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АНАЛИТИЧЕСКИЕ И ЧИСЛЕННЫЕ МЕТОДЫ РАСЧЕТА КОНСТРУКЦИЙ
ANALYTICAL AND NUMERICAL METHODS OF STRUCTURAL ANALYSIS

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Analytical Modeling of Reinforced Concrete Columns Under Lateral Impact with Shear Failure

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Abstract. The issue of ensuring the mechanical safety of load-bearing structures in buildings and facilities is currently of particular relevance. One critical aspect of this problem is the strength of compressed and compressed-bent elements under transverse impact loading. Several failure mechanisms can occur in reinforced concrete (RC) columns. This paper develops an analytical methodology for determining the ultimate load capacity of square cross-section elements under horizontal impact, specifically for the failure mode associated with diagonal shear. Such scenarios are possible in cases of vehicle collision with a column or impacts near the support zone of the structure. The analytical model is based on static equilibrium equations, which incorporate the ultimate mechanical characteristics of the materials, accounting for dynamic strengthening effects. The concrete deformation model considers the confining effect in the direction perpendicular to compression, which enhances the concrete's calculated resistance but induces additional stresses in the transverse reinforcement. A numerical example of the calculation for a building's RC column is provided, yielding specific numerical results. A comparison is made between the outcomes of the proposed methodology and those obtained from a detailed numerical simulation performed using a verified solid finite element model. The limitations of the proposed analytical method are identified, and its sufficiently high accuracy and efficiency are demonstrated. Finally, prospects for further development are outlined, and recommendations for the practical application of the method to ensure the mechanical safety of reinforced concrete columns are provided.

Keywords: lateral impact, reinforced concrete structures, dynamic loading, structural safety, shear strength

Conflicts of interest. The authors declare that there is no conflict of interest.

Authors' contribution: *Alekseytsev A.V.* — research concept, development of analytical model dependencies, preparation of figures, preparation of finite element model, scientific editing; *Yurysov K.V.* — development of analytical model dependencies, verification of finite element model, execution of numerical calculation examples, preparation of text, analysis of calculation results. Both of the authors read and approved the final version of the article.

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Аналитическая модель динамического расчета железобетонных колонн при горизонтальном ударе с разрушением по наклонному сечению

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Аннотация. Проблема обеспечения механической безопасности несущих конструкций зданий и сооружений представляется особенно актуальной. Одним из важных аспектов этой проблемы является прочность сжатых и сжато-изогнутых элементов при действии поперечной ударной нагрузки. При этом для железобетонных колонн может реализовываться несколько схем разрушения. Разработана методика аналитического определения предельной нагрузки от горизонтального удара для элементов квадратного поперечного сечения при реализации схемы разрушения по наклонному сечению. Характерными аварийными ситуациями, соответствующими этой схеме разрушения, являются столкновения с колонной автомобиля или удары вблизи зоны опорного закрепления конструкции. Основой аналитического расчета являются уравнения статического равновесия, в которых используются предельные механические характеристики материалов с учетом динамического упрочнения. При деформировании бетона учитывается эффект от стеснения в направлении, перпендикулярном сжатию, что повышает расчетное сопротивление бетона, но вызывает дополнительные напряжения в поперечной арматуре. Выполнен пример расчета железобетонной колонны здания, доведенный до числовых значений, и приведено сопоставление результатов предлагаемой методики с численным расчетом. В качестве инструмента численного расчета использовалась верифицированная объемная конечно-элементная модель. Выявлены ограничения предлагаемого аналитического метода и установлена его достаточно высокая точность и эффективность. Сформулированы перспективы дальнейших разработок и даны рекомендации по конкретному использованию метода для обеспечения механической безопасности железобетонных колонн.

Ключевые слова: поперечный удар, железобетонные конструкции, динамическое нагружение, механическая безопасность, сдвиговая прочность

Заявление о конфликте интересов. Авторы заявляют об отсутствии конфликта интересов.

Вклад авторов: *Алексейцев А.В.* — концепция исследования, разработка зависимостей аналитической модели, подготовка рисунков, подготовка конечноэлементной модели, научное редактирование; *Юрусов К.В.* — разработка зависимостей аналитической модели, верификация конечноэлементной модели, выполнение числовых примеров расчета, подготовка текста, анализ результатов расчета. Оба автора ознакомлены с окончательной версией статьи и одобрили ее.

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1. Introduction

An important aspect of ensuring the safety of buildings and structures with reinforced concrete frames is checking the strength of columns and beam-columns against lateral impact. This issue has received considerable attention, which indicates the relevance of this research area. Factors such as the load-bearing capacity of reinforced concrete columns, taking into account the effect of confinement of concrete (confined concrete) [1], the deformability of composite reinforced concrete columns in carbon fiber reinforced plastic (CFRP) tubes [2], and the dynamic response and energy absorption capacity of structures with modified concrete, for example, with the addition of rubber crumb [3]. A large amount of experimental research, analytical and numerical modeling has been carried out, which shows interest in a comprehensive study of the behavior of compressed columns under impact loads. This includes corrosion damage [4], considering the longitudinal (vertical) coupler splicing of rebar along the length [5], and considering combinations of bending and compressive actions [6]. One of the most important aspects of research in column dynamics is its response and damage assessment due to horizontal impact [7]. The point of application of the impact load is important. For example, in [8], impact on a part of a column near its support node is considered. The authors of this study prove the effect of the type of supports on the load-bearing capacity of columns.

A number of papers are devoted to the study of compressed structures under low-velocity impact, for example [9]. For such an impact, the damping properties of the column and its ability to absorb energy can be increased by strengthening it with CFRP [10]. Low-velocity impact can be modeled in calculations as both quasi-static [11] and impulse load [12], where it is noted that the limit state of columns can be reached either by fracture or buckling. Buckling usually prevails for slender reinforced concrete columns with a slenderness ratio of more than 50. The complexity of describing the deformation and modeling of columns under horizontal impact has necessitated the development of simplified analytical approaches to calculations and engineering methods. Various features of these methods are presented in the following papers. Article [13] compares simplified approaches to assessing the load-bearing capacity of conventional and CFRP-strengthened reinforced concrete columns, paper [14] describes the probabilistic nature of the loading effect and assesses the reliability of structures, and study [15] reveals the influence of the percentage of longitudinal reinforcement on the load-bearing capacity. Analytical models for determining the load-bearing capacity under dynamic impact are based on quasi-static equilibrium equations [16], which have been refined for cases of random corrosion damage [17] and various compressive and impact force ratios [18]. There are significant differences in calculation methods depending on the impact velocity, the stiffness of the impactor body, the stiffness of the column itself, and the penetration of the impactor tip into it. In [19], a significant reduction in the stiffness and load-bearing capacity of a compressed column as a result of corrosion is noted, but stiffness can also decrease due to cracking under combined loads, including static moments and shear forces [20]. A number of studies focus on the stiffness of the impactor. If the stiffness of the impactor body is high, then all the kinetic energy is transferred to the impacted structure ("hard" impact), and if the impactor itself can absorb energy during impact, such an impact is considered soft [21]. In addition to the stiffness of the impact, the shape of the cross-section has a significant effect on the dynamics of columns. Thus, when comparing the results of studies [22] (square cross-section) and [23] (round cross-section), it can be seen that round columns are more vulnerable to brittle shear failure than square ones.

A number of considered studies [7; 8; 18; 22] involve the use of intensive computational procedures and require highly qualified researchers, which significantly hinders the practical application of the proposed developments.

Therefore, the purpose of this article is to develop an engineering method for calculating compressed columns under impact causing shear failure. To achieve this goal, it is necessary to solve the problem of constructing a simplified method for evaluating the load-bearing capacity and to verify it on the basis of experiments and numerical modeling. The object of the study is a compressed reinforced concrete column subjected to horizontal impact at the support, and the subject of the study is the ultimate shear bearing capacity.

2. Methods

When a horizontal force is dynamically applied to the column, most researchers, including [24], identified the following failure patterns: local crushing, which may be accompanied by chipping or punching (local failure), bending failure (along normal sections), and shear failure (along inclined sections) (Figure 1).

Considering experimental and theoretical studies of column shear failure, as well as qualitative results of numerical modeling, it is noted that the load-bearing capacity is significantly affected by the level of compressive loading, as well as the magnitude and direction of the static shear force present in the frame structure. As a result of dynamic impact, the column is loaded by both shear force and bending moment. Therefore, similar to normal operation, two strength conditions must be formed: strength along the inclined section under shear force taking into account added stress from dynamic loading; strength along the inclined section under bending moment taking into account its change under dynamic loading:

$$\begin{cases} k_{d1} Q_{\max} \le Q_{cd,ult}, \\ k_{d2} M_{\max} \le M_{cd,ult}, \end{cases} \tag{1}$$

where k_{d1}, k_{d2} are the dynamic load coefficients of the system; $Q_{\max}$, $M_{\max}$ are the internal forces due to load for the considered inclined section; $Q_{cd,ult}$, $M_{cd,ult}$ are the ultimate force values determined by the resistance of concrete and reinforcement in case of shear failure.

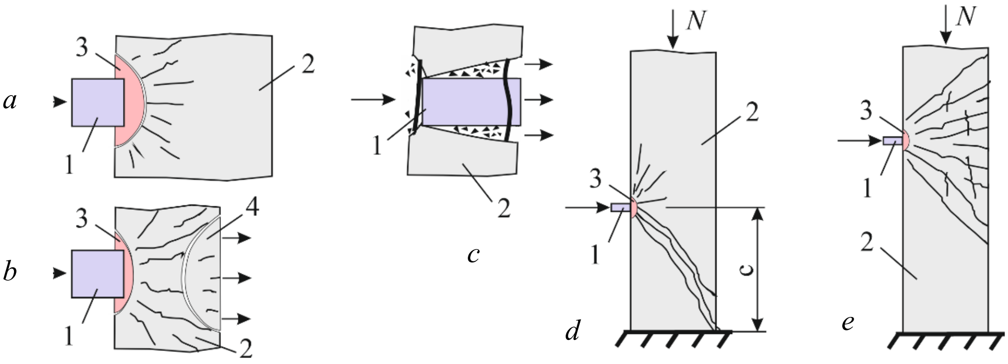

Figure 1. Failure modes under horizontal impact:
a, b, c — local crushing at various impact velocities; *d* — diagonal shear failure of a compressed column;
e — flexural failure: *1* — impactor; *2* — column; *3* — local crushing zone;
4 — concrete spall on the side opposite to the contact area between the impactor and the column
S o u r c e: made by A.V. Alekseytsev.

The method chosen for solving the system of inequalities (1) consists of compiling static equilibrium equations with respect to the lateral axis of the column and with respect to the point passing through the beginning of the inclined section. Cases of local failure (Figure 1, *a*–*c*) are not considered in this study. For numerical verification, the method of direct integration using the implicit scheme of differential equations of motion of the system, discretized using the finite element method (FEM), was adopted. For a reinforced concrete frame, this equation can be represented in the form of (2).

$$M\,\ddot{\vec{y}} + C\,\dot{\vec{y}} + K_{\tau}\vec{y} = \vec{F}, \qquad (2)$$

where $\ddot{\vec{y}}, \dot{\vec{y}}, \vec{y}$ are the acceleration, velocity and displacement vectors respectively, $\vec{F}$ is the vector of nodal forces, M is the mass matrix. Damping matrix C and global shear stiffness matrix K_{τ} in formula (2) are determined using formula (3):

$$C = \beta K_{\tau};\; K_{\tau} = K_{co} + K_{ro} + K_{so}, \qquad (3)$$

where β is the structural damping constant; K_{co}, K_{ro}, K_{so} are the shear coefficient matrices for concrete, reinforcement and supports (in case when the column rests on a deformable base).

3. Results and Discussion

3.1. Calculation Procedure

The *first strength condition* is obtained from expression (1). If there is static load present in the system before the dynamic impact, the following formula is proposed:

$$\left(\frac{\Delta Q_d}{k_N^d Q_d^{ult}} \pm \frac{Q_{st}}{Q_{bN} + Q_{swN}} \right) \le 1, \qquad (4)$$

where ΔQ_d is the added shear force caused by the horizontal impact; k_N^d is the coefficient accounting for the restraints and the column stress state under service load; Q_d^{ult} is the ultimate shear force in the section taking into account dynamic strengthening of concrete and reinforcement and the confinement effect; Q_{st}, Q_{bN}, Q_{swN} are respectively: the shear force from the design static load, the shear force resisted by concrete, and the shear force resisted by transverse reinforcement during normal operation, taking into account the presence of longitudinal force N_e.

Coefficient k_N^d is determined according to formula (5):

$$k_N^d = \begin{cases} \frac{1}{\sqrt{1.5-\mu}}\left(k_e + \left(\frac{N_e}{N_{ult}}\sqrt{\frac{P_{Ne}^{ult}}{P_{ult}}}\right)\right)^{-1}, & \frac{N_e}{N_{ult}} < 0.6, \\ \left(\frac{P_{Ne}^{ult}}{P_{ult}} + k_e\right)^{-1}, & \frac{N_e}{N_{ult}} \geq 0.6, \end{cases} \tag{5}$$

where P_{Ne}^{ult} is the static equivalent of the lateral impact load, at the value of the service longitudinal force equal to Ne and when bending failure of the column occurs; P_{ult} is the same for longitudinal force $Ne = 0$; μ is the coefficient for converting the design column length to the geometric length. When one end of the column is fixed and the other is pinned, $\mu = 0.7$; when both ends are fixed, but compression remains possible, $\mu = 0.5$. Evaluation of P_{Ne}^{ult}, P_{ult} is performed taking into account the influence of bending moments and deflections caused by these forces. The value of N_{ult} is determined for the case of small eccentricity (virtually axially compressed bar) using the formula from SP 63.13330[1]:

$$N_{ult} = \varphi(R_b A_b + R_{sc} A_{sc}), \tag{6}$$

where φ is the buckling coefficient; R_b, R_{sc} are the design compressive strengths of concrete and reinforcement respectively; A_b, A_{sc} are the areas of concrete and reinforcement.

The value of k_e refers to the level of the confinement effect. For a square cross-section based on the basic recommendations of J. Mander's model [25], it is determined based on the percentage of longitudinal reinforcement and the geometry of transverse reinforcement:

$$k_e = \frac{1}{1-\mu_{sc}}\left(1 - \frac{S_w - d_w}{2d}\right)^2, \tag{7}$$

where μ_{sc}, d_w are the percentage of longitudinal reinforcement taking into account the area of the confinement contour (рис. 2) and the transverse rebar diameter respectively; d is the transverse rebar length along the side, which is parallel to the plane of impact *P* within the bounds of the centers of gravity of the transverse rebars perpendicular to *P*.

Ultimate force Q_d^{ult} can be determined based on the expression that is valid for shear failure with a projection length of the inclined section $c = 2h_0$:

$$Q_d^{ult} = \sqrt{3k_b R_{bt}(1+k_e)^{\left(1+\sqrt{N_e/N_{ult}}\right)} b h_0^2 q_{sw}^d}, \tag{8}$$

[1] SP 63.13330. Concrete and reinforced concrete structures. General provisions. JSC Research Center of Construction — A.A. Gvozdev Research Institute of Concrete and Reinforced Concrete (NIIZHB). 2019.

where R_{bt} is the design tensile strength of concrete; k_b is the coefficient of dynamic strengthening of concrete; b, h_0 are the width and the design height of the cross-section; q_{sw}^d is the intensity of the load resisted by the transverse reinforcement, taking into account dynamic strengthening of steel and the stress from confined concrete core (Figure 2).

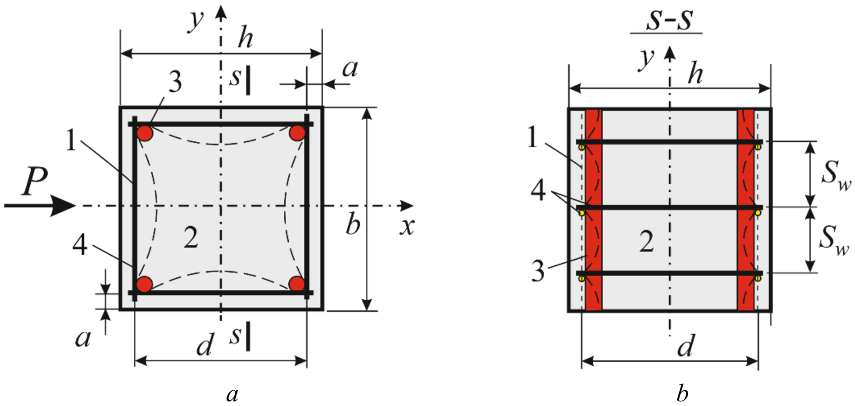

Figure 2. Determination of the confinement level:
a — column cross-section; *b* — section *s–s*: *1* — confinement boundary;
2 — core zone of the confinement; *3* — longitudinal rebars; *4* — transverse rebars
Source: made by A.V. Alekseytsev.

The value of q_{sw}^d is determined as

$$q_{sw}^d = k_s \frac{R_{sw} A_{sw}}{S_w} \left(1 - (0.2 + k_e) \frac{N_e}{N_{ult}}\right), \tag{9}$$

where R_{sw}, A_{sw} are the design strength and the area of transverse reinforcement; k_s is its coefficient of dynamic strengthening.

The value of the shear force resisted by concrete, taking into account compression by the longitudinal force, is calculated as follows, based on the methodology described in SP 63.13330 for the design of prestressed reinforced concrete structures:

$$Q_{bN} = \frac{1.5\varphi_n R_{bt} b h_0^2}{c},\ 0.5\varphi_n R_{bt} b h_0 \le Q_{bN} \le 2.5 R_{bt} b h_0, \tag{10}$$

where c is the projection of the inclined section onto the vertical axis, and the value of φ_n accounts for the presence of normal stress caused by the compressive force. It is determined as follows:

$$\varphi_n = \begin{cases} 1.25,\ 0.25R_b \le \sigma_b < 0.5R_b \\ 2.5 - \left(1 - \dfrac{\sigma_b}{R_b}\right),\ 0.5R_b \le \sigma_b \le R_b \end{cases},\ \sigma_b = \frac{N_e}{A_b + \dfrac{\alpha E_b \varepsilon_{b0}}{R_b} A_{sc}}, \tag{11}$$

where in the case of small eccentricity (all of the section is non-uniformly compressed) A_{sc} is the area of the longitudinal reinforcement; A_b is the area of concrete; $\alpha = E_s / E_b$ is the ratio of the elastic moduli of concrete and reinforcement, determined taking into account the stress state of the column; R_b is the design compressive strength of concrete; $\varepsilon_{b0} = 0.002$ is the concrete strain for short-term load.

The value of c can be determined as:

$$c = \sqrt{\frac{1.5\varphi_n R_{bt} b h_0^2}{0.75 q_{sw}^c}}, c \le 2h_0, \tag{12}$$

where the intensity q_{sw}^c of the load resisted by the transverse reinforcement is determined according to (5) at $k_s = 1$.

The second strength condition from expression (1) has the form given in SP 63.13330 and, in general, if the required development length of longitudinal reinforcement is achieved, it is satisfied

$$k_{d2} M_{\max} \le M_{cd,ult} = M_s + M_{sw}, \tag{13}$$

where M_s, M_{sw} are the bending moments resisted by the longitudinal and transverse reinforcement of the column, respectively.

3.2. Analytical Calculation

A column with the following parameters is considered: cross-section of 400×400 mm, length of 4.0 m, B25 grade concrete ($R_b = 11.5$ MPa, $R_{bt} = 0.9$ MPa), A500 grade longitudinal reinforcement with $R_s = 435$ MPa, transverse reinforcement of the same grade, but with $R_{sw} = 300$ MPa, coefficients of dynamic strengthening taking into account [26] $k_1 = 1.1$ for concrete, and $k_2 = 1.2$ for reinforcement, fixed at the base, pinned connection to the upper floor slab, $\mu = 0.7$, the distance from the exterior face of concrete (horizontal and vertical) to the center of gravity of the longitudinal reinforcement $a = 5$ cm (Figure 3).

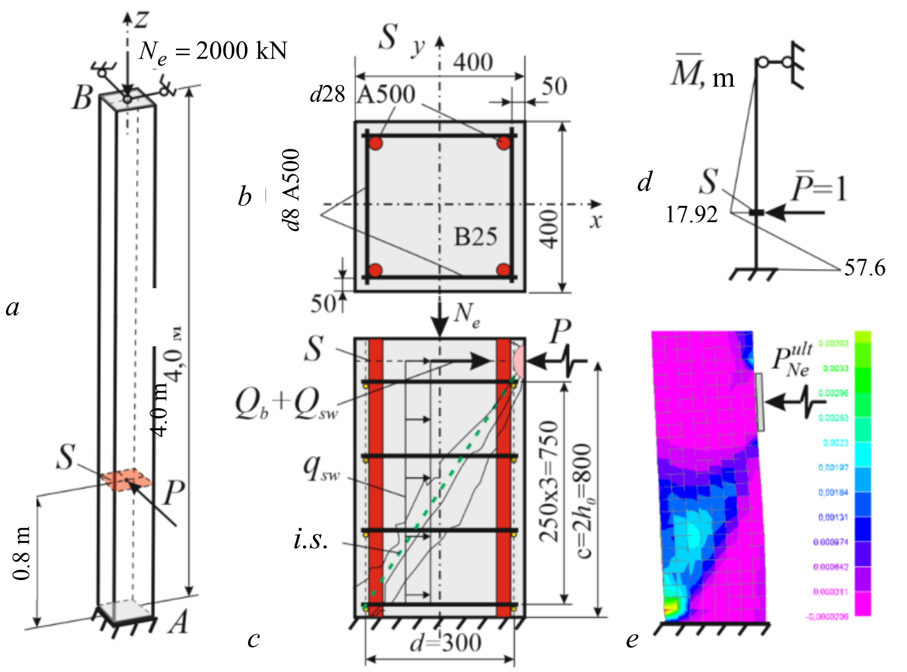

Figure 3. Calculation example for a column under horizontal impact:
a — initial structure; *b* — cross–section *S*; *c* — design model with an inclined section (i.s.);
d — bending moment diagram from a unit impact load; *e* — principal tensile strains in concrete
at the peak dynamic force under service load *Ne*

Source: made by A.V. Alekseytsev.

The column is reinforced with $4d28$ longitudinal bars, $A_{sc0}^{4d28} = 24.63\text{ cm}^2$, symmetrically at the corners. The transverse reinforcement represents a closed frame made of $4d8$, and this frame is arranged at a constant spacing of 250 mm along the height of the column, rebar area $A_{sw}^{d8} = 0.503\text{ cm}^2$. The column is assumed to be virtually axially compressed under service load $N_e = 2000\text{ kN}$, shear force $Q_{st} = 0\text{ kN}$.

The column is subjected to dynamic action in the form of a horizontal impact from car collision. This impact is simulated by a mechanical force applied at a distance of 0.8 m from the base support. It is necessary to determine the maximum value of this dynamic force, assuming that the shape of the impact impulse is rectangular.

In the calculation, it is assumed that the failure occurs along an inclined section with a projection length of $c = 2h_0$, and the strength condition for the inclined section under the action of bending moment is satisfied (the value of the moment is small compared to the beam elements).

Condition (4) takes the form $\Delta Q_d \le k_N^d Q_d^{ult}$. For determining k_N^d, the following values are calculated: $\mu_{sc} = 24.63/30 \cdot 30 = 2.73\%$, where d = 30 cm is the size of the square confinement region (Figure 3, *b, c*).

Then the confinement level

$$k_e = \frac{1}{1-\mu_{sc}}\left(1-\frac{S_w - d_w}{2d}\right)^2 = \frac{1}{1-0.0273}\left(1-\frac{25-2.8}{2\cdot 30}\right)^2 = 0.408,$$

and

$$N_{ult} = \varphi(R_b A_b + R_{sc} A_{sc0}) = 0.9(1.15\cdot 40\cdot 40 + 43.5\cdot 24.64) = 2620\text{ kN}.$$

Ratio $N_e / N_{ult} = 2000/2620 = 0.763 \ge 0.6$, the equilibrium equation at the beginning of the inclined section passing through section S (Figure 3, *c*), is used, and the conditional ultimate horizontal forces at $N_e = 0\text{ kN}$, $N_e = 2000\text{ kN}$, which would cause bending failure, are calculated. The maximum moment from the action of these forces will be at the fixed support (Figure 3, *d*), an equilibrium equation is formed for this section. The deflection from the horizontal force at the fixed support is zero, so the equation will take the form:

$M_{\max} + N_e e_f = k_1 R_b \cdot b \cdot h_0^2 \cdot \alpha_R + k_2 R_{sc} \cdot A_{sc}^{2d28} \cdot (h_0 - a)$, the maximum moment is determined using the displacement method under the condition that it is caused by force P_{Ne}^{ult}, eccentricity $e_f = ((35-5)/2) = 15$ cm. Constant α_R, associated with ensuring plastic failure mode, is determined, for which the boundary value of the relative height of compressed concrete under impact is calculated: $\xi_R = 0.8 \div \left(1 + \left(435 \div 2\cdot 10^5\right) \div 0.0035\right) = 0.493$, then $\alpha_R = 0.493(1 - 0.493/2) = 0.37$. Substituting all the obtained values into the equilibrium equation yields:

$$0.144\cdot 400 P_{Ne}^{ult} + 2000\cdot 15 = 1.1\cdot 1.15\cdot 40\cdot 35^2\cdot 0.37 + 1.2\cdot 43.5\cdot 12.32\cdot(35-5),$$

$$57.6 P_{Ne}^{ult} = 22934 + 19293 - 30000,\ P_{Ne}^{ult} = 212.74\text{ kN. When } N_e = 0\text{ kN},\ P_{ult} = 733.1\text{ kN.}$$

Then $k_N^d = (0.403 + 212.74/733.1)^{-1} = 1.4427$. The intensity of the impact load resisted by two $d8$ transverse reinforcement bars taking into account the confinement effect level and the presence of service compressive force (5) is calculated:

$$q_{sw}^d = 1.2\cdot\frac{30\cdot 1.01}{25}\left(1-(0.2+0.403)\left(\frac{2000}{2620}\right)\right) = 1.4544(1-0.422) = 0.78493\text{ kN/cm.}$$

The adopted diameter of the transverse reinforcement, taking into account the presence of longitudinal force and the adopted reinforcement spacing, must be checked for developing its full strength in concrete. Elasticity coefficient:

$$\nu_b = R_b \div E_{b0}\varepsilon_b = 11.5 \div \left(27.5 \cdot 0.002 \cdot 10^3\right) = 0.209 .$$

Average stress is calculated as

$$\sigma = N / A_{red} = 2000 / \left(40 \cdot 40 + 0.209^{-1} \cdot (2 / 2.75) \cdot 10 \cdot 24.64\right) = 1.017 \text{ kN/cm}^2,$$

coefficient

$$\varphi_n = 2.5\left(1 - \frac{\sigma}{R_в}\right) = 2.5\left(1 - \frac{1.017}{1.15}\right) = 0.289 ,$$

$$q_{sw}^d \ge q_{sw,\min} = 0.25\varphi_n R_{bt} b = 0.25 \cdot 0.289 \cdot 0.09 \cdot 40 = 0.2601 .$$

The condition is satisfied. Then, according to (4):

$$Q_d^{ult} = \sqrt{3 \cdot 1.1 \cdot 0.09 \cdot (1 + 0.403)^{\left(1+\sqrt{2/2.62}\right)} \cdot 40 \cdot 35^2 \cdot 0.7849} = 146.8 \text{ kN}.$$

The value of $\Delta Q_d = Q_d^{ult} \cdot k_N^d = 146.8 \cdot 1.4427 = 211.8$ kN.

That is, with an impact time of 1 second, the column can withstand a mechanical force of 211.8 kN.

3.3. Numerical Verification of Calculation Results

Due to the complexity of setting up and conducting a full-scale experiment that reproduces the calculated situation, the problem is verified using a three-dimensional finite element model. The Drucker — Prager plasticity model [27] was used for concrete, with the possibility of material softening under shear stress, which simulates shear failure. The reinforcement was modeled using a bilinear diagram with a limit on the rupture strain. The parameters of the concrete and reinforcement deformation models, as well as the hyperparameters of the calculation algorithm, are given in Table 1.

Verification of the finite element method (FEM) model with respect to bending strain with concrete deformation model parameters was carried out in [28]. For compressive strain, a design comparison at the ultimate force $N_{ult} = 2620$ kN is performed. Numerical simulation of this process, taking into account slow (close to static) loading, yielded a value of 2778 kN. Numerical and analytical calculations with other material parameters also yielded similar results for the ultimate compressive force. However, there are the following differences in the strength parameters obtained on the basis of the numerical and analytical models. The compressive stress in concrete obtained in the finite element model corresponds to the value of R_b, but the stress in the longitudinal reinforcement at the limit state is at the level of $\sigma = (0.4 \div 0.7) R_{sc}$, and the main criterion for stopping the load growth is the stress in the transverse reinforcement reaching its limit $\sigma = R_{sw}$. It has been established that the growth of strain in the transverse reinforcement leads to concrete failure. It is evident that analytical models need to take into account the concrete confinement effect and the formation of strength criteria in terms of strain in the presence of transverse reinforcement, as well as the need to consider the aspects of concrete deformation in the area surrounding the reinforcement, which has been done in [29].

Using the verified finite element model, calculations of the considered column were made at various values of N_e, and these were compared with the results obtained using the proposed analytical method (Table 2).

Table 1. **FEM-analysis constants**

Material / Hyperparameters	Parameters				
	Cohesion stress	**Internal friction angle**	**Dilation angle**	**Yield strength**	**Ultimate strain**
B25 concrete	3.3 MPa	38 deg.	28 deg.	0.9 MPa (tension) 11.5 MPa (compression)	0.0001 (tension) 0.0035 (compression)
A500 longitudinal rebars	–	–	–	435	0.025
A500 transverse rebars	–	–	–	300	0.025
General damping ratio	5%				
Convergence criterion	Load convergence tolerance 0.1%				
Nonlinear iteration	25 Newton — Raphson iterations per step, stiffness matrix updates each 5 iterations				
Integration step and time	$\Delta t = 0.05$ s , $t = 1.5$ s				

S o u r c e: made by A.V. Alekseytsev.

Table 2. **Comparison of the ΔQ_d values**

Method	ΔQ_d, with axial compression force Ne equal to				
	2500 kN	**2000 kN**	**1500 kN**	**1000 kN**	**500 kN**
FEA	109	195	256	287	309
Proposed method (M)	**257.28**	211.8	272.4	275.3	298.2
Tolerance $\delta =$(FEA-M)/M	**–91 %**	–0.07	–0.048	0.026	0.022

S o u r c e: made by A.V. Alekseytsev.

The FEA results in this problem agree well with the analytical ones. As the table shows, the applicability of the proposed methodology should be limited by the level $N_e / N_{ult} < 0.8$, since the error of the method as $N_e \to N_{ult}$ becomes very large. Calculations have shown that in the presence of service transverse force Q_{st} condition (1) yields results, which correspond with numerical modeling quite satisfactorily.

3.4. Discussion and Perspectives

The presented method is based on one of the possible scenarios of column failure, when the dynamic force increases at a relatively low rate, so that the stress in the concrete does not exceed the crushing stress, and its strain rate does not exceed the critical values leading to the formation of crack fans. If the load application rate is high, then during contact interaction, a failure pattern in the form of punching or chipping may occur. The calculation models for compressed elements given in a number of regulatory documents do not currently take into account the effects of concrete dilation, confinement effect, and the emergence of stress in the transverse reinforcement due to this confinement, but numerical models and experiments confirm these effects. Numerical models show that the load-bearing capacity of a compressed column along an inclined cross-section, taking into account lateral impact, significantly depends on the stress state of the concrete. The proposed model indirectly takes into account the effects of the emergence of stress in the transverse reinforcement when calculating parameter q_{sw} (5), however, microcracking and, as a result, concrete dilation are not taken into account, nor is the resistance of concrete on the descending branch. Therefore, it is assumed that a significant error arises as $N_e \to N_{ult}$.

The developed method can be used as an additional tool for solving problems related to assessing the survivability of buildings and structures [30; 31] in the event of man-induced accidents of mechanical nature. Prospects for improving this method include refining it for calculating elongated rectangular cross-sections (pylons) and slender reinforced concrete columns ($\lambda \geq 50$). It is also interesting to introduce various types of initial or acquired damage into the model, as well as to adapt the method to the calculation of reinforced concrete columns strengthened with steel or carbon fiber.

4. Conclusion

1. A method for analytical calculation of the lateral impact load capacity for reinforced concrete beam-columns causing shear failure has been developed. Verification showed satisfactory agreement with the obtained results based on the calculation of the solid FEM model in the range of compressive force values corresponding to characteristic loading of columns in civil buildings. The difference in results at $N_e < 0.8N_{ult}$ is no more than 5%.

2. The analytical model takes into account the confinement effect of concrete under compression, considering different spacing, diameter, and grade of transverse reinforcement. Limitations in the application of the method have been identified in terms of the compressive force, which must be less than 80% of the ultimate value.

3. A possibility has been established to quickly, compared to a solid FEM model, assess the safety of reinforced concrete columns in accidental situations such as collisions of technological or other transport with columns, man-induced mechanical impacts on columns in support areas, when failure occurs due to the action of a lateral force with a projection length of the inclined section $2h_0 \leq c \leq 3h_0$.

4. The proposed relationships are recommended for use in the design of preventive measures aimed at improving the mechanical safety of buildings and structures, including their protection against progressive collapse.

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