2024. 20(2). 146–158
СТРОИТЕЛЬНАЯ МЕХАНИКА ИНЖЕНЕРНЫХ КОНСТРУКЦИЙ И СООРУЖЕНИЙ
STRUCTURAL MECHANICS OF ENGINEERING CONSTRUCTIONS AND BUILDINGS
ISSN 1815-5235 (Print), 2587-8700 (Online)
HTTP://JOURNALS.RUDN.RU/STRUCTURAL-MECHANICS

РАСЧЕТ ТОНКИХ УПРУГИХ ОБОЛОЧЕК
ANALYSIS OF THIN ELASTIC SHELLS

DOI: 10.22363/1815-5235-2024-20-2-146-158
UDC 624.04:624.074:514
EDN: DSRDSR

Research article / Научная статья

Geometric Investigation of Three Thin Shells with Ruled Middle Surfaces with the Same Main Frame

Gerard L. Gbaguidi Aisse[1], **Olga O. Aleshina**[2] ✉, **Iraida A. Mamieva**[2]

[1] Verechaguine AK School of Civil Engineering, *Cotonou, Republic of Benin*
[2] RUDN University, *Moscow, Russia*
✉ xiaofeng@yandex.ru

Article history

Received: November 23, 2023
Revised: January 21, 2024
Accepted: February 14, 2024

Conflicts of interest

The authors declare that there is no conflict of interest.

Authors' contribution

Undivided co-authorship.

Abstract. It is proved and illustrated that by taking the main frame of the surface, consisting of three plane curves placed in three coordinate planes, three different algebraic surfaces with the same rigid frame can be designed. For the first time, one three of new ruled surfaces in a family of five threes of ruled surfaces, formed on the basis of some shapes of hulls of river and see ships, which, in turn, are projected in the form of algebraic surfaces with a main frame of three superellipses or of three other plane curves, is under consideration in detail with a standpoint of differential geometry. The geometrical properties of the ruled surfaces taken as the middle surfaces of thin shells for industrial and civil engineering are presented. Analytical formulas for determination of force resultants with using the approximate momentless theory of shells of zero Gaussian curvature given by non-orthogonal conjugate curvilinear coordinates are offered for the first time. The results derived using these formulae will help to correct the results obtained by numerical methods.

Keywords: thin shell, ruled surface, algebraic surface, main frame of the surface, superellipse

For citation

Gbaguidi Aisse G.L., Aleshina O.O., Mamieva I.A. Geometric Investigation of three thin shells with ruled middle surfaces with the same main frame. *Structural Mechanics of Engineering Constructions and Buildings.* 2024;20(2):146–158. http://doi.org/10.22363/1815-5235-2024-20-2-146-158

Gerard L. Gbaguidi Aisse, PhD of Technical Sciences, Director, Verechaguine AK School of Civil Engineering, Cotonou, Republic of Benin; ORCID: 0000-0002-8557-1392; E-mail: gbaguidi.gerard@yahoo.fr

Olga O. Aleshina, Candidate of Technical Sciences, Assistant of the Department of Civil Engineering, Engineering Academy, RUDN University; Moscow, Russia; eLIBRARY SPIN-code: 6004-2422; ORCID: 0000-0001-8832-6790; E-mail: xiaofeng@yandex.ru

Iraida A. Mamieva, Assistant of the Department of Civil Engineering, Academy of Engineering, RUDN University, Moscow, Russia; eLIBRARY SPIN-code: 3632-0177; ORCID: 0000-0002-7798-7187; E-mail: i_mamieva@mail.ru

Геометрическое исследование трех оболочек с линейчатыми срединными поверхностями с одинаковым главным каркасом

Ж.Л. Гбагуиди Аиссе[1], О.О. Алёшина[2]✉, И.А. Мамиева[2]

[1] Высшая школа гражданского строительства им. А.К. Верещагина, *Котону, Республика Бенин*
[2] Российский университет дружбы народов, *Москва, Россия*

✉ xiaofeng@yandex.ru

История статьи

Поступила в редакцию: 23 ноября 2023 г.
Доработана: 21 января 2024 г.
Принята к публикации: 14 февраля 2024 г.

Заявление о конфликте интересов

Авторы заявляют об отсутствии конфликта интересов.

Вклад авторов

Нераздельное соавторство.

Аннотация. Показано и проиллюстрировано, что, взяв основной каркас поверхности, состоящий из трех плоских кривых, расположенных в трех координатных плоскостях, можно спроектировать три различные алгебраические поверхности с одним и тем же жестким каркасом. Рассмотрена одна тройка новых линейчатых поверхностей в семействе из пяти троек линейчатых поверхностей, сформированных на основе некоторых форм корпусов речных и морских судов, которые, в свою очередь, проецируются в виде алгебраических поверхностей с основным каркасом из трех суперэллипсов или из трех других плоские кривые подробно рассматриваются с точки зрения дифференциальной геометрии. Приводятся геометрические свойства линейчатых поверхностей, взятых в качестве средних поверхностей тонких оболочек для промышленного и гражданского строительства. Предложены аналитические формулы для определения результирующих сил с использованием приближенной безмоментной теории оболочек нулевой гауссовой кривизны, заданных неортогональными сопряженными криволинейными координатами. Результаты, полученные с использованием этих формул, помогут скорректировать результаты, полученные численными методами.

Ключевые слова: тонкая оболочка, линейчатая поверхность, алгебраическая поверхность, главный каркас поверхности, суперэллипс

Для цитирования

Gbaguidi Aisse G.L., Aleshina O.O., Mamieva I.A. Geometric investigation of three thin shells with ruled middle surfaces with the same main frame // Строительная механика инженерных конструкций и сооружений. 2024. Т. 20. № 2. С. 146–158. http://doi.org/10.22363/1815-5235-2024-20-2-146-158

1. Introduction

The geometry and shaping of surfaces, design, calculation and application of thin-walled structures based on different types of surfaces have been the subject of many scientific works. At the same time, there are always questions to be considered and new results to be obtained. The purpose of this research is to investigate the possibility of shaping surfaces with a framework of three plan curves in the superellipse type and to investigate the stress-strain state of these shells.

Гбагуиди Аиссе Жерар Леопольд, доктор наук, директор Высшей школы гражданского строительства им. А.К. Верещагина, Котону, Республика Бенин; ORCID: 0000-0002-8557-1392; E-mail: gbaguidi.gerard@yahoo.fr

Алёшина Ольга Олеговна, кандидат технических наук, ассистент департамента строительства инженерной академии, Российский университет дружбы народов, Москва, Россия; eLIBRARY SPIN-код: 6004-2422; ORCID: 0000-0001-8832-6790; E-mail: xiaofeng@yandex.ru

Мамиева Ираида Ахсарбеговна, ассистент департамента строительства инженерной академии, Российский университет дружбы народов, Москва, Россия; eLIBRARY SPIN-код: 3632-0177; ORCID: 0000-0002-7798-7187; E-mail: i_mamieva@mail.ru

It was proved that taking the main frame of the surface, consisting of three plane curves placed in three coordinate planes (Figure 1), it is possible to design three different algebraic surfaces with the same rigid frame [1–3]. In Figure 1, the plane curve in section with *yOz* plane coincides with midsection, in section with *xOz* plane coincides with main buttock section and in section with *xOy* plane the plane curve coincides with waterline. These three plane curves lie in mutually perpendicular cross-sections of the ship's hull. The geometric parameters of the hull (see Figure 1) are defined as follows: T — hull draft, $2W$ — hull width, $2L$ — hull length. The surfaces constructed in this way are used for forming hulls of river and sea ships (Figure 2) and underwater vehicles [1; 3]. It was first offered in [4; 5] to use these surfaces as middle surfaces of building shells (Figures 3, 4).

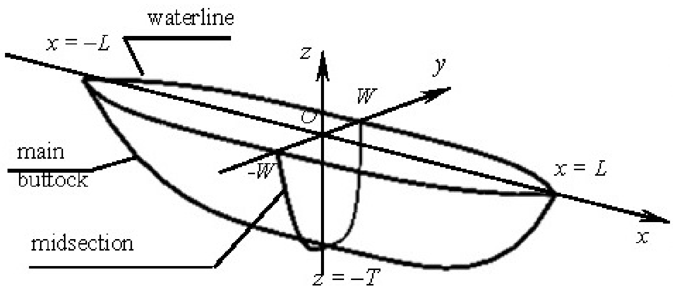


Figure 1. Hydrodynamic surface skeleton consisting of three plane curves [2]
S o u r c e: made by S.N. Krivoshapko

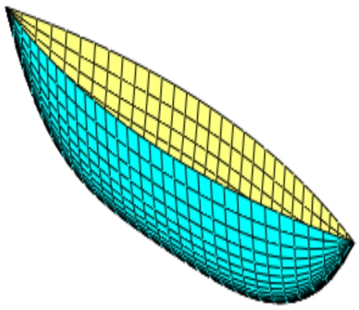


Figure 2. A surface of ship hull formed by a family of midship sections [2]
S o u r c e: made by S.N. Krivoshapko

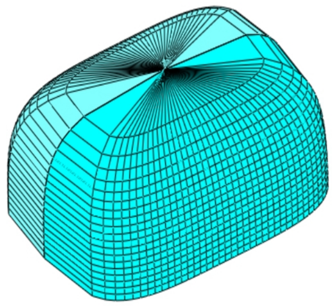


Figure 3. The surface with a main frame from three superellipses formed by plane lines at horizontal planes [4]
S o u r c e: made by O.O. Aleshina

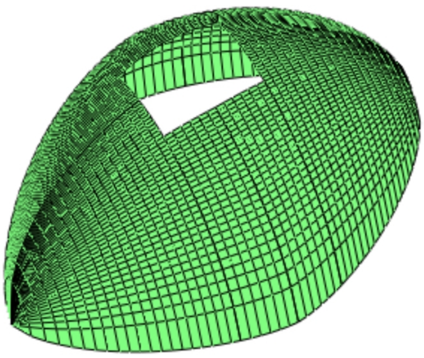


Figure 4. The surface with a main frame from three superellipses formed by plane lines at vertical plane [5]
S o u r c e: made by O.O. Aleshina

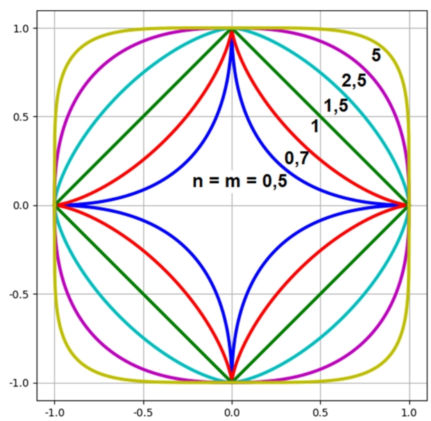


Figure 5. Lame curves at different values of parameters $n = m = 0,5; 0,7; 1; 1,5; 2,5; 5$ [6]
S o u r c e: https://mathworld.wolfram.com/Superellipse.html

In [3] three plane curves (see Figure 1) were taken as Lamé curves, also known as superellipses [6]. In [3] the waterline, the main buttock and the midsection were accepted in the form of superellipses. Parameters r, t, n, m, s, k are positive degrees of superellipse equations. The application of superellipses as plane curves gave the opportunity to simplify the visualization process of studied surfaces.

Assume that plane curves of the main frame of studied surfaces represent superellipses [6] and are given in the form:

the first curve is placed in the plane $z = 0$:

$$|y|^r = W^r\left(1-|x|^t / L^t\right), \tag{1}$$

the second curve is placed in the plane $x = 0$:

$$|z|^n = T^n\left(1-|y|^m / W^m\right). \tag{2}$$

the third curve is placed in the plane $y = 0$:

$$|z|^s = T^s(1-|x|^k / L^k), \tag{3}$$

where $r = t$, $n = m$, $s = k$, for convex curves $r; t, n, m, s, k > 1$; for concave curves $r; t, n, m, s, k < 1$. If to take $r = t = 1$, $n = m = 1$ (see Figure 5), $s = k = 1$, then curves (1)–(3) degenerate into straight lines, and superellipses degenerate into rhombs. Arbitrary parameters n, m, r, t, s, k make it possible to obtain a large number of different surface shapes.

Using the method set forth in [1; 2], we can derive explicit equations of three algebraic surfaces with the same main frame (1)–(3):

- with generatrix family of the section x = const:

$$|z| = T\left(1-|x|^k / L^k\right)^{1/s}\left[1-|y/W|^m / \left(1-|x/L|^t\right)^{m/r}\right]^{1/n}; \tag{4}$$

- with generatrix family of the section y = const:

$$|z| = T\left(1-|y|^m / W^m\right)^{1/n}\left[1-|x/L|^k / \left(1-|y/W|^r\right)^{k/t}\right]^{1/s}; \tag{5}$$

- with generatrix family of the section z = const:

$$|y| = W\left(1-|z|^n / T^n\right)^{1/m}\left[1-|x/L|^t / \left(1-|z/T|^s\right)^{t/k}\right]^{1/r}, \tag{6}$$

where $-L \le x \le L$, $-W \le y \le W$, $0 \le z \le T$.

The explicit equations of surfaces (4)–(6) can be transformed into parametrical equations:

$$x = x(u) = \pm uL,\quad y = y(u,\text{v}) = \text{v}W\left[1-u^t\right]^{1/r};$$

$$z = z(u,\text{v}) = T\left[1-u^k\right]^{1/s}\left[1-|\text{v}|^m\right]^{1/n}; \tag{4a}$$

$$x = x(u,\nu) = \nu L\left[1-u^{r}\right]^{1/t}, \quad y = y(u) = \pm uW;$$

$$z = z(u,\nu) = T\left[1-u^{m}\right]^{1/n}\left[1-|\nu|^{k}\right]^{1/s}; \tag{5a}$$

$$x = x(u,\nu) = \nu L\left[1-u^{s}\right]^{1/k};$$

$$y = y(u,\nu) = \pm W\left[1-u^{n}\right]^{1/m}\left[1-|\nu|^{t}\right]^{1/r}, \quad z = x(u) = uT, \tag{6a}$$

where $0 \le u \le 1$, $-1 \le \nu \le 1$; u, ν are dimensionless parameters and are the curvilinear coordinates lines of the surfaces.

I.A. Mamieva in [7] proposed to introduce ruled surfaces given by equations (4)–(6) or (4a)–(6a). It is established that due to the equations presented above, it is possible to construct five groups of ruled surfaces, and each group contains three surfaces.

The aim of the investigation is to study the geometry and carry out a static analysis of shells with two types of new ruled middle surfaces, first presented in [7]. This research will help to choose the optimal shapes of ruled shells and extend the opportunities of their form-building for architects within the modern architectural styles [8].

2. Method

2.1. Geometry of ruled surfaces based on algebraic surfaces with the main frame of three superellipses (1)–(3)

Let the superellipse placed in the xOy plane be given by formula (1), and let the other two superellipses of the main frame degenerate in straight lines, i.e. $n = m = s = k = 1$, then we shall have three surfaces on the oval plane:

$$z = T\left(1-|x|/L\right)\left[1-|y/W|/\left(1-|x/L|^{t}\right)^{1/r}\right]; \tag{7}$$

$$z = T\left(1-|y/W|\right)\left[1-|x/L|/\left(1-|y/W|^{r}\right)^{1/t}\right]; \tag{8}$$

$$|y| = W\left(1-z/T\right)\left[1-|x/L|^{t}/\left(1-z/T\right)^{t}\right]^{1/r}, \tag{9}$$

where $-L \le x \le L$, $-W \le y \le W$, $0 \le z \le T$.

The explicit equations of the surfaces (7)–(9) can be transformed into parametric equations:

$$x = x(u) = \pm uL, \quad y = y(u,\nu) = \nu W\left[1-u^{t}\right]^{1/r};$$

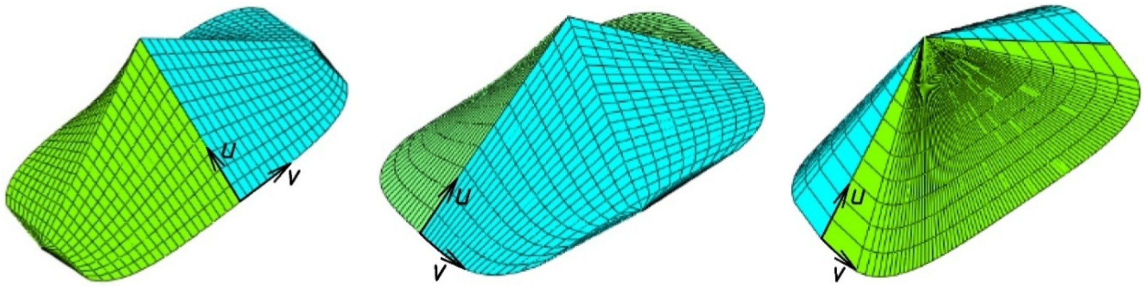
$$z = z(u,\nu) = T\left[1-u\right]\left[1-|\nu|\right], \quad \text{(Figure 6, } a\text{)}; \tag{7a}$$

$$x = x(u,\nu) = \nu L\left[1-u^{r}\right]^{1/t}, \quad y = y(u) = \pm uW;$$

$$z = z(u,\nu) = T\left[1-u\right]\left[1-|\nu|\right], \quad \text{(Figure 6, } b\text{)}; \tag{8a}$$

$$x = x(u,\nu) = \nu L\left[1-u\right], \quad y = y\left(u,\nu\right) = \pm W\left[1-u\right]\left[1-\left|\nu\right|^{t}\right]^{1/r};$$
$$z = x\left(u\right) = uT, \quad \text{(Figure 6, } c\text{)}. \tag{9a}$$

Three surfaces with $r = t = 4$, but with the same main frame are shown in Figure 6, where u, ν are the curvilinear coordinates lines of the surfaces.



a *b* *c*

Figure 6. The ruled surfaces on a plane oval base with the same main frame
S o u r c e: made by G.L. Gbaguidi Aisse, O.O. Aleshina, I.A. Mamieva

It is obviously that the first two surfaces (7) and (8) are the cylindroids [9].

Rewrite equations (7)–(9) in the detailed form:

$$\left[z - T\left(1-\left|x\right|/L\right)\right]^{r}\left(1-\left|x\right|^{r}/L^{r}\right) = \left[-T\left|y\right|\left(1-\left|x\right|/L\right)/W\right]^{r}; \tag{7b}$$

$$\left[z - T\left(1-\left|y\right|/W\right)\right]^{r}\left(1-\left|y\right|^{r}/W^{r}\right) = \left[-T\left|x\right|\left(1-\left|y\right|/W\right)/L\right]^{r}; \tag{8b}$$

$$\left|y\right|^{r}/W^{r} + \left|x\right|^{r}/L^{r} - \left[1-z/T\right]^{r} = 0. \tag{9b}$$

So, the ruled surfaces (7), (8) are algebraic surfaces of the $2r$ order. The ruled surface (9) is the r order algebraic surface.

Taking into account that only the shells shown in Figure 6, i.e. with $r = 4$, will be considered below, let us determine the coefficients of the basic quadratic forms [9] (Gaussian quantities of the first and second orders in theory of surfaces) for their middle surfaces. The coefficients of the first quadratic form E, G, F characterize the internal geometry of the shell, the coefficients of the second quadratic form $\underline{L}$, M, N characterize the curvature of the surface in space.

Each surface defined by parametric equations can be given by the vector equation

$$\boldsymbol{r} = \boldsymbol{r}\left(u,\nu\right) = x\left(u,\nu\right)\boldsymbol{i} + y\left(u,\nu\right)\boldsymbol{j} + z\left(u,\nu\right)\boldsymbol{k}.$$

In this case, the coefficients of the basic quadratic forms [9] of the surface (7a) are expressed as

$$E = A^{2} = \boldsymbol{r_u}^{2} = L^{2} + \nu^{2}u^{6}W^{2}/\left(1-u^{4}\right)^{3/2} + T^{2}\left(1-\nu\right)^{2};$$
$$G = B^{2} = \boldsymbol{r}_{\nu}^{2} = W^{2}\left(1-u^{4}\right)^{1/2} + T^{2}\left(1-u\right)^{2} = B^{2}\left(u\right);$$
$$F = \boldsymbol{r_u}\boldsymbol{r}_{\nu} = -\nu u^{3}W^{2}/\left(1-u^{4}\right)^{1/2} + T^{2}\left(1-u\right)\left(1-\nu\right); \tag{10}$$

$$\underline{L} = \mp \frac{3LTW\text{v}u^2(1-u)}{\sqrt{A^2B^2-F^2}\left(1-u^4\right)^{7/4}};$$

$$M = \pm \frac{LTW\left(1-u^3\right)}{\sqrt{A^2B^2-F^2}\left(1-u^4\right)^{3/4}};$$

$$N = 0, \qquad (11)$$

where A and B are the Lame parameters of the surface.

The curvature coefficient $N = 0$ of the undeformed middle surface shows that the coordinate lines v coincide with straight generatrixes of the surface (7a). The metric coefficient $F \neq 0$ shows that the curvilinear coordinates u, ν are non-orthogonal, and the curvature coefficient $M \neq 0$ of the undeformed middle surface shows that the coordinate lattice u, ν is non-conjugate.

The coefficients of the basic quadratic forms [9] of the surface (8a) are expressed as

$$E = A^2 = \boldsymbol{r_u}^2 = W^2 + v^2u^6 L^2 / \left(1-u^4\right)^{3/2} + T^2(1-\text{v})^2;$$

$$G = B^2 = \boldsymbol{r}_\text{v}^2 = L^2\left(1-u^4\right)^{1/2} + T^2(1-u)^2 = B^2(u);$$

$$F = \boldsymbol{r_u}\boldsymbol{r}_\text{v} = -vu^3L^2 / \left(1-u^4\right)^{1/2} + T^2(1-u)(1-\text{v}); \qquad (12)$$

$$\underline{L} = \pm \frac{3LTWvu^2(1-u)}{\sqrt{A^2B^2-F^2}\left(1-u^4\right)^{7/4}};$$

$$M = \mp \frac{LTW\left(1-u^3\right)}{\sqrt{A^2B^2-F^2}\left(1-u^4\right)^{3/4}};$$

$$N = 0. \qquad (13)$$

The replacement of constant geometrical parameters $W \leftrightarrow L$ in formulas (12), (13) permits to obtain formulas (10), (11). Comments to formulas (12), (13) will be analogous to comments to formulas (10) and (11).

The two ruled surfaces shown in Figure 6, *a* and Figure 6, *b* are surfaces of negative Gaussian curvature because

$$K = \left(\underline{L}N - M^2\right) / \left(A^2B^2 - F^2\right) = -M^2 / \left(A^2B^2 - F^2\right) < 0. \qquad (14)$$

The differentials of the corresponding arcs of coordinate lines u and ν can be calculated using the formulas $ds_u = Adu$ and $ds_\text{v} = Bd\text{v}$.

The coefficients of the basic quadratic forms [9] of the surface (9a) are expressed as

$$E = A^2 = \boldsymbol{r_u}^2 = W^2\left(1-\text{v}^4\right)^{1/2} + v^2L^2 + T^2 = A^2(\text{v});$$

$$G = B^2 = \boldsymbol{r}_\text{v}^2 = (1-u)^2\left[L^2 + \text{v}^6W^2 / \left(1-\text{v}^4\right)^{3/2}\right];$$

$$F = \boldsymbol{r_u}\boldsymbol{r}_\text{v} = -\text{v}(1-u)\left[L^2 + \text{v}^2W^2 / \left(1-\text{v}^4\right)^{1/2}\right]; \qquad (15)$$

$$E = A^2 = \left[\frac{W^2}{\left(1-\nu^4\right)^{3/2}} \left(L^2 + T^2\nu^6\right) + L^2T^2 \right] = (1-u)^2 \left[f(\nu) \right];$$

$$N = \mp \frac{3TLW\nu^2 (1-u)^2}{\sqrt{A^2B^2 - F^2}\left(1-\nu^4\right)^{7/4}};$$

$$L = 0, \; M = 0. \tag{16}$$

The coefficients of the basic quadratic forms (15), (16) of the surfaces (9a) show that the coordinate lines u are straight lines, and the surface in question is a surface of zero Gaussian curvature

$$K = \left(\underline{L}N - M^2\right) / \left(A^2B^2 - F^2\right) = 0. \tag{17}$$

The curvilinear coordinate lines u are principal curvature lines of the surface (9a).

The curvature coefficient $M = 0$ (16), therefore, curvilinear coordinate lines u, ν are conjugate, and this is naturally, as every family of lines intersecting a family of straight coordinate lines on surfaces forms conjugate nets on them.

2.2. Preconditions for choosing a method for determining the parameters of the stress-strain state of the ruled shells

The article [10] presents four stages of creation and development of the theory of plates and shells, which gave rise to the mechanism of analysis of spatial roof structures of large-span buildings and structures at the modern level. The beginning of the fourth stage in the development of the shell theory, design and construction of large-span structures has been laid since the very beginning of the 21st century.

At present, there is a great variety of analytical, semi-analytical, and numerical methods for analyzing shells and shell structures. In the previous part, it is shown that middle surfaces of the shell in question are given in Cartesian coordinates using algebraic equations (4)–(6) or parametric equations (4a)–(6a). The curvilinear coordinate lines u, v on these surfaces are non-orthogonal ($F \neq 0$) and non-conjugate ($M \neq 0$) coordinate lines. One family of coordinate lines coincides with the rectilinear generatrixes of the surfaces ($\underline{L} = 0$ or $N = 0$).

Taking these conditions into consideration, one can use the system of 20 governing equations of Goldenveiser [11] of thin shell theory for arbitrary curvilinear coordinates containing internal "pseudo-forces" and "pseudo-moments" or the system of governing equations suggested by Krivoshapko [12] containing internal forces and moment recalculated per unit length of curvilinear coordinates or the governing equations of Grigorenko and Timonin [13] written in tensor form. The governing equations offered by these scientists contain the coefficients of the basic quadratic forms of surfaces, which for the considered ruled shells are presented in this paper for the first time.

The analysis of published works has shown that these three groups of governing equations of the linear theory of thin shells have been used only for simplified momentless theory of shells or for the analysis of ruled shells with assumption of some simplifications in the geometry or in the governing equations of the shell theory [14].

2.3. The momentless theory of analysis of the conical shell presented in Figure 6, c

The equilibrium equations of the approximate momentless thin shell theory for an arbitrary coordinate system are obtained from the equilibrium equations of the moment theory. Eliminating the bending and twisting moments and retaining the normal and tangent internal forces, we can write (Figure 7) [12]:

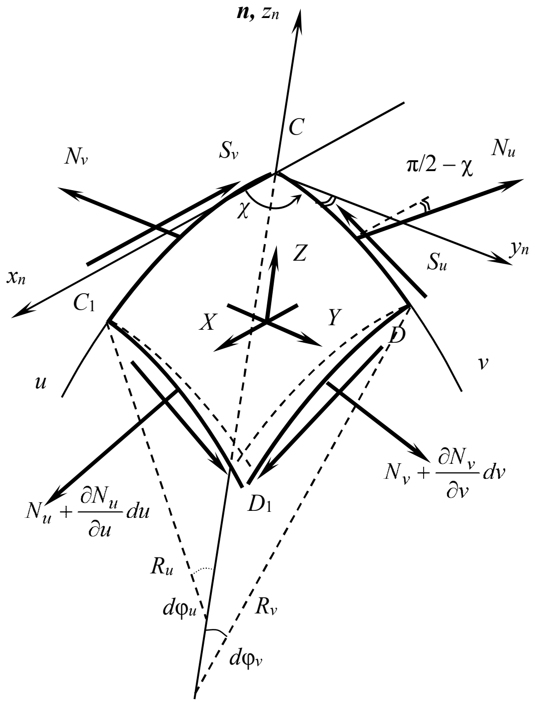


Figure 7. Internal forces in a momentless shell [12]
S o u r c e: made by S.N. Krivoshapko

$$\frac{\partial}{\partial \text{v}}\left(AS_\text{v}\right)+\frac{N_u-N_\text{v}}{\sin\chi}\left(\frac{\partial B}{\partial u}-\frac{\partial A}{\partial \text{v}}\cos\chi\right)+\frac{\partial A}{\partial \text{v}}S_u+B\frac{\partial S_u}{\partial u}\cos\chi+B\frac{\partial N_u}{\partial u}\sin\chi+ABX\sin\chi=0; \quad (18)$$

$$\frac{\partial}{\partial \text{v}}\left(AN_\text{v}\right)+\frac{S_u+S_\text{v}}{\sin\chi}\left(\frac{\partial B}{\partial u}-\frac{\partial A}{\partial \text{v}}\cos\chi\right)-\frac{\partial A}{\partial \text{v}}N_u+B\frac{\partial S_u}{\partial u}\sin\chi-B\frac{\partial N_u}{\partial u}\cos\chi=0; \quad (19)$$

$$\frac{N_\text{v}}{R_\text{v}\sin\chi}-Z\sin\chi=0; \quad (20)$$

$$\left(S_u-S_\text{v}\right)\sin\chi+\left(N_\text{v}-N_u\right)\cos\chi=0. \quad (21)$$

Thus, equations (19), (20) contain the resultant forces N_u, N_v and $S_u \neq S_v$ which are the forces per unit length. X and Z represent the external forces per unit area applied to the surface.

The equations (18)–(21) were written with allowance for $1/R_u = Y = 0$ for the problem in question. The full version of the equations (18)–(21) is presented in [12].

In addition, $\cos\chi = F/(AB)$, $\sin\chi$, $\partial B/\partial u$ are functions of the dimensionless parameter ν only.

From the equation (20) we get the normal force N_v:

$$N_\text{v}=Z\sin^2\chi R_\text{v}=\frac{Z\sin^2\chi B^2}{N}=Z\frac{\left(A^2B^2-F^2\right)}{A^2N}>0. \quad (22)$$

From the equation (21) we get the normal force N_u:

$$N_u=\left(S_u-S_\text{v}\right)\text{tg}\chi . \quad (23)$$

The external surface load X, Z is defined as $X = -q\cos\varphi$, $Z = q\sin\varphi$, $Y = 0$, where φ is the angle of the external load direction q with the direction opposite to the coordinate line direction u, there

$$\cos\varphi = T / A, \quad \sin\varphi = \left[A^2 - T^2\right]^{1/2} / A \tag{24}$$

are functions of the dimensionless parameter ν only.

The coefficient of the first fundamental form $A = A(\nu)$ is equal to the length of the straight coordinate line u from the vertex of the cone to the plane z = 0.

Taking into consideration that

$$F = -A(1-u)\frac{\partial A}{\partial \nu}, \qquad \frac{\partial B}{\partial u} - \frac{\partial A}{\partial \nu}\cos\chi = -\frac{\left(A^2B^2 - F^2\right)}{A^2B(1-u)}, \tag{25}$$

we substitute these expressions and the quantity of the normal force N_u, given by a formula (23), into equation (19). The result obtained is integrated into the following expression

$$S_\nu = \frac{A}{(1-u)\sqrt{A^2B^2 - F^2}}\int(1-u)\left(\frac{\partial N_\nu}{\partial \nu} + \frac{F}{A}\frac{\partial N_\nu}{\partial u}\right)du + \frac{V_1(\nu)}{(1-u)^2}, \tag{26}$$

where $V_1(\nu)$ is an arbitrary function of integration only over the parameter ν.

Substituting expressions (25) and the difference ($N_u - N_\nu$) defined by formulae (21) into equation (18) and integrating the results, we can find

$$S_u = \frac{\left(A^2B^2 - F^2\right)}{A^2B^2}S_\nu - \frac{F}{AB^2}\int\frac{\partial}{\partial \nu}(AS_\nu)du - \\ -\sqrt{A^2B^2 - F^2}\left[\frac{F}{A^2B^3}N_\nu + \frac{Fu}{(1-u)AB^2}\left(1 - \frac{u}{2}\right)X\right] + \frac{F}{AB^2}V_2(\nu), \tag{27}$$

where $V_2(\nu)$ is an arbitrary integration function. The unknown functions $V_1(\nu)$ and $V_2(\nu)$ are found from the boundary conditions acceptable for the momentless shell theory.

Thus, the momentless shell theory makes it possible to obtain approximate values of internal normal forces N_u and N_ν using formulas (23) and (22), and values of membrane shearing forces S_u and S_ν using formulas (27) and (26). Formulas (25), (26) can be easily integrated and can be written in the detailed form.

The derived analytical formulas can be applied to the approximate calculation of only one type of studied shells, shown in Figure 6, *c*. The other two ruled shells presented in Figure 6, *a* and Figure 6, *b* can only be analyzed using numerical methods.

2.4. Geometry of ruled surfaces constructed on the basis of algebraic surfaces with main frame of three degenerate superellipses

The simplest ruled surface is obtained if all three degenerated superellipses are rhombuses. In this case, it is necessary to take $r = t = n = m = s = k = 1$ in formulas (1)–(3). Then the surfaces (4)–(6) become identical:

$$z = T(1 - |x| / L - |y| / W). \tag{28}$$

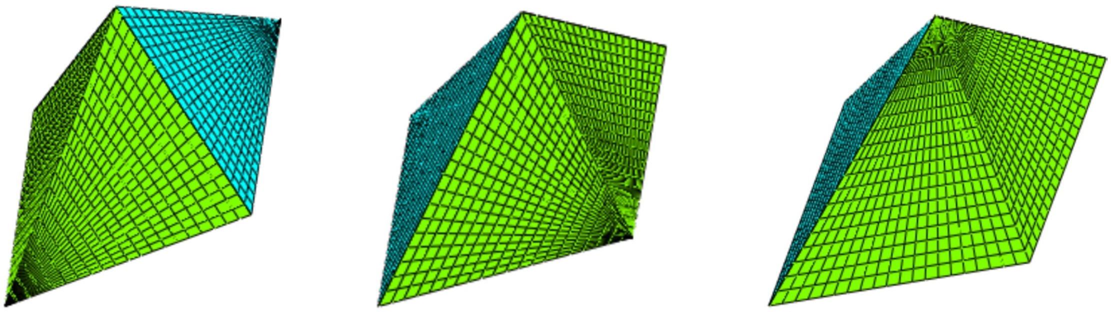
The parametrical equations (4a)–(6a) become

$$x = x(u) = \pm uL, \quad y = y(u,\nu) = \nu W[1-u];$$
$$z = z(u,\nu) = T[1-u][1-|\nu|], \quad \text{(Figure 8, } a\text{);} \tag{29}$$

$$x = x(u) = \pm uL, \quad y = y(u,\nu) = \nu W[1-u];$$
$$z = z(u,\nu) = T[1-u][1-|\nu|], \quad \text{(Figure 8, } b\text{);} \tag{30}$$

$$x = x(u,\nu) = \nu L[1-u];$$
$$y = y(u,\nu) = \pm W[1-u][1-|\nu|], \quad z = x(u) = uT, \quad \text{(Figure 8, } c\text{).} \tag{31}$$

Three identical surfaces with different curvilinear coordinates u, ν are presented in Figures 8.



a *b* *c*

Figure 8. The polyhedrons with four triangular fragments of plane and on the rhombic base [12]
S o u r c e: made by S.N. Krivoshapko

It is obviously from Figures 8 that the derived identical surfaces (polyhedrons [15]) consist of the same four fragments of planes with different position of surface coordinates.

3. Results and Discussion

In this paper the geometry of six new ruled surfaces belonging to two subgroups are studied. All of them are constructed on the basis of general translation surfaces of the velaroidal types. For the first time, the coefficients of the first and second fundamental forms in the theory of surfaces were obtained for these new ruled surfaces. These geometric results will help architects and designers to widen the possibilities of applications of the presented construction and engineering shells.

Analytical formulas for the determination of the force resultants using of the approximate momentless theory of shells of zero Gaussian curvature, given by non-orthogonal conjugate curvilinear coordinates, have been obtained. These formulas are presented for the first time.

The research in the article shows the complexity of studying the stress-strain state by an analytical method using the general moment theory of shells. In this regard, one of the numerical calculation methods can be used to further study the subclass of shells presented in the article. The finite element method [16] has proven itself as an effective method for studying the stress-strain state of various shell shapes [17; 18]. Moreover, at present time, there is only one work [19] devoted to the determination of the stress-strain state of super ellipsoidal shells of revolution.

The complexity of contemporary free-form architecture has been a driving force for the development of new digital design process over the last years [20]. An interesting class of ruled surfaces, generated by a continuously moving straight line, opens a wide range of advantageous options for support structures, mould production or facade elements [21]. Geometricians present many non-traditional methods for defining ruled surfaces not only with the classical means, but first of all with the help of a computer [22].

4. Conclusion

The introduction into practice of new geometric shapes of shells and shell structures gives an opportunity to expand the search for the most optimal forms that correspond to the selected criteria of optimality. The distinguished Spanish engineer E. Torroja supposed that it is very prospective direction for investigations carried out by experienced mechanical scientists, architects and young research. These conclusions are confirmed by the appearance of new architectural styles, directions and style flows in the 21st century.

The main results:

1. The parametric equations of new ruled surfaces on a plane oval base with the same main frame are obtained in the article (Figure 6). It is shown that, the ruled surfaces in Figure 6, *a* and Figure 6, *b* are algebraic surfaces of the 2r order. The ruled surface Figure 6, *c* is the r order algebraic surface.
2. The coefficients of the basic quadratic forms of the surfaces in Figure 6 are obtained in the article for the first time. The two ruled surfaces shown in Figure 6, *a* and Figure 6, *b* are surfaces of negative Gaussian curvature. The surface shown in Figure 6, *c* is a surface of zero Gaussian curvature.
3. The momentless shell theory makes it possible to obtain approximate values of internal normal forces N_u and N_v using formulas (23) and (22), and values of membrane shearing forces S_u and S_v using formulas (27) and (26).
4. The parametric equations of ruled algebraic surfaces with main frame of three degenerate superellipses are obtained in the article for the first time (Figure 8). The simplest ruled surface is obtained if all three degenerated superellipses are rhombuses. In this case, it is necessary to take $r = t = n = m = s = k = 1$ in formulas (1)–(3).

References

1. Krivoshapko S.N. Tangential developable and hydrodynamic surfaces for early stage of ship shape design. *Ships and Offshore Struct.* 2023;18(5):660–668. https://doi.org/10.1080/17445302.2022.2062165
2. Krivoshapko S.N. Algebraic ship hull surfaces with a main frame from three plane curves in coordinate planes. *RUDN Journal of Engineering Research.* 2022;23(3):207–212. https://doi.org/10.22363/2312-8143-2022-23-3-207-212
3. Karnevich V.V. Design of hydrodynamical surface by the frame from Lame curves on the example of submarine hull. *RUDN Journal of Engineering Research.* 2022;23 (1):30–37. https://doi.org/10.22363/2312-8143-2022-23-1-30-37
4. Krivoshapko S.N., Aleshina O.O., Ivanov V.N. Static analysis of shells with middle surfaces containing the main frame from three given superellipses. *Structural Mechanics and Analysis of Constructions.* 2022;6:18–27. https://doi.org/10.37538/0039-2383.2022.6.18.27
5. Aleshina O.O. Geometry and static analysis of thin shells in the form of a diagonal translation surface of the velaroidal type. *Structural Mechanics of Engineering Constructions and Buildings*. 2023;19(1):84–93. http://doi.org/10.22363/1815-5235- 2023-19-1-84-93
6. Weisstein E.W. Superellipse. From MathWorld — A Wolfram Web Resource. Available from: https://mathworld.wolfram.com/Superellipse.html (accessed: 12.05.2023)
7. Mamieva I.A. Ruled algebraic surfaces with a main frame from three superellipses. *Structural Mechanics of Engineering Constructions and Buildings*. 2022;18(4):387–395. https://doi.org/10.22363/1815-5235-2022-18-4-387-395
8. Krivoshapko S.N., Christian A.B.H., Gil-oulbé M. Stages and architectural styles in design and building of shells and shell structures. *Building and Reconstruction.* 2022;4(102):112–131. https://doi.org/10.33979/2073-7416-2022-102-4-112-131
9. Krivoshapko S.N., Ivanov V.N. *Encyclopedia of Analytical Surfaces*. Springer International Publishing Switzerland, 2015.
10. Sysoeva E.V. Scientific approaches to calculation and design of large-span structures. *Vestnik MGSU [Monthly J. on Construction and Architecture]*. 2017;12 2(101) :131–141. https://doi.org/10.22227/1997-0935.2017.2.131-141

11. Goldenveizer A.L. *Theory of Elastic Thin Shells*, Published by Pergamon Press, New York, USA, 1961.

12. Krivoshapko S.N., Razin A.D. Comparison of two systems of governing equations for the thin shell analysis. *AIP Conference Proceedings*. 2022;2559:020009. https://doi.org/10.1063/5.0099905

13. Steigmann D.J., Bîrsan M., Shirani M. Lecture Notes on the Theory of Plates and Shells: Classical and Modern Developments. *Part of the book series: Solid Mechanics and Its Applications* (SMIA, volume 274). Springer; 2023. https://doi.org/10.1007/978-3-031-25674-5

14. Tupikova E.M. Investigation of V.G. Rekatch's method of stress-strain analysis of the shell of long shallow oblique helicoid form. *Structural Mechanics and Analysis of Constructions*. 2016;1:14–19.

15. Markov I.J., Gabriel J.F. Spatial and structural aspects of polyhedral. *Engineering Structures.* 2001;23(1):4–11. https://doi.org/10.1016/S0141-0296(00)00016-X

16. Dinkler D., Kowalsky U. *Introduction to Finite Element Methods.* Springer Vieweg Wiesbaden; 2024. https://doi.org/10.1007/978-3-658-42742-9

17. Aleshina O., Cajamarca-Zuniga D., Ivanov V., Rekach F., Alborova L. Analytical and numerical stress state analysis of a shell with tangential developable middle surface. *AIP Conference Proceedings.* 2022;2559:020008. https://doi.org/10.1063/5.0099513

18. Aleshina O., Cajamarca D., Barbecho J. Numerical Comparative Analysis of a Thin-Shell Spatial Structure for the Candela's Cosmic Rays Pavilion. Volume 174 of the Advances in the Astronautical Sciences Series. *IAA/AAS SciTech Forum 2019 on Space Flight Mechanics and Space Structures and Materials*. 25–27 June 2019, Moscow, Russia. 2019:741–751. (IAA-AAS-SciTech2019-064-AAS 19-1017)

19. Ma Y.Q., Wang C.M., Ang K.K. Buckling of super ellipsoidal shells under uniform pressure. *Thin Walled Struct*. 2008;46(6):584–591. https://doi.org/10.1016/j.tws.2008.01.013

20. Mele T.V., Rippmann M., Lachauer L., et al. Geometry-based understanding of structures. *J. of the International Association for Shell and Spat. Structures.* 2012;53(174):285–296.

21. Flöry S., Pottmann H. Ruled surfaces for rationalization and design in architecture. *Proceedings ACADIA*. 2010;103–109. https://doi.org/10.52842/conf.acadia.2010.103

22. Kamil M., Dagmar S. A method for creating ruled surfaces and its modifications. *KoG*. 2002;6 (6):59–66.