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ANALYSIS OF THIN ELASTIC SHELLS

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Construction of Developable Surfaces with Two Director Curves

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Abstract. An analysis of a number of published materials regarding four types of developable surfaces with two director (supporting) algebraic curves of the second order lying in parallel or in intersecting planes has been conducted. Three types of developable surfaces are shortly described with references to sources, and visualizations of each type of developable surface are presented. For the developable surfaces with two supporting curves with intersecting axes in intersecting planes, the construction technique and the method of obtaining parametric equations are given. This method is illustrated with three examples. It is established that to date, there are no studies on the strength of thin shells in the form of the presented developable surfaces defined in curvilinear conjugate non-orthogonal coordinates that coincide with the external contour of the shells. It is shown that there are suggestions of application of the studied surfaces in architecture, shipbuilding, and agricultural machine engineering.

Keywords: parallel vectors, vector coplanarity, second-order algebraic curves, developable surface with two director curves, surface modelling, computer graphics

Conflicts of interest. The author declares that there is no conflict of interest.

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Построение торсовых поверхностей на двух направляющих кривых

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Аннотация. Проведен анализ ряда опубликованных материалов по четырем типам торсовых поверхностей с двумя направляющими (опорными) алгебраическими кривыми второго порядка, лежащими в параллельных или пересекающих-ся плоскостях. Три типа торсов описаны кратко со ссылками на источники и приведены графические иллюстрации для

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каждого типа торсов, а для торсовых поверхностей с двумя опорными кривыми с пересекающимися осями в пересекающихся плоскостях представлен порядок построения этой поверхности и методика получения параметрических уравнений. Методика проиллюстрирована на трех примерах. Установлено, что до настоящего времени нет ни одного исследования напряженно-деформированного состояния предложенных тонких торсовых оболочек, заданных в криволинейных неортогональных сопряженных координатах, которые совпадают с внешним контуром торсовых оболочек. Показано, что есть предложения по применению предложенных поверхностей в архитектуре, судостроении и сельскохозяйственном машиностроении.

Ключевые слова: параллельность векторов, компланарность векторов, алгебраические кривые второго порядка, торс с двумя направляющими кривыми, моделирование поверхностей, компьютерная графика

Заявление о конфликте интересов: Автор заявляет об отсутствии конфликта интересов.

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1. Introduction

Over the last five years, the author has published articles on the construction of developable surfaces containing two prespecified plane algebraic curves on opposite sides of rectangular [1], trapezoidal [2], and arbitrary quadrilateral [3] bases. Moreover, the generator lines of the resulting developable surfaces coincide with the opposite sides of the rectangular and trapezoidal bases. In the case of an arbitrary quadrilateral base, the generator lines do not coincide with the sides, but are only projected onto them.

The construction of the considered developable surfaces is based on the works of G. Monge, G.E. Pavlenko [4], J.N. Gorbatovich [5], B. Bhattacharya [6], V.G. Rekach and N.N. Ryzhov [7], V.N. Ivanov [8], M.E. Ershov, E.M. Tupikova [9], Fr. Perez-Arribas and L. Fernandez-Jambrina [10].

Despite the fact that many geometers and engineers have been creating and improving methods for constructing developable surfaces with two prespecified plane curves, there are very few illustrations of specific developable surfaces, literally only a handful.

The *purpose of the study* is to draw the attention of experts to the possibility of obtaining parametric equations of developable surfaces constructed with two prespecified supporting plane curves lying in parallel or intersecting planes. Until now, graphical methods have been mainly used to construct these surfaces [11; 12]. Implicit or parametric equations have been obtained only for 5–6 developable surfaces [13]. Engineers and designers in the mechanical and textile industries are interested in expanding the list of developable surfaces defined by analytical formulas, which is also the goal of the proposed study [14; 15].

2. Algebraic Curves as Director Curves for Construction of Developable Surfaces

In all publications [1; 2; 3], second- and fourth-order algebraic curves were used as director curves. These curves can be defined as follows:

- ***parabola:***

$$x = x(u) = au, y = y(u) = h\left(1-u^2\right), \tag{1}$$

- ***ellipse fragment:***

$$x = x(u) = au,\ y = y(u) = h_1\left(\sqrt{1-u^2a^2 / a_1^2} - \sqrt{1-a^2 / a_1^2}\right), \tag{2}$$

where a_1 and h_1 are the lengths of the semiaxes of a complete ellipse, $a_1 \geq a$. By assuming the value of a_1, the length of the other semiaxis of the complete ellipse h_1 can be determined,

- ***circle fragment:***

$$x_1 = x(u) = au,\; y_1 = y(u) = \sqrt{R^2 - a^2u^2} - \sqrt{R^2 - a^2}, \tag{3}$$

- ***hyperbola:***

$$x = x(u) = au,\; y = c + h - \sqrt{c^2 + hu^2(2c + h)}, \tag{4}$$

constant parameter c is chosen arbitrarily, but $c \neq 0$,

- ***biquadratic parabola:***

$$x = x(u) = au,\;\; y = y(u) = h\left(1 - u^4\right), \tag{5}$$

- ***superellipse:***

$$x = x(u) = au,\; y = y(u) = h\sqrt[r]{1 - |u|^t}\;. \tag{6}$$

The tangent to the curve at angle φ is determined by the formula:

$$\text{tg}\,\varphi = \frac{dy}{dx}, \tag{7}$$

$$-1 \le u \le 1.$$

The remaining geometric parameters are shown in Figure 1. More detailed information on curves (1)–(6) can be found in any reference book on analytical geometry or in publications [1–3].

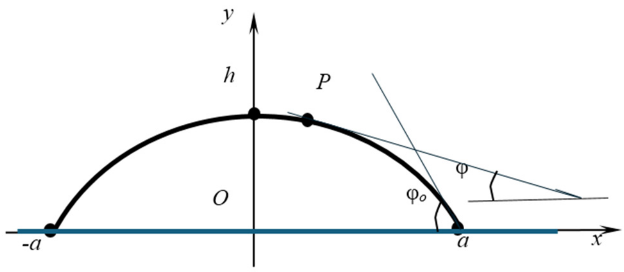


Figure 1. Constant geometrical parameters of curves
S o u r c e: compiled by S.N. Krivoshapko.

2.1. Examples of Developable Surfaces with Rectangular, Trapezoida and Quadrilateral Bases with Two Director Curves at Opposite Ends

2.1.1.Cylindrical surfaces with rectangular base (Figure 2)

All algebraic second-order cylindrical surfaces are considered in [16]. A cylindrical surface is an improper developable surface, in which the edge of regression is moved off to infinity. It is very easy to define a cylindrical surface with a rectangular base in parametric form. For example, if the identical director parabolas are specified as: $y = ax^2$, where $a = h/c^2$, then the parametric equations of the cylindrical surface will be: $x = x$, $y = ax^2$, $z = z$.

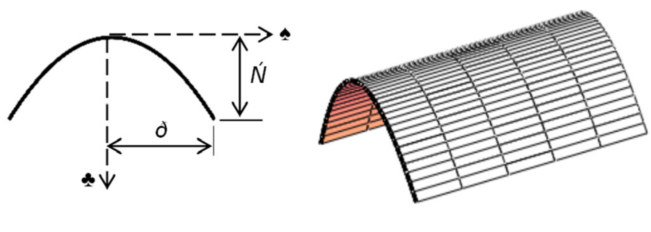


Figure 2. Cylindrical surface with parabolas at the ends
S o u r c e: compiled by V.N. Ivanov et al. [16].

2.1.2. Developable surfaces with two prespecified plane curves in parallel planes

For the construction of a developable surface, which contains plane curves in parallel xOy planes, i.e. at $z = 0$, and at $z = l$, and in which the opposite straight generator lines lie in the horizontal xOz plane parallel to the coordinate plane yOz, it is necessary to assume that angles φ_0 of both director curves are equal (Figure 1).

Given a pair of any director curves defined by equations (1)–(6), their vector equations can be represented as:

$$\boldsymbol{r}_1 = \boldsymbol{r}_1(u) \text{ and } \boldsymbol{r}_2 = \boldsymbol{r}_2(v) \tag{8}$$

with respect to origin O, where u, v are the corresponding parameters, then the equation of the developable surface can be represented in the form [6]:

$$\boldsymbol{r}(u,\lambda) = \boldsymbol{r}_1(u) + \lambda[\boldsymbol{r}_2(v) - \boldsymbol{r}_1(u)], \tag{9}$$

where λ is a dimensionless parameter, $0 \le \lambda \le 1$.

By defining the developable surface in the form of (9), coordinate lines λ = 0 and λ = 1 coincide with the director curves. The following relation must hold true between parameters u and v [4]:

$$\frac{y_1'(u)}{x_1'(u)} = \frac{y_2'(v)}{x_2'(v)}. \tag{10}$$

The geometric meaning of equation (10) is that the straight generator of the developable surface passes through two corresponding points of the plane curves, for which the angular coefficients of the tangents φ_0 are equal, i.e., the tangents drawn through the corresponding points of the two curves must be parallel.

Vector equation (9) may be represented in parametric form:

$$x = x(u,\ \lambda) = x_1(u)(1-\lambda) + \lambda x_2\left[v(u)\right],$$
$$y = y(u,\ \lambda) = y_1(u)(1-\lambda) + \lambda y_2\left[v(u)\right],$$
$$z = z(\lambda) = \lambda l. \tag{11}$$

The method described above for determining parametric equations (11) of the developable surfaces in [1] has been tested on five examples of pairs of plane curves as director curves: ellipse (2) + parabola (1), circle fragment (3) + parabola (1), hyperbola (4) + parabola (1) (Figure 3), parabola (1) + biquadratic parabola (5) (Figure 4), superellipse (6) with $r = t = 2$ + superellipse (6) with $r = t = 3$.

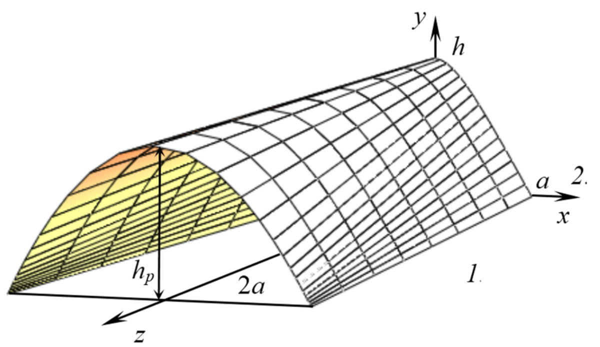


Figure 3. Developable surface with a parabola and a hyperbola at the ends
S o u r c e: compiled by S.N. Krivoshapko.

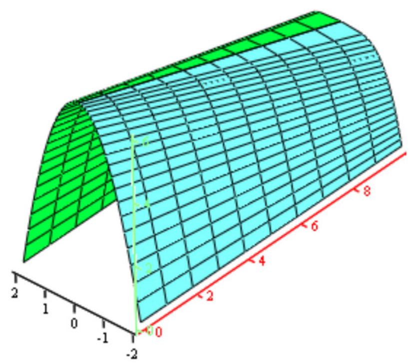


Figure 4. Developable surface with a second- and a fourth-order parabola at the ends
S o u r c e: compiled by S.N. Krivoshapko.

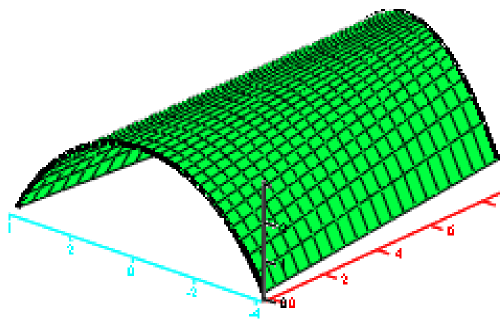
The developable surface with a circle and a parabola in parallel planes also attracted the attention of J.N. Gorbatovich [5]. The surface with an ellipse and a parabola in parallel planes was studied in article [17]. A developable surface with parabolas was used to illustrate the method of constructing its projection onto a plane [9]. There is an example of approximating a developable surface with parabolas of the 2nd and 4th orders in parallel planes with a folded structure [18]. These developments can be applied to the considered surfaces with rectangular base, but in the articles [5; 9; 18], the contour generator lines do not lie in the horizontal plane.

Article [2] considers developable surfaces with two prespecified plane curves (1)–(6) in parallel planes, but with a trapezoidal base. In this case, for the first curve (Figure 1) $-1 \le u \le 1$, i.e. $-a \le x \le a$, and for the second curve $-1 \le v \le 1$, i.e. $-b \le v \le b$. If the plane director curves (8) lie in parallel planes, relation (10) must hold between parameters u and v.

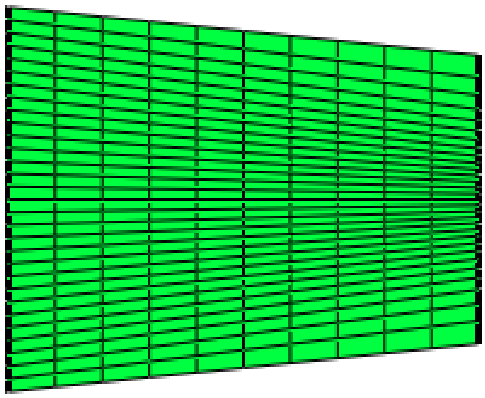
When constructing a developable surface, which has plane director curves of same rise h along axis Oz and two straight generator lines, which coincide with the sides of the trapezoidal base in the xOz plane, the following additional condition must be satisfied (Figure 1):

$tg\varphi_o$ of one curve at $x = \pm a$ must be equal to $tg\varphi_o$ of the other curve at $x = \pm b$.

After satisfying the above condition and condition (10), parametric equations (11) of the considered developable surface can be written. In article [2], the construction method is tested on examples of six pairs of plane curves as directors: ellipse (2) + parabola (1) (Figure 5), circle fragment (3) + parabola (1) (Figure 6), hyperbola (4) + parabola (1), parabola (1) + biquadratic parabola (5), superellipse (6) with $r = t = 2$ + + superellipse (6) with $r = t = 3$, hyperbola (4) + biquadratic parabola (5).



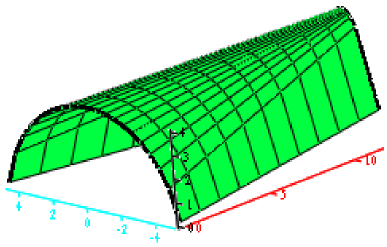
General view



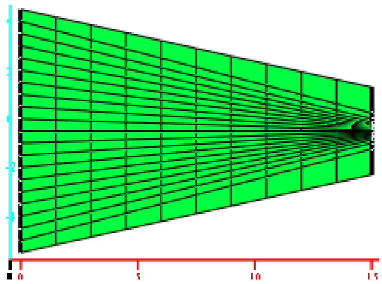
View in *yOz* plane

Figure 5. Developable surface with an ellipse fragment and a parabola at parallel ends

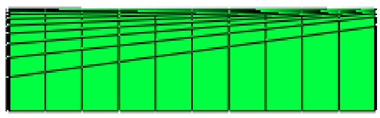
S o u r c e: compiled by S.N. Krivoshapko.



General view



View in *xOz* plane



View in *yOz* plane

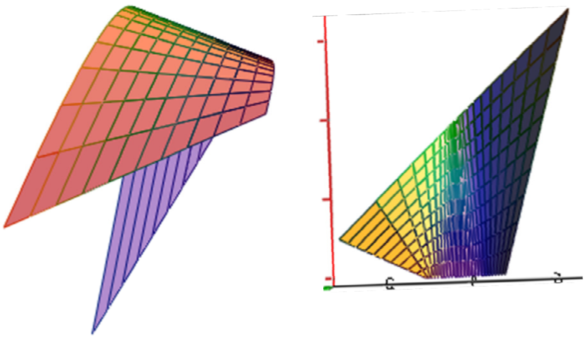
Figure 6. Developable surface with a circle fragment and a parabola at parallel ends

S o u r c e: compiled by S.N. Krivoshapko.

Developable surfaces with trapezoidal base with two prespecified plane curves at the two parallel edges and with straight generators resting on the lateral sides were considered only in article [7] from an architectural point of view.

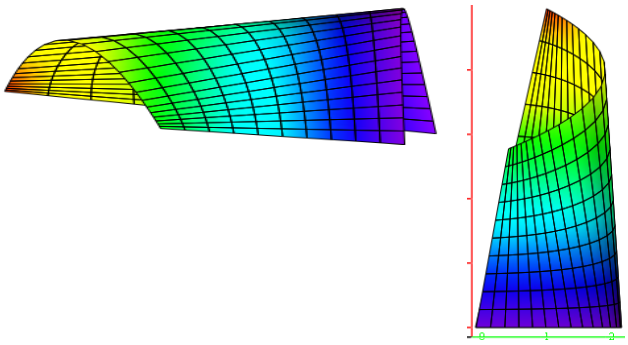
2.1.3. Developable Surfaces with Two Prespecified Plane Curves in Intersecting Planes

Under these conditions, there are two possible cases that are acceptable for practical application: when the axes of the director curves are parallel (case 1) and when the axes of the director curves intersect (case 2). The first case is discussed in detail in article [3]. Curves (1)–(6) are taken in pairs as director curves, and six developable surfaces are constructed. Two of them are shown in Figures 7 and 8.



a *b*

Figure 7. Developable surface with a second- and fourth-order parabola in intersecting planes: *a* — general view; *b* — view in xOz plane
S o u r c e: compiled by S.N. Krivoshapko.



a *b*

Figure 8. Developable surface with a parabola and a hyperbola in intersecting planes: *a* — general view; *b* — view in yOz plane
S o u r c e: compiled by S.N. Krivoshapko.

The second case is considered in more detail below. Assuming that the two curves lie in intersecting planes and their axes intersect (Figure 9), then their parametric equations can be represented as:

Curve 1: $x_1 = x_1(u), y_1 = u, z_1 = 0;$

Curve 2: $x_2 = x_2(v), y_2 = v, z_2 = x_2 \text{tg}\varphi.$ (12)

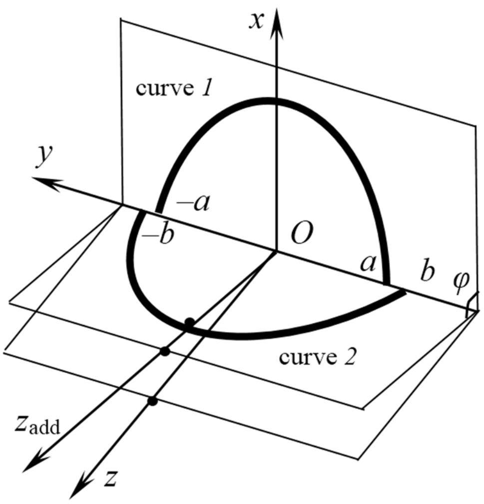


Figure 9. Two director curves with intersecting axes in intersecting planes
S o u r c e: compiled by S.N. Krivoshapko.

The coplanarity condition of the three vectors is written as:

$$\left(\boldsymbol{r}_2 - \boldsymbol{r}_1, \boldsymbol{r}_1^{/}, \boldsymbol{r}_2^{/}\right) = 0,$$

or

$$\begin{vmatrix} x_2 - x_1 & v - u & x_2 \text{tg}\varphi \\ x_1^{/} & 1 & 0 \\ x_2^{/} & 1 & x_2 \text{tg}\varphi \end{vmatrix} = 0,$$

or in expanded form:

$$x_2^{/} x_1^{/}\left(v - u\right) + x_1 x_2^{/} - x_2 x_1^{/} = 0, \quad (13)$$

by taking

$$y_1^{/}(u) = 1, y_2^{/}(v) = 1, z_2^{/} = x_2^{/} \text{tg}\varphi, x_2 = z_{add}(v)\cos\varphi,$$

φ is the angle between the intersecting planes.

Vector equation (9) for the considered case of director curves (12) can be converted into parametric form of definition of the desired developable surface:

$$x = x(u,\lambda) = x_1(u)(1-\lambda) + \lambda x_2[v(u)];$$

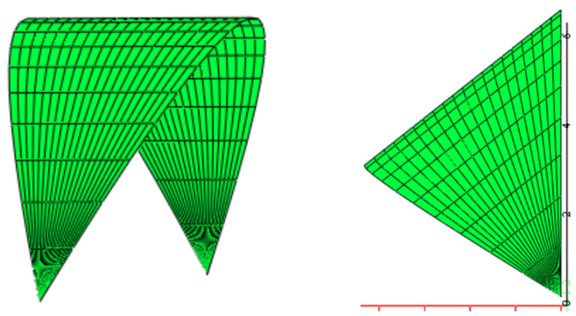
$$y = y(u,\lambda) = y_1(u)(1-\lambda) + \lambda y_2[v(u)] = u(1-\lambda) + \lambda v;$$

$$z = z(u,\lambda) = \lambda x_2[v(u)]\mathrm{tg}\,\varphi. \tag{14}$$

Example 1. Two square parabolas, which lie in planes intersecting at angle φ, are specified (Figure 9):

$$x_1 = x_1(u) = h\left[1 - u^2/a^2\right], y_1 = y_1(u) = u,\ z_1 = 0;$$

$$x_2 = x_2(v) = H\left[1 - v^2/b^2\right]\cos\varphi,\ y_2 = y_2(v) = v,\quad z_2 = z_2(v) = H\left[1 - v^2/b^2\right]\sin\varphi. \tag{15}$$



a *b*

Figure 10. Developable surface defined by equations (17):
a — general view; *b* — view in *xOz* plane
S o u r c e: compiled by S.N. Krivoshapko.

The relation between parameters *u* and *v* is determined by formula (13):

$$v_{1,2} = \frac{1}{2}\left(u + \frac{a^2}{u}\right) \pm \frac{1}{2}\sqrt{\left(u + \frac{a^2}{u}\right)^2 - 4b^2}. \tag{16}$$

By taking $a = b$, one obtains $v_1 = u$ and $v_2 = a^2/u$.

By further taking $v = v_1 = u$ and $a = b$, parametric equations (14) of the desired developable surface will be

$$x = x(u,\lambda) = \left(1 - \frac{u^2}{a^2}\right)\left[h(1-\lambda) + \lambda H\cos\varphi\right];$$

$$y = y(u) = u;$$

$$z = z(u,\lambda) = \lambda H\left(1 - \frac{u^2}{a^2}\right)\sin\varphi. \tag{17}$$

Figure 10 shows the surface defined by parametric equations (17), where

$h = 6$ m, $H = 5$ m, $a = 2$ m, $\varphi = 60^{\circ}$, $-a \le u \le a$, $0 \le \lambda \le 1$.

Example 2. By assuming that parabolas (15) lie in mutually perpendicular planes, then φ = 90°, and parametric equations (17) will take the following form:

$$x = x(u,\lambda) = \left(1 - \frac{u^2}{a^2}\right)\left[h(1-\lambda)\right];$$
$$y = y(u) = u;$$
$$z = z(u,\lambda) = \lambda H\left(1 - \frac{u^2}{a^2}\right). \quad (18)$$

Parametric equations (18) can be transformed into implicit form:

$$\frac{z}{H} + \frac{y^2}{a^2} + \frac{x}{h} - 1 = 0.$$

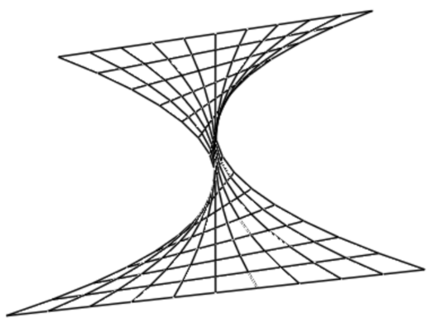


Figure 11. Parabolic developable surface
Source: compiled by S.N. Krivoshapko.

It is evident that this implicit equation describes a parabolic cylinder.

Encyclopedia [13] describes two developable surfaces: one with parabolas, the axes of which intersect, but the parametric equations of which differ from equations (15), and another developable surface containing two ellipses in mutually perpendicular planes.

V.S. Obukhova and R.I. Vorobkevich [19] proposed using two parabolas in mutually perpendicular coordinate planes as director curves, with their vertices touching one of the coordinate axes and the axes of the parabolas perpendicular to this axis (Figure 11).

Example 3. A semiellipse (curve *1*) and a parabola (curve *2*) are taken as director curves:

$$x_1 = x_1(u) = h\sqrt{1 - \frac{u^2}{a^2}}, \quad y_1 = u, \quad z_1 = 0;$$
$$x_2 = x_2(v) = H\left(1 - \frac{v^2}{b^2}\right)\cos\varphi, \quad y_2 = v, \quad z_2(v) = H\left(1 - \frac{v^2}{b^2}\right)\sin\varphi.$$

The relation between parameters u and v is determined by formula (13):

$$u = \frac{2va^2}{b^2 + v^2}. \quad (19)$$

Parametric equations (14) of the desired developable surface will be written as

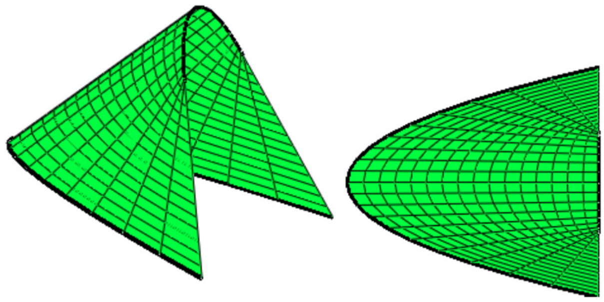
$$x = x(v,\lambda) = h(1-\lambda)\sqrt{1 - \frac{4v^2a^2}{\left(b^2+v^2\right)^2}} + \lambda H\left(1 - \frac{v^2}{b^2}\right)\cos\varphi;$$
$$y = y(v,\lambda) = (1-\lambda)\frac{2va^2}{\left(b^2+v^2\right)} + \lambda v;$$
$$z = z(v,\lambda) = \lambda H\left(1 - \frac{v^2}{b^2}\right)\sin\varphi. \quad (20)$$

Figure 12 shows the surface defined by parametric equations (20), where

$h = 4$ m, $H = 5$ m, $a = 1.2$ m, $b = 1.8$ m;

$\varphi = 60^{\circ}$, $-b \le v \le b$, $0 \le \lambda \le 1$.

In further introduction of developable surfaces defined by parametric equations (14) for practical application, the resulting surface can be rotated around the *y*-axis, so that the extreme straight generator lines rest on the specified base. In this case, the intersecting planes with director curves will be inclined to the base at corresponding angles (Figure 10, *a*).



a *b*

Figure 12. Developable surface defined by equations (20): *a* — general view; *b* — view in *yOz* plane
Source: compiled by S.N. Krivoshapko.

3. Review of Studies on Strength Analysis of Four Types of Developable Shells with Proposed Middle Surfaces

G. Monge laid the foundation for geometric research on proper developable surfaces in 1805. Since then, hundreds of scientific papers have been published on the geometry and application of these surfaces. Less than two dozen papers are devoted to the study of the stress-strain state of proper developable thin shells, with the exception of developable helicoids [20] and shells of equal slope [21]. All known developable shells have their middle surfaces defined in a non-orthogonal conjugate system of curvilinear coordinates, which significantly complicates the analytical calculation of these shells.

The system of 20 equations for determining 19 two-dimensional parameters, presented by A.L. Goldenveiser, provided that the mid-surface is specified in the arbitrary system of curvilinear coordinates, contains internal “pseudo-forces” and “pseudo-moments” as opposed to internal forces and moments adopted in the system of 20 equations containing 19 unknowns obtained by the author [22]. These two systems of equations were used in a simplified version only for the momentless analysis of two types of developable shells. G.Ch. Bajoria [23] applied A.L. Goldenveiser’s equilibrium equations for the momentless analysis of a developable shell defined in the form:

$$\boldsymbol{r} = \boldsymbol{r}(u,v) = \boldsymbol{\rho}(v) + u\boldsymbol{l}(v), \tag{21}$$

where $\boldsymbol{\rho}(v)$ is the current position vector of the edge of regression; $\boldsymbol{l}(v)$ is the unit tangent vector to the edge of regression. B. Bhattacharya [24] also applied A.L. Goldenveiser’s equilibrium equations, but on the premise of defining the middle surface of the developable shell in the form of (9).

A developable shell with an arbitrary quadrangular base with two plane parabolas lying in intersecting planes with parallel axes is calculated according to the momentless theory [25]. The results of calculating a developable shell with a circle and an ellipse in parallel planes, subjected to a linear load on the circular edge, are presented in [26]. The same shell, but loaded with self-weight, is considered in [27].

4. Results

1. When constructing a developable surface with two *n*-order algebraic director curves in parallel planes (Figures 3, 4) passing through opposite sides of a rectangular base $2a \times l$, any algebraic curves can be taken as director curves, with the rise *h* (Figure 1) of one of the two curves being arbitrary, and the rise of the second curve being calculated based on the geometric parameters of the two specified curves from the condition of equality of angles φ_0. Distance *l* between the planes with the curves does not affect the rise values of the curves.

2. The rise of two superellipses (6) in parallel planes can be any value.

3. To construct developable surfaces with two n-order algebraic director curves in parallel planes (Figures 3, 4) passing through opposite sides of a rectangular base $2a \times l$, and developable surfaces with two specified plane curves (1)–(6) in parallel planes, but with trapezoidal base with the parallel sides equal to $2a$ and $2b$, the same parametric equations (11) can be used.

4. The rise values h and H of the two director curves lying in intersecting planes, and the magnitude of angle φ between these planes, do not affect the relationship between parameters u and v (see, for example, formulas (16) and (19)).

5. It is shown that to date, 6 developable surfaces with director curves in parallel planes and with specified boundary conditions on the contours of rectangular bases [1], 8 developable surfaces with director curves in parallel planes and with specified boundary conditions on the contours of trapezoidal bases [2], 5 developable surfaces with director curves with parallel axes in intersecting planes [3], and only 3 developable surfaces with director curves with intersecting axes in intersecting planes have been studied. In this article, 3 more developable surfaces with director curves with intersecting axes in intersecting planes are introduced.

6. The literature review has shown that there are currently no studies on the strength analysis of thin shells with the considered developable middle surfaces, specified in curvilinear non-orthogonal conjugate coordinates u, λ in the form (11) or (14) using the moment theory of shells. Researchers from the Academy of Engineering of RUDN University, Moscow have published a large number of studies on geometry, application, approximation of developable surfaces by folds, unfolding developable surfaces onto a plane and their parabolic bending, and determination of the strength parameters of some special cases of developable shells. In addition to their studies, some of which are listed in the “References” section, most of the scientific articles published over the last 25 years are devoted to the implementation of methods for unfolding developable surfaces with two specified director curves onto a plane with maximum use of computers [14; 28; 29] and the practical application of developable surfaces [30] in avant-garde architecture, agricultural machine engineering, shipbuilding [10], the fashion industry [15], as well as the solution of mathematical problems related to developable surfaces [31].

7. It is established that the only study on finding the optimal cylindrical shell with two variable supporting curves at the ends is the article by V.N. Ivanov, O.O. Aleshina, E.A. Larionov [16]. Cylindrical surfaces are improper developable surfaces in which the edge of regression is moved off to infinity.

5. Conclusion

Scientific and technical literature proposes 10 methods for constructing developable surfaces. The most well-known of them are constructing developable surfaces based on two specified director curves, based on the specified edge of regression, and the kinematic method of winding a plane with a straight line onto a cylinder and cone. The first method listed above is recommended mainly for designing large-area roofs in construction, the second is widely used to create screw and helical products in mechanical engineering, and the third method is used when studying the trajectory of a straight line in space and when studying the carved ruled Monge surface.

Despite the fact that there are sketches of architectural objects in the form of developable surfaces with specified supporting plane curves, they are most commonly used in the design of river and sea vessel hulls. Virtually all publications on the manufacture of ship hulls use graphic representations of the ideas for design of these developable surfaces.

The article offers analytical solutions to the problems posed. For convenience of studying developable surfaces with two specified curves, they are divided into four types, for each of which the procedure for obtaining explicit or parametric equations is shown, according to which the corresponding developable surfaces with specified geometric parameters are constructed using computer graphics.

The presented material may encourage architects and practicing engineers to make wider use of the proposed developable surfaces in the forms of real products, structures, and buildings.

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