parameters of the rational outline of timber beams and the relative cross-section height L/h . 1st grade timber

Source: made by A.V. Repin.

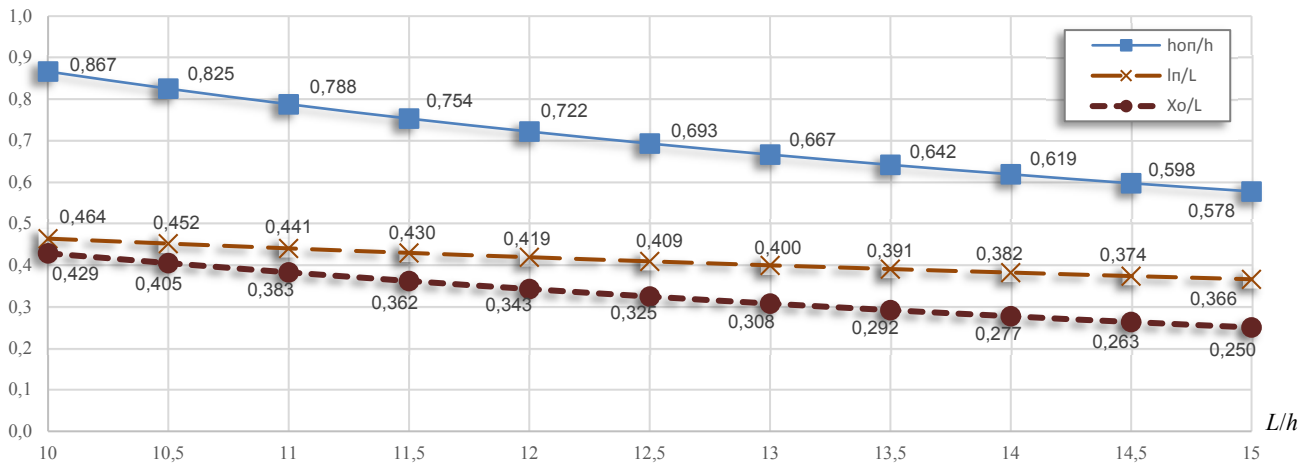


Figure 4. Diagram of the relationship between the relative parameters of the rational outline of timber beams and the relative cross-section height L/h . 2nd grade timber

Source: made by A.V. Repin.

Calculations show that including a safety factor and/or changing the service condition factors has no influence whatsoever on the relative values of the rational beam parameters. The absence of influence indicates that the dominant factor is the ratio between the values of the design strengths to bending and horizontal shear, while the service condition factors and safety factors, judging from negligible influence, are similarity factors.

A techno-economic evaluation of the RTB structural designs shows that their economic efficiency increases as the section height decreases from $L/h = 10$ to $L/h = 15$: from 6.2 to 18.1% for grade 1 timber and from 6.6 to 18.3% for grade 2 timber (Figure 5). At the same time, the difference between the corresponding economic efficiency values for grade 1 and grade 2 timber is within 0.4%. Thus, the influence of timber grade on the economic efficiency of RTB structural designs is insignificant. Given the presence of a large number of factors with insignificant influence on the parameters of the rational beam, it follows that the RTB analysis method will reduce to determining the relative section height L/h and subsequently selecting the RTB parameters from the corresponding table. Intermediate values are determined by interpolation.

Future research will focus on developing the methodology for RTB analysis and conducting a comparative analysis with the results of numerical studies using FEM.

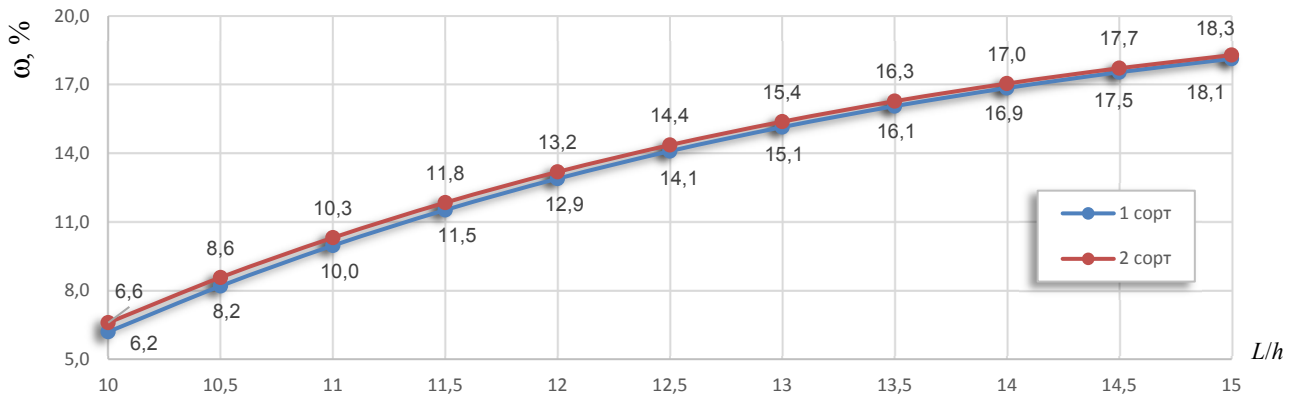


Figure 5. Diagram of the relationship between the economic efficiency (ω) of the RTB and the relative height of the section L/h

Source: made by A.V. Repin.

4. Conclusion

The following conclusions were drawn as a result of the study:

1. Theoretical studies of single-span timber beams were conducted to determine their rational outline based on the uniform strength trajectories of beams with constant cross-sectional widths in bending and horizontal shear. Expressions were derived to determine the coordinates of the nodal points of the rational beam contour, and calculations were performed to determine the absolute and relative values of the geometric parameters of rational timber beams for various spans and cross-sectional dimensions;

2. Analysis of the calculation results showed that the relative values of the shape parameters of a rational beam depend on the relative section height h at midspan. Other characteristics — length L , cross-sectional width b , etc. — have virtually no effect. It has been established that as the section height h increases, the values of the following relative values also increase:

- the length of the undercut l_{cut} / L ;
- the height of the support section h_{sup} ;
- the coordinate of the critical section in zones of variable rigidity.

3. A techno-economic evaluation of structural designs for rational timber beams shows that the economic efficiency (wood consumption relative to a beam of constant rigidity) increases as the cross-sectional height decreases: from 6.2 to 18.1% for grade 1 timber and from 6.6 to 18.3% for grade 2 timber; the difference between the corresponding values is within 0.4%. The insignificant difference in wood consumption indicates that the grade of timber has a negligible effect on the economic efficiency of the studied structures.

References

1. Zapoev M.A., Romanov N.P., Belyaeva S.V. Features of glued laminated timber anisotropic structure. *Alfabuild*. 2018;4(6):83–91. <https://doi.org/10.34910/ALF.6.8> EDN: MYLBYQ
2. Xiong W., Xu J., Li Z., Xiao Y. Use of glued laminated bamboo (glubam), and cross-laminated bamboo and timber (CLBT) in the environmentally-friendly construction: A study from the perspective of fatigue properties. *Engineering Structures*. 2025;342:120979. <https://doi.org/10.1016/j.engstruct.2025.120979> EDN: MKIFAU
3. Gong Y., Liu R., Yao L., Ren H., Xu J. Innovation analysis of carbon emissions from the production of glued laminated timber in China based on real-time monitoring data. *Journal of Cleaner Production*. 2024;469:14314. <https://doi.org/10.1016/j.jclepro.2024.143174> EDN: CVPZKW
4. Bremner A., Poudyal N.C., Nepal P., Brandeis C., Taylor A., Bergman R. Social, economic, and policy aspects of cross-laminated timber: A review of emerging literature and future research needs. *Forest Policy and Economics*. 2026;183:103695. <https://doi.org/10.1016/j.forpol.2025.103695>
5. Tsai M.-T., Soegiono P.D., Lee W.-L. Life cycle assessment and environmental impact of novel cross-laminated timber composite façade applied in renovation of aged reinforced concrete residential building in subtropical area. *Energy and Buildings*. 2026;357:117085. <https://doi.org/10.1016/j.enbuild.2026.117085>

6. Hassan O.A., Johansson C. Glued laminated timber and steel beams. *Journal of Engineering, Design and Technology*. 2018;16(3):398–417. <https://doi.org/10.1108/JEDT-12-2017-0130>
7. Zhadanov V.I., Nesterenko M.A., Pinaykin I.P. Analysis of structural solutions of bearing ribs of large-sized combined panels on a wooden frame. *Expert: Theory and Practice*. 2024;4:31–36. (In Russ.) https://doi.org/10.51608/26867818_2024_4_31 EDN: IBQWHQ
8. Nemirovsky Yu.V., Boltaev A.I. Calculation and design of hybrid wooden beams. *PNRPU Mechanics Bulletin*. 2017;(3):129–152. (In Russ.) <https://doi.org/10.15593/perm.mech/2017.3.11> EDN: ZJZRUX
9. Toktoraliev E.T., Asrankulov T., Karoolbek K.A., Abutalipov E.A. Determination of optimal slope of top chord for the timber structures. *Science and Innovative Technologies*. 2020;(1):221–228. (In Russ.) <https://doi.org/10.33942/sit.nes031> EDN: FKSZPD
10. Shorstov R.A., Yaziev S.B., Chepurnenko A.S., Klyuev A.V. Flat bending shape stability of rectangular cross-section wooden beams when fastening the edge stretched from the bending moment. *Construction Materials and Products*. 2022;5(4):5–18. (In Russ.) <https://doi.org/10.58224/2618-7183-2022-5-4-5-18> EDN: GIVPBC
11. Vafadar F., Jaaranen J., Fink G. Probabilistic numerical modeling of glued laminated timber beams — Experimental validation and insights into the potential of layout modification. *Engineering Structures*. 2025;344:121310. <https://doi.org/10.1016/j.engstruct.2025.121310> EDN: VBZODI
12. Kandler G., Lukacevic M., Zechmeister Ch., Wolff S., Füssl J. Stochastic engineering framework for timber structural elements and its application to glued laminated timber beams // *Construction and Building Materials*. 2018. Vol. 190. P. 573–592. <https://doi.org/10.1016/j.conbuildmat.2018.09.129>
13. Nemirovskiy Yu.V., Boltaev A.I. Features of deformation and destruction of hybrid timber beams. *Russian Forestry Journal*. 2018;(4):118–131. (In Russ.) <https://doi.org/10.17238/issn0536-1036.2018.4.118> EDN: XUBIZF
14. Sciomenta M., Spera L., Peditto A., Ciuffetelli E., Savini F., Bedon Ch., Romagnoli M., Nocetti M., Brunetti M., Fragiaco M. Mechanical characterization of homogeneous and hybrid beech-Corsican pine glue-laminated timber beams. *Engineering Structures*. 2022;264:114450. <https://doi.org/10.1016/j.engstruct.2022.114450> EDN: LNBVHQ
15. Pech S., Kandler G., Lukacevic M., Füssl J. Metamodel assisted optimization of glued laminated timber beams by using metaheuristic algorithms. *Engineering Applications of Artificial Intelligence*. 2019;79:129–141. <https://doi.org/10.1016/j.engappai.2018.12.010>
16. Novitsky Ya.Ya., Khanko O.V., Yushkevich I.V. Theoretical studies of proportioning of sections of glued beams with a thin wall. *Innovative regional development: The potential of science and modern education. Proceedings of the VI National Scientific and Practical Conference with international participation*. Astrakhan, February 08–09, 2023. p. 43–47. (In Russ.) EDN: JIEGKL
17. Vida Ch., Lukacevic M., Eberhardsteiner J., Füssl J. Modeling approach to estimate the bending strength and failure mechanisms of glued laminated timber beams. *Engineering Structures*. 2022;255:113862 <https://doi.org/10.1016/j.engstruct.2022.113862> EDN: DIUQOE
18. Vafadar F., Jaaranen J., Fink G. Experimental stiffness investigation of finger joints in glued laminated timber beams using digital image correlation. *Construction and Building Materials*. 2024;438:137095. <https://doi.org/10.1016/j.conbuildmat.2024.137095> EDN: UNJELC
19. Roshchina S.I., Lisyatnikov M.S., Griбанov A.S., Glebova T.O. Calculation and strengthening of maximum stressed near support zones of high glue-wood beam constructions. *Forestry Engineering Journal*. 2015;(5):187–197. (In Russ.) <https://doi.org/10.12737/11276> EDN: TVXUFV
20. Lisyatnikov M.S. Improving the technology of glued wooden structures with increased support zones. *Forestry Engineering Journal*. 2015;5(2):137–148. (In Russ.) <https://doi.org/10.12737/111988> EDN: TZCARL
21. Lukina A., Lisyatnikov M., Lukin M., Vatin N., Roshchina S. Strength properties of raw wood after a wildfire. *Magazine of Civil Engineering*. 2023;(3):11907. <https://doi.org/10.34910/MCE.119.7> EDN: JIUHQK
22. Ding M., Hua Zh., Lin Y., Zhong X., Yi J., Wei Y. Experimental analysis and finite element simulation of the flexural performance of glued laminated bamboo beam string structures // *Construction and Building Materials*. 2026. Vol. 519. Article no. 145854. <https://doi.org/10.1016/j.conbuildmat.2026.145854>
23. Shein A.I., Zemtsova O.G. Closed solution of the problem of optimizing multi-story frame systems from the stability conditions. *Transport Facilities*. 2018;5(2):6. (In Russ.) <https://doi.org/10.15862/06SATS218> EDN: UUCBHQ
24. Shevchenko A.V., Shapovalov S.M. Calculation of composite wood beams is based on variational method. *Bulletin of Bstu Named After V.G. Shukhov*. 2017. № 1. С. 88–91. (In Russ.) <https://doi.org/10.12737/23928> EDN: XHLEJZ
25. Repin V.A., Lukina A.V., Usov A.S. Rational structural solutions for triangular trusses. *Structural Mechanics of Engineering Constructions and Buildings*. 2023;19(2):199–209. (In Russ.) <http://doi.org/10.22363/1815-5235-2023-19-2-199-209> EDN: CUKIGR
26. Vida Ch., Lukacevic M., Hochreiner G., Pech S., Füssl J. Numerical modeling of glued laminated timber beams without finger joints: Identifying load-bearing capacity and analyzing failure mechanisms. *Engineering Structures*. 2025;345:121489. <https://doi.org/10.1016/j.engstruct.2025.121489> EDN: YTJPSB

27. Jaaranen J., Fink G. A finite element simulation approach for glued-laminated timber beams using continuum-damage model and sequentially linear analysis. *Engineering Structures*. 2024;304:117679. <https://doi.org/10.1016/j.engstruct.2024.117679> EDN: GNKZQAQ
28. Miryaev B.V., Sorokina E.A. Mesh wooden dome frame optimization. *Modeling and Mechanics of Structures*. 2023;(17):95–104. (In Russ.) EDN: RKUTVQ
29. Yaziev B.M., Chepurnenko A.S., Karamysheva A.A., Yaziev S.B. Optimization of wooden beams of variable cross-section Certificate of registration of the computer program RU 2016615085, 05.16.2016. Application No. 2016612477 dated 03/23.2016. (In Russ.) EDN: YGMPRJ
30. Kryukova A.A., Suzyumov A.V., Yartsev V.P. About coefficient of compliance, error and bearing capacity of composite wood elements. *Ecological and resource-saving technologies in science and technology : proceedings of the All-Russian Scientific and Technical Conference*. October 19-20, 2021. Voronezh: Voronezh State Forestry Engineering University named after G.F. Morozov; 2021. p. 107–111. (In Russ.) https://doi.org/10.34220/ERSTST2021_107-111 EDN: REJFX Y
31. Capellán G., Sánchez-Haro Ja., de Celis P., Ramos-Gavilán A.B. Theoretical, experimental and numerical study of lateral buckling in glued laminated timber beams. *Engineering Structures*. 2025;338:120558. <https://doi.org/10.1016/j.engstruct.2025.120558> EDN: UQKEQJ
32. Shorstov R.A., Yaziev S.B., Chepurnenko A.S. Optimization of compressed wooden bars of variable section according to the criterion of maximum critical load. *Construction and Architecture*. 2023;11(1):4. (In Russ.) <https://doi.org/10.29039/2308-0191-2022-11-1-5-5> EDN: NRRYQL
33. Li H., Wang L., Wei Ya., Wang B.J., Jin H. Bending and shear performance of cross-laminated timber and glued-laminated timber beams: A comparative investigation. *Journal of Building Engineering*. 2022;45:103477. <https://doi.org/10.1016/j.jobe.2021.103477> EDN: OGEXZG
34. Serov E.N., Belov V.V. Modern assessment of glued wooden design durability. *Bulletin of Civil Engineers*. 2016;(6):109–113. (In Russ.) EDN: XGRIZP
35. Karamysheva A.A., Yazyeva S.B., Chepurnenko A.S. Calculation of plane bending stability of beams with variable stiffness. *Bulletin of Higher Educational Institutions. North Caucasus Region. Technical Sciences*. 2016;(1):95–98. (In Russ.) <https://doi.org/10.17213/0321-2653-2016-1-95-98> EDN: VPF BMP
36. Popov E., Labudin B., Konovalov A., Karelskiy A., Sopilov V., Bobyleva A., Stolypin D. Numerical buckling calculation method for composite rods with semi-rigid ties. *Magazine of Civil Engineering*. 2023;(3):11904. <https://doi.org/10.34910/MCE.119.4> EDN: AGVRHV
37. Schmidt A.B. Calculation of a double-slope curved glued beam of variable cross-section height. Certificate of state registration of a computer program. No. RU 2019610152. Date of publication: 09.01.2019. Registration date: 12.18.2018. Saint Petersburg: SPbGASU, 2019. EDN: FOZMJT