Строительная механика инженерных конструкций и сооружений
Structural Mechanics of Engineering Constructions and Buildings
2024. 20(3). 265–275
ISSN 1815-5235 (Print), 2587-8700 (Online)
HTTP://JOURNALS.RUDN.RU/STRUCTURAL-MECHANICS

РАСЧЕТ ТОНКИХ УПРУГИХ ОБОЛОЧЕК
ANALYSIS OF THIN ELASTIC SHELLS

DOI: 10.22363/1815-5235-2024-20-3-265-275
UDC 624.04:624.074
EDN: RDDWZD

Research article / Научная статья

Analytical Calculation of Momentless Conical Shell with Elliptical Base

Sergey N. Krivoshapko

RUDN University, *Moscow, Russia*
✉ sn_krivoshapko@mail.ru

Article history
Received: February 28, 2024
Revised: April 17, 2024
Accepted: April 26, 2024

Conflicts of interest
The author declares that there is no conflict of interest

Abstract. Differential equilibrium equations of the momentless shell theory are very easily integrated in cases of cylindrical and right circular conical shells. Shells of zero Gaussian curvature defined in arbitrary curvilinear coordinates are more difficult to analyze, which was reaffirmed by the case of elliptical conical shells. For the first time, analytical expressions of normal and tangential internal forces in a momentless right elliptical conical shell defined in non-orthogonal conjugate system of curvilinear coordinates are obtained. The derived results can be used for approximation of the stress state of thin conical shells with elliptical base and also for the investigation of stability of these shells. Four internal tangential forces obtained by integration of the system of four equilibrium equations of a shell element contain two unknown integration functions, which are determined by satisfying given boundary conditions. The application of obtained analytical equations is demonstrated by an example of analysis of a truncated elliptical conical shell with free upper edge. A uniformly distributed surface load in the direction of the vertical axis of the shell was assumed as external load. The presented formulae are easily adapted for the analysis of a right circular conical shell.

Keywords: elliptical cone, momentless shell theory, non-orthogonal curvilinear coordinates, truncated elliptical cone

For citation
Krivoshapko S.N. Analytical calculation of momentless conical shell with elliptical base. *Structural Mechanics of Engineering Constructions and Buildings.* 2024;20(3):265–275. http://doi.org/10.22363/1815-5235-2024-20-3-265-275

Sergey N. Krivoshapko, Doctor of Technical Sciences, Professor of the Department of Civil Engineering, Academy of Engineering, RUDN University, Moscow, Russia; eLIBRARY SPIN-code: 2021-6966; ORCID: 0000-0002-9385-3699; E-mail: sn_krivoshapko@mail.ru

Аналитический расчет конической оболочки на эллиптическом основании по безмоментной теории

С.Н. Кривошапко

Российский университет дружбы народов, *Москва, Россия*
✉ sn_krivoshapko@mail.ru

История статьи
Поступила в редакцию: 28 февраля 2024 г.
Доработана: 17 апреля 2024 г.
Принята к публикации: 26 апреля 2024 г.

Заявление о конфликте интересов
Автор заявляет об отсутствии конфликта интересов.

Аннотация. Дифференциальные уравнения равновесия безмоментной теории оболочек легче всего интегрируются для цилиндрических и прямых конических круговых оболочек. Труднее задача решается для оболочек нулевой гауссовой кривизны, заданных не в линиях кривизны. Это еще раз подтверждено на примере конической эллиптической оболочки. Впервые получены аналитические формулы для определения нормальных и касательных внутренних усилий в прямой конической эллиптической оболочке по безмоментной теории оболочек, заданных в неортогональной сопряженной системе криволинейных координат. Полученные результаты могут быть использованы для приближенной оценки напряженного состояния тонких конических оболочек на эллиптическом основании, а также при исследовании устойчивости этих оболочек. Четыре внутренних тангенциальных усилия, полученные интегрированием системы четырех уравнений равновесия элемента оболочки, содержат две неизвестные функции интегрирования, которые находятся при выполнении поставленных граничных условий. Использование полученных аналитических формул проиллюстрировано на примере расчета усеченной конической эллиптической оболочки со свободным верхним краем. Внешняя нагрузка — поверхностная равномерно распределенная нагрузка в направлении вертикальной оси оболочки. Приведенные формулы легко адаптируются для случая расчета прямой круговой конической оболочки.

Ключевые слова: эллиптический конус, безмоментная теория оболочек, неортогональные криволинейные координаты, усеченный эллиптический конус

Для цитирования
Krivoshapko S.N. Analytical calculation of momentless conical shell with elliptical base // Строительная механика инженерных конструкций и сооружений. 2024. Т. 20. № 3. С. 265–275. http://doi.org/10.22363/1815-5235-2024-20-3-265-275

1. Introduction

The momentless theory of the analysis of rigid thin shells is an approximate theory, but in some cases it gives fairly accurate values of tangential internal forces, which can be used for preliminary analysis of the stress state of a thin shell [1]. This data can be useful, for example, when assigning the thickness of the shell. It has been established that the momentless theory of shells yields reasonable results in comparison with exact results when fulfilling well-known requirements for supports, the type of external loads, boundary conditions

Кривошапко Сергей Николаевич, доктор технических наук, профессор, профессор департамента строительства, инженерная академия, Российский университет дружбы народов, Москва, Россия; eLIBRARY SPIN-код: 2021-6966; ORCID: 0000-0002-9385-3699; E-mail: sn_krivoshapko@mail.ru

and the shape of the shell [2; 3]. Research on the momentless theory of rigid thin shells was being actively developed especially until the 1980s [4]. Then came the fascination with numerical methods for calculating moment shells using linear and physically or geometrically nonlinear calculation theories. However, in many monographs of famous scientists of the last years of the 20th century, there was a necessary chapter devoted to the momentless theory of calculating thin shells [1–3; 5], and textbooks contained information about the application potential of this theory [6]. The momentless theory is used in the study of stability of thin shells [7].

At the present day, the amount of studies on the momentless theory of shells has been significantly reduced, but they are available [8]. Generally, a comparative analysis of the results obtained by the momentless theory and by more precise methods using numerical methods is carried out [9]. There are studies containing comparative analysis of the calculation results obtained using the momentless theory of shells and the results obtained experimentally, for example, in the process of designing the conical shell foundations [10].

The conical shape of shells is currently widely used in civil [11] and mechanical engineering [12]. Shells in the form of right thin (Figure 1) and thick [13] circular cones have been applied in most cases, but conical shells with elliptical base are also used [14–16]. Moreover, truncated conical shells with elliptical base have found application even in medicine [17].

The purpose of this work is to obtain analytical expressions for determining tangential internal forces in an elliptical conical shell according to the momentless theory.

The symmetric equation of a right elliptical conical surface may be expressed in the following form:

$$\frac{x^2}{L^2}+\frac{y^2}{W^2}-\frac{z^2}{T^2}=0,$$

where T is the height of the conical surface.

Parametric equations of the elliptical cone (Figure 2) are known [18]:

$$x = x(u,\nu) = \nu L[1-u],\ y = y(u,\nu) = \pm W[1-u]\left[1-|\nu|^2\right]^{1/2},\ z = z(u) = uT, \tag{1}$$

where $-L \le x \le L$, $-W \le y \le W$, $0 \le z \le T$; $2L$, $2W$, T are the dimensions of the surfaces under consideration.

Figure 1. Circular cone, “City of Arts and Sciences”, Valencia, Spain
S o u r c e : photo by S.L. Shambina

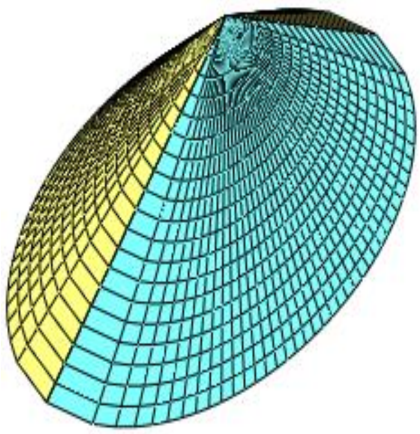

Figure 2. Right elliptical conical surface
S o u r c e : compiled by S.N. Krivoshapko

Lateral area of an elliptical cone can be calculated with the following formula [19]:

$$A_{\text{cone}} \approx \frac{\pi}{2}\left[W\sqrt{L^2+T^2}+L\sqrt{W^2+T^2}\right].$$

Volume of an elliptical cone is determined by the following formula:

$$V_{\text{cone}} = \frac{\pi}{3}TLW.$$

The surface depicted in Figure 2 is generated by a family of z = const sections (ellipses).

By adopting a new variable parameter $0 \ge \beta \ge 2\pi$, so that $\nu = \sin\beta,\ 1-\nu^2 = \cos^2\beta$, parametric equations (1) can be expressed in the following form:

$$\begin{aligned} x &= x(u,\nu) = L[1-u]\sin\beta;\\ y &= y(u,\nu) = W[1-u]\cos\beta;\\ z &= z(u) = uT. \end{aligned} \tag{2}$$

Curvilinear coordinates u, ν of the elliptical cone defined by parametric equations (1), (2) are non-orthogonal and conjugate [18]. Coordinate lines u are straight generatrices of the cone, lines ν are ellipses lying in parallel planes. Coordinate lines v intersect coordinate lines u at right angles only along the straight generatrices $\nu = 0$ and $\nu = \pm 1$.

2. Momentless Theory of Calculating Right Conical Surface with Elliptical Base

Paper [18] contains the derivation of analytical equations for determining the normal and tangential internal forces of a thin shell, the middle surface of which is defined by equations (1). Using the momentless shell theory, the system of three equilibrium equations of a shell element is obtained from the general equations of equilibrium for shells defined in curvilinear non-orthogonal conjugate coordinates [5].

The equations for determining normal forces N_u, N_ν and tangential forces $S_u \ne S_\nu$ per unit length of the corresponding coordinate lines, obtained in article [18], can be expanded in the form, convenient for computer-assisted calculations:

$$\begin{aligned} N_\nu &= (1-u)\,f_5(\nu);\\ N_u &= \frac{(1-u)\sqrt{f_8}}{\left(L^2-W^2\right)}\left(f_9-f_{10}\right)+\frac{\left(1-\nu^2\right)^{\frac{1}{2}}}{2(1-u)^2 f_0}\left[\frac{3\nu}{A^2}\left(L^2-W^2\right)V_1+\frac{dV_1}{d\nu}+\frac{2(1-u)}{A}V_2\right]+(1-u)\,f_5;\\ S_\nu &= (1-u)\,f_6(\nu)+\frac{V_1(\nu)}{(1-u)^2};\\ S_u &= (1-u)\left(1-\nu^2\right)^{\frac{1}{2}}\nu f_{10}+\frac{\left(1-\nu^2\right)}{(1-u)^2 f_0}\left[\frac{f_8V_1}{\left(1-\nu^2\right)A^2}-\frac{\nu^2}{2A^2}\left(L^2-W^2\right)^2V_1-\frac{\nu}{2}\left(L^2-W^2\right)\frac{dV_1}{d\nu}\right]-\\ &-\frac{\nu\left(1-\nu^2\right)}{(1-u)Af_0}\left(L^2-W^2\right)V_2(\nu); \end{aligned} \tag{3}$$

where $f_i = f_i(\nu)$ are known values, $V_1(\nu)$, $V_2(\nu)$ are arbitrary functions of integration, which are determined by satisfying boundary conditions defined in forces.

Equations (3) contain constant geometric dimensions L, W, T of the shell middle surface, mentioned in the comments to equations (1). In addition, new constants K and R have been introduced, which allowed to slightly reduce the formulas given below for expressing the known functions $f_i(\nu)$:

$$K = L^2 - W^2, \qquad R = \left(W^2 + T^2\right)\left(T^2 + L^2\right).$$

The known functions $f_i(\nu)$, contained in equations (3), obtained by integration of the three equilibrium equations in paper [18], can be written as follows:

$$f_3(\nu) = \sqrt{A^2 - T^2} = \sqrt{\nu^2 L^2 + W^2\left(1-\nu^2\right)}; \qquad A^2 = A^2(\nu) = \nu^2 L^2 + W^2\left(1-\nu^2\right) + T^2;$$

$$f_0(\nu) = L^2 - \nu^2 K;$$

$$f_5(\nu) = \frac{q\sqrt{A^2-T^2}\left[L^2\left(W^2+T^2\right) - \nu^2 T^2 (L^2 - W^2)\right]^{\frac{3}{2}}}{LTWA^3} = \frac{q\sqrt{A^2-T^2}\, f_8^{\frac{3}{2}}(\nu)}{LTWA^3};$$

$$f_6(\nu) = \left(1-\nu^2\right)^{\frac{1}{2}} \nu f_9(\nu);$$

$$f_7(\nu) = \left(1-\nu^2\right)^{\frac{1}{2}} \nu f_{10}(\nu);$$

$$f_8(\nu) = L^2\left(W^2+T^2\right) - \nu^2 T^2\left(L^2 - W^2\right);$$

$$f_9(\nu) = \frac{qK}{3ALTW\sqrt{A^2-T^2}}\left[R\left(4\frac{T^2}{A^2} - 3\right) - T^4\right];$$

$$f_{10}(\nu) = \frac{1}{2Af_0(\nu)}\left\{2\frac{f_8 f_9}{A} - \frac{qK^2}{3LTWf_3(\nu)}\left[\left(1-\nu^2 - \frac{\nu^2 L^2}{f_3^2(\nu)}\right)\left(4R\frac{T^2}{A^2} - 3R - T^4\right) - \right.\right.$$
$$\left.\left. - \frac{\left(1-\nu^2\right)8\nu R T^2 K}{A^4}\right] + \frac{\sqrt{f_8(\nu)}}{A} K\left[f_5(\nu) + qT\right]\right\}. \qquad (4)$$

In equations (4), q denotes the external surface load, such as self-weight, in the direction opposite to the fixed coordinate axis z.

Thus, the momentless shell theory allows to obtain approximate values of normal forces N_u, N_ν and tangential forces S_u, S_ν using analytical equations (3).

3. Truncated Elliptical Conical Shell with Free Upper Edge

Let the upper edge $u = u_0$ of the thin shell be free and the lower edge $u = 0$ be simply supported, with the direction of the supports coinciding with the direction of the straight generatrices of the cone. The shell is smooth, without fractures, and of constant thickness. External load q = const is a constant distributed load, such as self-weight. Thus, all the requirements for the application of the momentless theory for the shell are fulfilled.

Since the upper edge $u = u_0$ is free, the following two boundary conditions can be defined at this edge:

$S_u = 0$, $N_u = 0$ at $u = u_0$.

The expression for normal force N_u is taken from paper [18] and is equated to zero:

$$N_u = -\left(S_u - S_\nu\right)\frac{\sqrt{f_8}}{\left(1-\nu^2\right)^{\frac{1}{2}}\nu\left(L^2 - W^2\right)} + N_\nu = 0, \tag{5}$$

and by considering that $S_u = 0$, the presented condition is simplified:

$$N_u = S_\nu \frac{\sqrt{f_8}}{\left(1-\nu^2\right)^{\frac{1}{2}}\nu\left(L^2 - W^2\right)} + N_\nu = 0.$$

By substituting the second and the fourth formula from the system of equations (3) into the last expression, it is possible to determine integration constant $V_1(\nu)$:

$$V_1(\nu) = -q\frac{\left(1-u_0\right)^3\left(L^2-W^2\right)}{ALTW\sqrt{A^2-T^2}}\nu\left(1-\nu^2\right)^{\frac{1}{2}}\left\{\frac{A^2-T^2}{A^2}f_8(\nu) + \frac{1}{3}\left[R\left(4\frac{T^2}{A^2}-3\right)-T^4\right]\right\} =$$

$$= -\nu\left(1-\nu^2\right)^{\frac{1}{2}}\left(1-u_0\right)^3\left[f_9(\nu) + \frac{qK\sqrt{A^2-T^2}}{LTWA^3}f_8(\nu)\right] = -\nu\left(1-\nu^2\right)^{\frac{1}{2}}\left(1-u_0\right)^3 V_{01}(\nu). \tag{6}$$

The second integration constant $V_2(\nu)$ is determined from the fourth formula of the system of equations (3) by satisfying the boundary condition of $S_u = 0$:

$$\frac{\nu\left(1-\nu^2\right)K}{Af_0}V_2(\nu) = \left(1-u_0\right)^2\nu\left(1-\nu^2\right)^{1/2} f_{10} + \frac{1}{\left(1-u_0\right)f_0}\left\{\frac{V_1(\nu)}{A^2}\left[f_8 - \frac{\nu^2\left(1-\nu^2\right)K^2}{2}\right] - \right.$$

$$\left. - \frac{\nu\left(1-\nu^2\right)K}{2}\frac{dV_{1(\nu)}}{d\nu}\right\}. \tag{7}$$

Now, by using equations (3), internal tangential forces per unit length of coordinate lines can be calculated:

$$S_\nu = \frac{\nu\left(1-\nu^2\right)^{1/2}}{\left(1-u\right)^2}\left\{\left[\left(1-u\right)^3 - \left(1-u_0\right)^3\right]f_{9(\nu)} - \frac{qK\left(1-u_0\right)^3\sqrt{A^2-T^2}}{A^3LTW}f_8(\nu)\right\}; \tag{8}$$

$$S_u = \nu\left(1-\nu^2\right)^{1/2}\left\{\frac{\left(1-u\right)^2 - \left(1-u_0\right)^2}{\left(1-u\right)}f_{10} + \frac{\left(u-u_0\right)\left(1-u_0\right)^2}{\left(1-u\right)^2 f_0(\nu)}\left[\frac{f_8(\nu)}{A^2}V_{01}(\nu) + \right.\right.$$

$$\left.\left. + \frac{K\left(1-\nu^2\right)\left(W^2+T^2\right)}{2A^2}V_{01}(\nu) - \frac{K}{2}\nu^2 V_{01}(\nu) + \frac{K\nu\left(1-\nu^2\right)}{2}\frac{dV_{01}(\nu)}{d\nu}\right]\right\}. \tag{9}$$

Normal force N_ν is calculated with the first formula of the system of equations (3). Normal force N_u is determined according to formula (5):

$$N_u = \frac{\sqrt{f_8}}{K}\left\{\frac{\left[(1-u)^3-(1-u_0)^3\right]}{(1-u)^2} f_9 - \frac{qK(1-u_0)^3\sqrt{A^2-T^2}}{(1-u)^2 A^3 LTW} f_8 - \frac{(1-u)^2-(1-u_0)^2}{(1-u)} f_{10} - \right.$$

$$-\frac{\left[(1-u)-(1-u_0)\right](1-u_0)^2}{(1-u)^2 f_0} V_{01}\left[\frac{f_8}{A^2}+\frac{K(1-\nu^2)(W^2+T^2)}{2A^2}-\frac{K\nu^2}{2}\right]+$$

$$\left.+\frac{\nu(1-\nu^2)(u-u_0)(1-u_0)^2}{2(1-u)^2 f_0} K\frac{dV_{01}}{d\nu}\right\}+(1-u)\frac{q\sqrt{A^2-T^2}}{A^3 LTW} f_8^{\frac{3}{2}}. \qquad (10)$$

Coefficients $f_i = f_i(\nu)$ are defined by equations (4); coordinate u varies in the range of $0 \le u \le u_0$, and $u_0 < 1$; coordinate ν varies from –1 to +1 (Figure 3).

It should be noted that truncated conical shells are widely used in mechanical engineering (Figure 4, *a*) and civil engineering (Figures 4, *b*, *c*). The momentless shell theory can be useful in studying the stability of truncated conical shells [20].

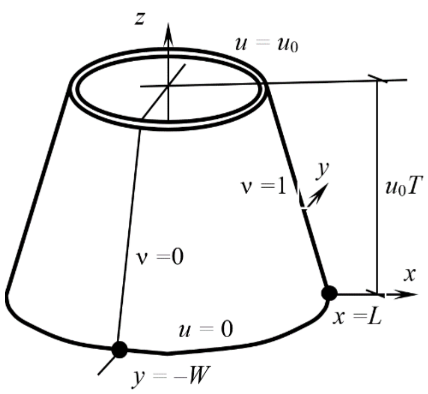

Figure 3. Truncated elliptical conical shell
Source: compiled by S.N. Krivoshapko

a *б* *c*

Figure 4. Truncated conical shells in mechanical and civil engineering:
a — metal product; *b* — conical roofs at the Bundeskunsthalle in Bonn Germany;
c — water tower in Midrand, South Africa
Source: *a* — http://molodec-kyznec.ru/market/izgotovlenie_obechaek/val_covka_konusov/; *б* — [11]; *c* — [11]

3.1. Example of Calculation

As an example, let us determine the internal tangential forces along the straight generatrices of the truncated conical elliptical shell considered above. Let us examine the straight generatrices coinciding with coordinate lines $\nu = 0$ and $\nu = \pm 1$. For these lines, parameters $f_i(\nu)$ are calculated and summarized in Table.

It was noted earlier that the curvilinear coordinate system u, ν is non-orthogonal conjugate, so $S_\nu \neq S_u$. Coordinate lines u = const intersect coordinate lines ν at right angles only at $\nu = 0$ and $\nu = \pm 1$. Therefore, due to the law of reciprocity of tangential stresses according to equations (8) and (9), it turns out that $S_\nu = S_u$.

The straight generatrices coinciding with coordinate lines v = 0 and ν = ±1

v = 0 (Figure 3)	v = ±1 (Figure 3)
$f_0 = L^2$	$f_0 = W^2$
$f_3 = \left(A^2 - T^2\right)^{1/2} = W$	$f_3 = \left(A^2 - T^2\right)^{1/2} = L$
$A^2 = W^2 + T^2$	$A^2 = L^2 + T^2$
$f_5 = qL^2 / T$	$f_5 = qW^2 / T$
$f_6 = 0$	$f_6 = 0$
$f_7 = 0$	$f_7 = 0$
$f_8 = L^2\left(W^2 + T^2\right)$	$f_8 = W^2\left(L^2 + T^2\right)$
$f_9 = \dfrac{qK\left[T^2L^2 - 3W^2T^2 - 3W^2L^2\right]}{3LTW^2\sqrt{W^2 + T^2}}$	$f_9 = \dfrac{qK\left[T^2W^2 - 3W^2L^2 - 3T^2L^2\right]}{3WTL^2\sqrt{L + T^2}}$
$f_{10} = \dfrac{qK\left[L^2T^2\left(L^2 + W^2\right) - 3W^4\left(L^2 + T^2\right)\right]}{6L^3TW^2\sqrt{W^2 + T^2}}$	$f_{10} = \dfrac{qK\left[W^2T^2\left(L^2 + W^2\right) - 3L^4\left(W^2 + T^2\right)\right]}{6L^2TW^3\sqrt{L^2 + T^2}}$
$V_{01} = \dfrac{qKT\left(L^2 - 3W^2\right)}{3LW^2\sqrt{W^2 + T^2}}$	$V_{01} = \dfrac{qKT\left(W^2 - 3L^2\right)}{3WL^2\sqrt{L^2 + T^2}}$
$N_v = \left(1 - u\right)\dfrac{qL^2}{T}$,	$N_v = \left(1 - u\right)\dfrac{qW^2}{T}$,
$S_u = S_v = 0$	$S_u = S_v = 0$

S o u r c e : compiled by S.N. Krivoshapko

Normal forces N_v along coordinate lines u, i.e. at ν = 0 and ν = ±1, are calculated according to equations presented in Table. Normal forces N_u along the same coordinate lines can be calculated using formula (10) at ν = 0 and ν = ±1. For example, formula (10) takes the following form when evaluated at ν = 0:

$$N_u\left(\nu - 0\right) = \frac{q}{6L^2W^2T}\left\{\left(1 - u\right)\left(T^2L^4 - 7L^2W^2T^2 + 3L^2W^4 + 3W^4T^2\right) - \right.$$

$$\left. - \frac{\left(1 - u_0\right)^3}{\left(1 - u\right)^2}\left[T^2\left(L^2 - 3W^2\right)\left(5L^2 - W^2\right)\right] + \frac{\left(1 - u_0\right)^2}{\left(1 - u\right)}[4T^2L^4 - 3L^2W^4 - 9L^2W^2T^2\right\}. \qquad (11)$$

One can perform a check that normal force N_u, calculated with formula (11), must be zero at the free edge $u = u_0$ (Figure 3).

3.2. Right Circular Conical Surface

A right circular conical surface may be analysed using the formulas presented above by setting $K = L^2 - W^2 = 0$ and $L = W = r$, where r is the radius of the base of the cone.

Thus, the following may be obtained for a truncated circular conical shell with a free upper edge:

$S_u = S_v = 0;$

$$N_v = (1-u)\frac{qr^2}{T};$$

$$N_u = \frac{q}{6T}\left\{3(1-u)\left(r^2 - T^2\right) + 8\frac{(1-u_0)^3}{(1-u)^2}T^2 - \frac{(1-u_0)^2}{(1-u)}\left(3r^2 + 5T^2\right)\right\}.$$

4. Results and Discussion

It is known that the differential equilibrium equations of the momentless shell theory are most easily integrated for shells of zero Gaussian curvature, the middle surfaces of which are defined in lines of curvature, e.g., cylindrical and right circular conical shells. The problem is more difficult to solve for shells of zero Gaussian curvature defined in arbitrary coordinate lines [1]. This is further confirmed by the example of an elliptical conical shell. The main difficulty in the calculation was the application of a curvilinear non-orthogonal conjugate coordinate system, in which the middle surface of the thin shell under consideration is defined. The equilibrium equations of the shell element defined in an arbitrary curvilinear coordinate system were used for the analysis. These equations were obtained by the author earlier.

1. For the first time, analytical expressions for the determination of normal and tangential internal forces in a right elliptical conical shell according to the momentless theory of shells defined in a non-orthogonal conjugate system of curvilinear coordinates are obtained. The obtained approximate results can be used to evaluate the stress state of thin conical shells with an elliptic base, as well as to study the stability of these shells.
2. The application of the formulas obtained for the first time for the calculation of tangential forces according to the momentless shell theory is demonstrated by the example of a truncated elliptical conical shell with a free upper edge subjected to a distributed load such as self-weight.
3. It is shown that the obtained equations can be used for the analysis of right circular conical shells according to the momentless theory.

5. Conclusion

Despite the development of powerful computer hardware and the creation of powerful software systems capable of calculating the stress-strain state of complex thin-walled spatial structures on the basis of the moment theory of shells, the momentless theory has not lost its relevance. Computational systems are most often based on the application of numerical methods of calculation. The possibility of obtaining analytical solutions is always preferable, as it allows to review the obtained solution. The momentless stress state occupies an honorable place in the calculation of shells, being the ideal to which one should strive when designing shells. The conditions of the momentless stress state cannot always be fulfilled structurally. Even so, the results of the calculation, at some distance from problematic areas, may be acceptable for preliminary design decisions.

References

1. Goldenveizer A.L. *Theory of Elastic Thin Shells*. New York: Pergamon Publ.; 1961.

2. Novozhilov V.V., Chernikh K.F., Mikhailovskiy E.I. *Linear Theory of Thin Shells*. Leningrad: Politechnika Publ.; 1991. (In Russ.) ISBN 5-7325-0127-4

3. Jawad M.H. *Design of Plate and Shell Structures*. New York: ASME Publ.; 2004. ISBN: 0-7918-0199-3

4. Bajoriya G.Ch. The application of the system of Goldenveiser's equations for analysis of momentless torse shells. *Issledovaniya po Raschetu Elementov Prostranstvennih Sistem* [Investigation on Analysis of Fragments of Space Systems]. Moscow: UDN; 1987. P. 65–72 (In Russ.)

5. Ivanov V.N., Krivoshapko S.N. *Analytical Methods of Analysis of Shells of Non-canonical Shape*. Moscow: RUDN; 2010. (In Russ.) EDN: TINZYP

6. Rekach V.G. *Manual for Solution of Problems of Applied Theory of Elasticity*. Moscow: Vysshaya Shkola Publ.; 1973.

7. Tovstik P.E. *Stability of Thin Shells: Asymptotic Methods*. Moscow: Nauka, Fizmatlit Publ.; 1995. (In Russ.) ISBN 5-02-014522-X

8. Tyurikov E.V. On the construction of mathematical models of the membrane theory of convex shells. *Advanced Engineering Research (Rostov-on-Don)*. 2023;23(1):17–25. https://doi.org/10.23947/2687-1653-2023-23-1-17-25

9. Ivanov V.N., Alyoshina O.O. Comparative analysis of the stress-strain state's parameters of equal slope shell with the director ellipse using three calculation methods. *Structural Mechanics and Analysis of Constructions*. 2020;3:37–46. In Russ.) https://doi.org/10.37538/0039-2383.2020.3.37.46

10. Fattah M.Y., Waryosh W.A., Al-Hamdani M.A.E. Experimental and theoretical studies of bearing capacity of conical shell foundations composed of reactive powder concrete. *Acta Geodyn. Geometer*. 2015;12(4):411–420. https://doi.org/10.13168/AGG.2015.0037

11. Mamieva I.A., Razin A.L. Symbol spatial structures in the form of conic surfaces. *Industrial and Civil Engineering.* 2017;10:5–11. (In Russ.) EDN: ZSMVFN

12. Golzan B.S., Showkati H. Buckling of thin-walled conical shells under uniform external pressure. *Thin-Walled Structures*. 2008;46(5):516–529. https://doi.org/10.1016/j.tws.2007.10.011

13. Chung S.W., Ju Gisu. A conical shell theory of hybrid anisotropic materials. *International Journal of Composite Materials.* 2019;9(3):53–59. https://doi.org/10.5923/j.cmaterials.20190903.01

14. Kantor B.Ya., Mellerovich G.M., Naumenko V.V. Investigation of stress state of shells of elliptic cone. *Dynamics and Strength of Mashines.* 1982;31:19–34.

15. Krivoshapko S.N., Timoshin M.A. Static calculation of an elliptical shell of the same slope, two conical shells with a guiding ellipse and a torso with two ellipses lying in parallel planes for overall stability. *Proceedings of the International Scientific and Practical Conference "Engineering Systems — 2012." Moscow*, 2012. C. 40–46. (In Russ.) EDN: TYKBOV

16. Drozdov S.M., Zhirnikov B.L., Kosykh A.P., Nersesov G.G., Yumashev V.L. Investigation of streamlining and aero-dynamical characteristics of blunted elliptical cones when supersonic speed. *Scientific notes of TSAGI*, 2008. Vol. 39. No. 1–2. P. 39–51. (In Russ.) EDN: JWVOIZ

17. Hueston S. *Anthropometric Analysis of the Cervical Spine*. Wright State University. Browse all Theses and Dissertations. 2011. 83 p. Available from: https://corescholar.libraries.wright.edu/etd_all/487 (accessed: 22.05.2023).

18. Krivoshapko S.N. Ruled shells of conical type on elliptical base. *Structural Mechanics of Engineering Constructions and Buildings*. 2024;20(1):40–56. http://doi.org/10.22363/1815-5235-2024-20-1-40-56

19. Weisstein E.W. Elliptic Cone. Available from: https://mathworld.wolfram.com/EllipticCone.html (accessed: 22.05.2023).

20. Gizatulina G.S. *Investigation of Stability of Truncated Conical Shells of Elliptical Cross Section under Different Types of Loading*: dissertation for the degree of Candidate of Physical and Mathematical Sciences. Kazan; 1999. EDN: QCZCNB

Список литературы

1. *Goldenveizer A.L.* Theory of Elastic Thin Shells. New York: Pergamon Publ.; 1961. 662 p.

2. *Новожилов В.В., Черных К.Ф., Михайловский Е.И.* Линейная теория тонких оболочек. Л.: Политехника, 1991. 656 с. ISBN 5-7325-0127-4

3. *Jawad M.H.* Design of Plate and Shell Structures. NY: ASME PRESS, 2004. 476 p. ISBN: 0-7918-0199-3

4. *Баджория Г.Ч.* Применение системы уравнений А.Л. Гольденвейзера для расчета торсовых оболочек по безмоментной теории // Исследования по расчету элементов пространственных систем. М.: Изд-во УДН, 1987. С. 65–72.

5. *Иванов В.Н., Кривошапко С.Н.* Аналитические методы расчета оболочек неканонической формы. М.: РУДН, 2010. 542 с. EDN: TINZYP

6. *Рекач В.Г.* Руководство к решению задач прикладной теории упругости. М.: Высшая школа, 1973. 384 с.

7. *Товстик П.Е.* Устойчивость тонких оболочек: асимптотические методы. М.: Наука; Физматлит, 1995. 320 с. ISBN 5-02-014522-X

8. *Тюриков Е.В.* К вопросу о построении математических моделей мембранной теории выпуклых оболочек // Advanced Engineering Research (Rostov-on-Don). 2023. Vol. 23. No. 1. P. 17–25. https://doi.org/10.23947/2687-1653-2023-23-1-17-25

9. *Иванов В.Н., Алёшина О.О.* Сравнительный анализ параметров напряженно-деформированного состояния торса с направляющим эллипсом с помощью трех методов расчета // Строительная механика и расчет сооружений. 2020. № 3. С. 37–46. https://doi.org/10.37538/0039-2383.2020.3.37.46

10. *Fattah M.Y., Waryosh W.A., Al-Hamdani M.A.E.* Experimental and theoretical studies of bearing capacity of conical shell foundations composed of reactive powder concrete // Acta Geodyn. Geometer. 2015. Vol. 12. No. 4. P. 411–420. https://doi.org/10.13168/AGG.2015.0037

11. *Мамиева И.А., Разин А.Д.* Знаковые пространственные сооружения в форме конических поверхностей // Промышленное и гражданское строительство. 2017. № 10. С. 5–11. EDN: ZSMVFN

12. *Golzan B.S., Showkati H.* Buckling of thin-walled conical shells under uniform external pressure // Thin-Walled Structures. 2008. Vol. 46. Iss. 5. P. 516–529. https://doi.org/10.1016/j.tws.2007.10.011

13. *Chung S.W., Ju Gisu*. A conical shell theory of hybrid anisotropic materials // International Journal of Composite Materials. 2019. Vol. 9. Iss. 3. P. 53–59. https://doi.org/10.5923/j.cmaterials.20190903.01

14. *Кантор Б.Я., Меллерович Г.М., Науменко В.В.* Исследование напряженного состояния оболочек типа эллиптического конуса // Динамика и прочность машин. 1982. Вып. 31. С. 19–34.

15. *Кривошапко С.Н., Тимошин М.А.* Статический расчет эллиптической оболочки одинакового ската, двух конических оболочек с направляющим эллипсом и торса с двумя эллипсами, лежащими в параллельных плоскостях на общую устойчивость // Труды Международной научно-практической конференции «Инженерные системы — 2012». М., 2012. С. 40–46. EDN: TYKBOV

16. *Дроздов С.М., Жирников Б.Л., Косых А.П., Нерсесов Г.Г., Юмашев В.Л.* Исследование обтекания и аэродинамических характеристик затупленных эллиптических конусов при сверхзвуковых скоростях // Ученые записки ЦАГИ. 2008. Т. 39. № 1–2. С. 39–51. EDN: JWVOIZ

17. *Hueston S.* Anthropometric Analysis of the Cervical Spine. Wright State University. Browse all Theses and Dissertations. 2011. 83 p. URL: https://corescholar.libraries.wright.edu/etd_all/487(дата обращения: 22.05.2023).

18. *Krivoshapko S.N.* Ruled shells of conical type on elliptical base // Строительная механика инженерных конструкций и сооружений. 2024. Т. 20. № 1. С. 40–56. http://doi.org/10.22363/1815-5235-2024-20-1-40-56

19. *Weisstein E.W.* Elliptic Cone. URL: https://mathworld.wolfram.com/EllipticCone.html (дата обращения: 22.05.2023).

20. *Гизатулина Г.С.* Исследование устойчивости усеченных конических оболочек эллиптического поперечного сечения при различных типах нагружения: дис. ... канд. физ.-мат. наук. Казань, 1999. 131 с. EDN: QCZCNB