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Parameterization of Maxwell — Cremona Diagram for Determining Forces in Elements of a Scissors Truss

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Abstract. An analysis of changing patterns of the values of member forces in a scissors truss, depending on the position of connections of its lower chords to the upper chords, is performed. Exploring effective truss structure designs in terms of balanced combination of maximum strength and minimum weight is a sustainable approach to a more rational use of building materials and the development of green construction. This determines the relevance of this area of research. The analysis of configurations of the truss under study was performed using the parameterized Maxwell — Cremona diagram. Such diagram is a visually informative tool in presenting the calculation results and it fully reflects the relationship between the member forces and the parameters of the structure. The research process was performed using the MS Excel spreadsheet editor. This eventually developed into a software tool for finding effective scissors truss designs, which has full potential for further improvement and development. Thus, the functionality of the tool can be easily expanded to designing scissors trusses made of various structural materials, as well as with various cross-sectional shapes of its elements. The proposed approach to the calculation of such structures can serve as a basis for parameterization of trusses with other types of web.

Keywords: construction, design, buildings, Maxwell — Cremona diagrams, truss

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Параметризация диаграммы Максвелла — Кремоны для определения усилий в элементах треугольной фермы типа «ножницы»

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Аннотация. Выполнен расчет по определению характера изменения значений усилий в элементах треугольной фермы типа «ножницы» в зависимости от положения точек примыкания ветвей её нижнего пояса к элементам верхнего пояса. Изыскание эффективных конструктивных решений ферм в контексте гармоничного сочетания максимальной прочности и минимального веса конструкции является устойчивым подходом к более рациональному использованию строительных материалов и развитию зеленого строительства. Это обуславливает актуальность данного направления исследований. Анализ вариантов конфигурации треугольной исследуемой фермы выполнен с помощью параметризованной диаграммы Максвелла — Кремоны, которая является наглядным инструментом в представлении результатов расчета и полноценно отражает зависимость усилий в элементах конструкции от ее параметров. Процесс исследований был воплощен с помощью табличного процессора MS Excel, что сложилось в программное средство для поиска эффективных конструктивных решений ферм типа «ножницы», которое в полной мере обладает потенциалом к дальнейшему совершенствованию и развитию. Функционал программы может быть расширен до возможности проектирования ферм типа «ножницы» из различных конструкционных материалов, а также для различных форм поперечного сечения ее элементов. Предлагаемый подход к расчёту таких конструкций может послужить основой для параметризации ферм с другими типами стержневой решётки.

Ключевые слова: строительство, проектирование, здания, диаграммы Максвелла — Кремоны, ферма

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1. Introduction

By the early 18-th century there was no reliable method of designing structures. For a long time, the experience of masonry and wooden construction was not generalized and fixed by any calculation methods [1]. When designing, one had to use a system of rough empirical calculations developed on the basis of centuries-old practice [2; 3].

Graphical structural analysis emerged in the second half of the 19-th century as a method for solving engineering problems containing equilibria of forces [4]. Graphical structural analysis is an intuitive and powerful design tool that allows the structural engineer to analyze and control forces in planar lattice structures — trusses [5; 6]. Trusses are widely used in a variety of engineering structures because of their relatively high strength to mass ratio. They are used both as internal support for other structures and as independent structures [7; 8]. Before the emergence of computers, graphical structural analysis was the only viable means of designing arches and trusses, the elements of which experience axial forces only. The resulting structures were often material efficient [9–11].

2. Method

With the advent of computers (around the 1980s), analytical and manual calculation methods gradually lost their relevance. Nevertheless, there are cases when during the design of a structure it was necessary to visualize the calculation results and their dependence on its parameters, in this case on the roof slope. In such situations, it is reasonable to use the Maxwell — Cremona diagram [12; 13]. The method of constructing the Maxwell — Cremona diagram allows to quickly and illustratively determine the forces in the truss members and find the relationships between the forces and the position of the connections between the lower and upper chords.

The advantages of this method include: compactness of constructing force-vectors, possibility of quickly checking against errors and assessment of the overall stress-strain state of the truss. This paper is devoted to the qualitative analysis of modeling a particular kind of vaulted (triangular) truss — the scissors truss. Usually, trusses with bottom chord elevation are installed over specific indoor facilities: living rooms, sports halls, often used for canopies [14; 15].

3. Results and Discussion

Paper [16] presents a study of the relationship between the forces in the members of the truss shown in Figure 1 and the ratio of the rise to the span of the structure. The study was based on the analysis of the Maxwell — Cremona diagram (hereinafter *M–C*) for different values of the f to L ratio.

The results showed that:

- the change in the horizontal coordinates of the points of the diagram is inversely proportional to the change in rise, i.e. the shape of the diagram changes only its length;
- its height is directly proportional to the value of the external load;
- the values of forces obtained from unit loads are valid for any span of the structure, but at the constant rise to span ratio.

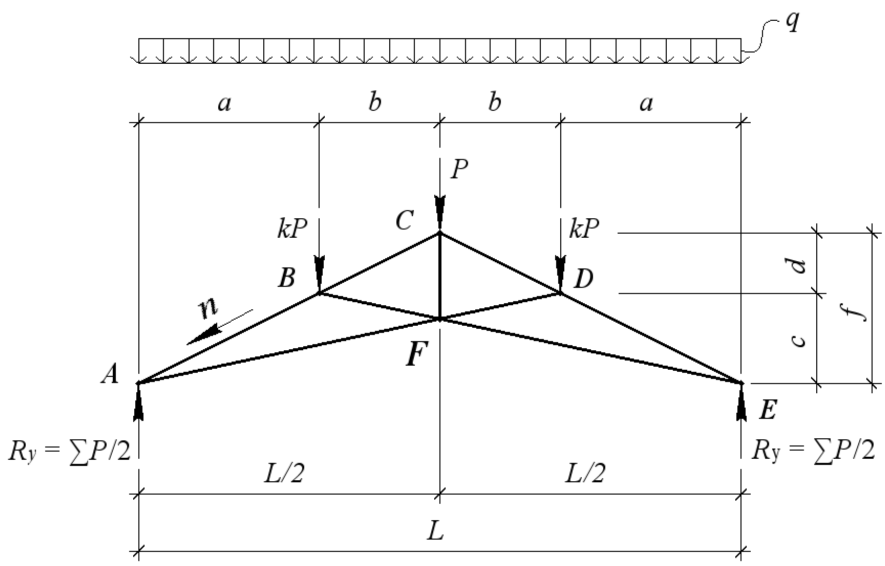


Figure 1. The studied truss model in general form
S o u r c e: made by V.A. Repin

Thus, to obtain the values of forces in the elements of a similar structure, it was sufficient to scale the original diagram according to the degree of change in the rise, and then multiply the obtained forces by the value of the load.

This paper presents the results of an additional study, which consists of determining the relationship between the force values and the position of the connections of the lower chords to the upper chords, which is controlled by the ratio of distances *a* and *b* (see Figure 1). This ratio is denoted as

$$m = \frac{a}{b}. \tag{1}$$

Assumptions of the analysis:

- loads are applied to the nodes of the upper chords of the structure in the form of concentrated forces;
- external loads act only vertically (horizontal components of the loads are expected to be considered in further studies);
- the connection of the elements is hinged, thus excluding the emergence of bending moments.

The results of the analysis of *M–C* diagrams plotted in a CAD software for different values of *m* show that they differ from each other radically (Figure 2). Thus, for obtaining forces at different ratios of *a* and *b*, scaling of the diagram is not suitable.

This raises a problem of finding an original approach to the construction of the force diagram in the elements of the investigated structure depending on parameter *m*. Such approach involves parameterization of the configuration of the scissors truss members and, as a consequence, of the force diagram graphs. An algorithm is developed on the basis of the obtained mathematical expressions. The algorithm is further implemented with the help of the *MS Excel* spreadsheet editor, which has a charting tool.

The magnitudes of external forces *P* applied at point *C*, in addition to distributed load *q*, are also determined by the width of the loaded region equal to *b*:

$$P = qb.$$

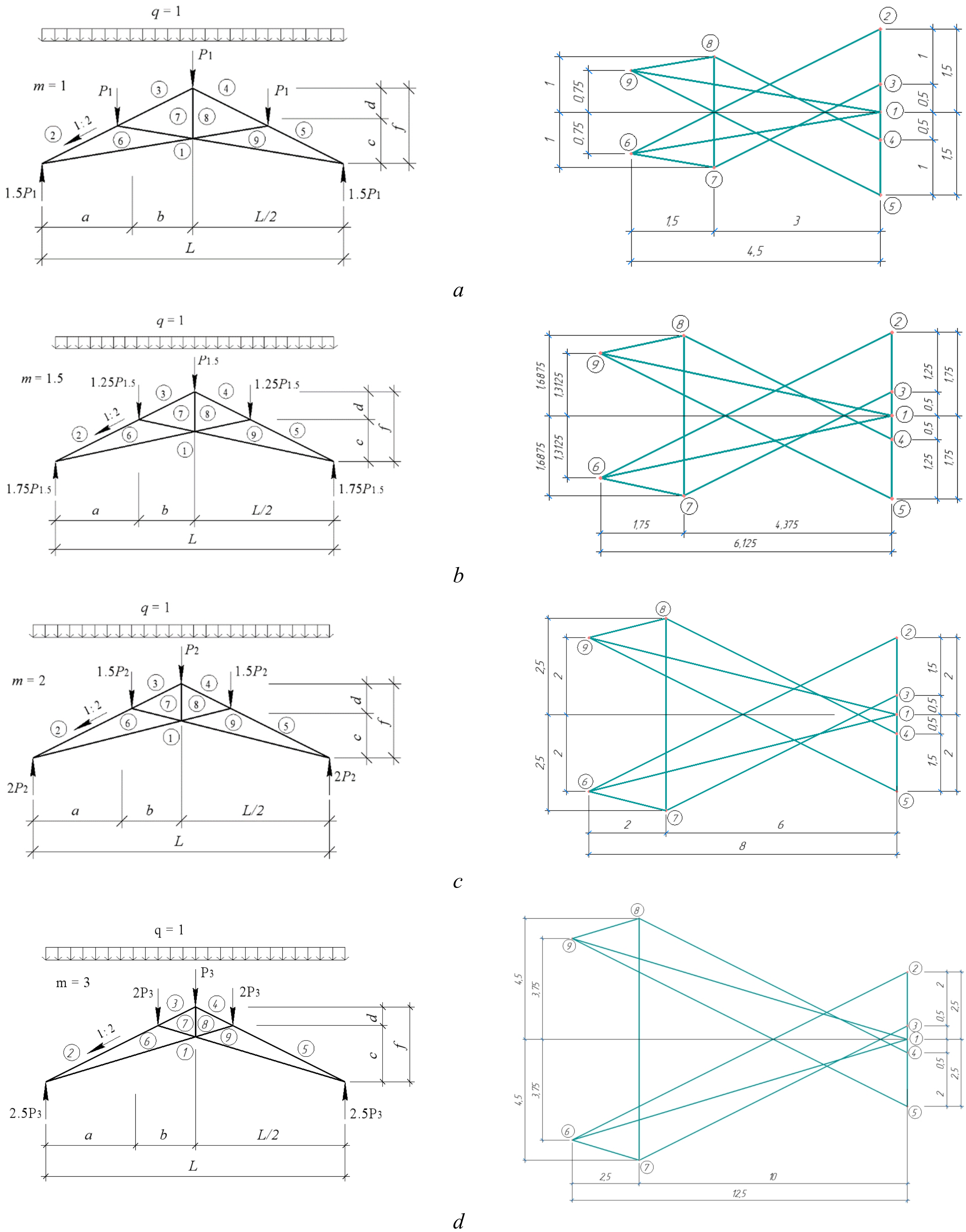


Figure 2. The changing pattern of force diagrams when varying the position of connections of the lower chords to the upper chords in the truss

S o u r c e: made by V.A. Repin

In turn, the width of the loaded region for the loads at points B and D is equal to $(a + b)/2$. Coefficient k reflects the difference in magnitude of the concentrated forces at points B and D with respect to the load at point C:

$$k = \frac{a+b}{2b}.$$

Considering relation (1), one may obtain:

$$k = \frac{m+1}{2}.$$

Support reactions R_y, therefore, are equal to

$$R_y = \frac{\sum P}{2} = \frac{P(2k+1)}{2} = \frac{P(m+2)}{2}.$$

The position of the upper chords of the truss is determined by the roof slope, which is the ratio of the rise to the span, or rather to its half, which is denoted as follows:

$$n = \frac{2f}{L}.$$

Let us assume the position of the coordinate origin at point A. Then, the following equation is valid for the configuration of element A–B–C (Figure 3):

$$y = nx.$$

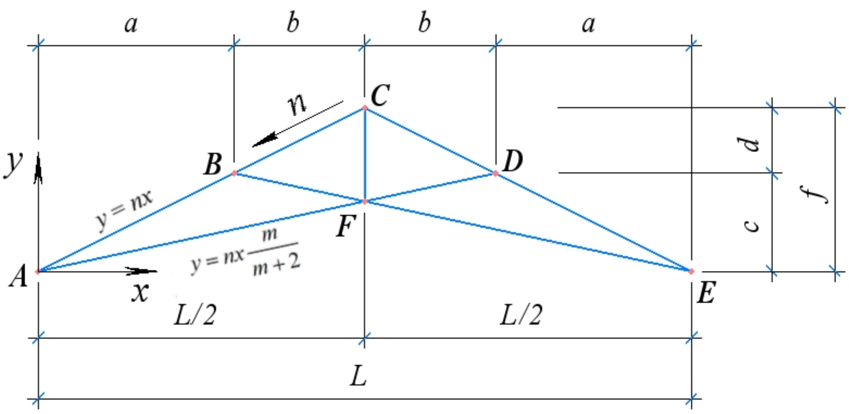


Figure 3. Parameterization of the configuration of the scissors truss elements
S o u r c e: made by V.A. Repin

Correspondingly, for the lower chord A–F–D:

$$y = \frac{cx}{a+2b}. \tag{2}$$

Now this equation needs to be expressed in terms of variable parameters n and m. Since $a + b = L/2$ and $a = bm$, then:

$$b = \frac{L}{2(m+1)},\ a = \frac{mL}{2(m+1)}.$$

Since $c + d = f$ and $c/d = m$, then

$$d = \frac{f}{m+1},\ c = \frac{mf}{m+1}.$$

Hence, expression (2) can be represented as:

$$y = nx\frac{m}{m+2}. \quad (3)$$

Parameterization of the *M–C* force diagram is performed similarly. The graphs of the diagram are constructed parallel to the truss members, and the nodal points of the diagram are located at their intersection. Point 1 is taken as the coordinate origin.

Hence, graph *2–6* is parallel to element *A–B* and its segment belongs to the line described by the following function (Figure 5):

$$y_{2-6} = nx + R_y = nx + P\frac{m+2}{2}.$$

Graph *1–6* is parallel to element *A–F* and is described by a straight line equation, effectively coinciding with expression (3):

$$y_{1-6} = nx\frac{m}{m+2}.$$

The position of point *6* is determined by the intersection of lines *1–6* and *2–6*. Thus, the *x*-coordinate value is found from the following equation:

$$nx\frac{m}{m+2} = nx + P\frac{m+2}{2},$$

which is equal to

$$x_6 = -P\frac{(m+2)^2}{4n} \ \text{or}\ x_6 = -\frac{R_y^2}{nP}.$$

Then, the *y*-coordinate value is equal to

$$y_6 = -P\frac{m(m+2)}{4} \ \text{or}\ y_6 = -\frac{mR_y}{2}.$$

Hence, the coordinates of point *6* on the force diagram are equal to

$$6\left(-\frac{R_y^2}{nP}; -\frac{mR_y}{2}\right).$$

Point *7* is located at the intersection of lines *3–7* and *6–7*. Their equations are derived by focusing on the points of their intersection with the vertical axis, as in the previous case. Thus, line *3–7* is parallel to element *A–B–C* (see Figure 3) and intersects the *y*-axis at point *3* (0; 0.5*P*). Then its equation takes the form of

$$y_{3-7} = nx + 0.5P.$$

Line *6–7* is parallel to *B–F–E*, which is symmetric to *A–F–D* (i.e. its sign is opposite), and intersects the vertical axis at the point with coordinates (0; $-m \cdot R_y$). Then the equation of this line takes the following form:

$$y_{6-7} = -nx\frac{m}{m+2} - mR_y.$$

The horizontal coordinate of point *7* is determined from the following equation:

$$nx + 0.5P = -nx\frac{m}{m+2} - mR_y,$$

which results in

$$x_7 = -P\frac{(m+1)(m+2)}{4n} \text{ or } x_7 = -\frac{kR_y}{n}.$$

Then, the vertical coordinate is equal to

$$y_7 = 0.5P - kR_y.$$

The parametrization results are presented in Figure 4.

Points *8* and *9* are located symmetrically to points *6* and *7*, so their vertical coordinate values are the same, but with the opposite sign.

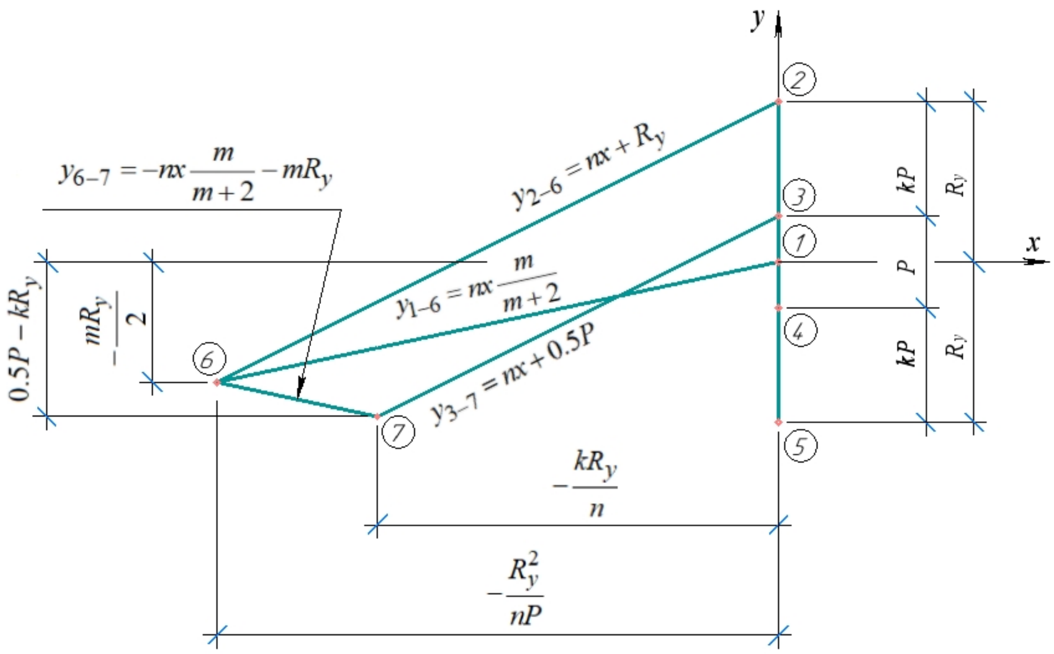
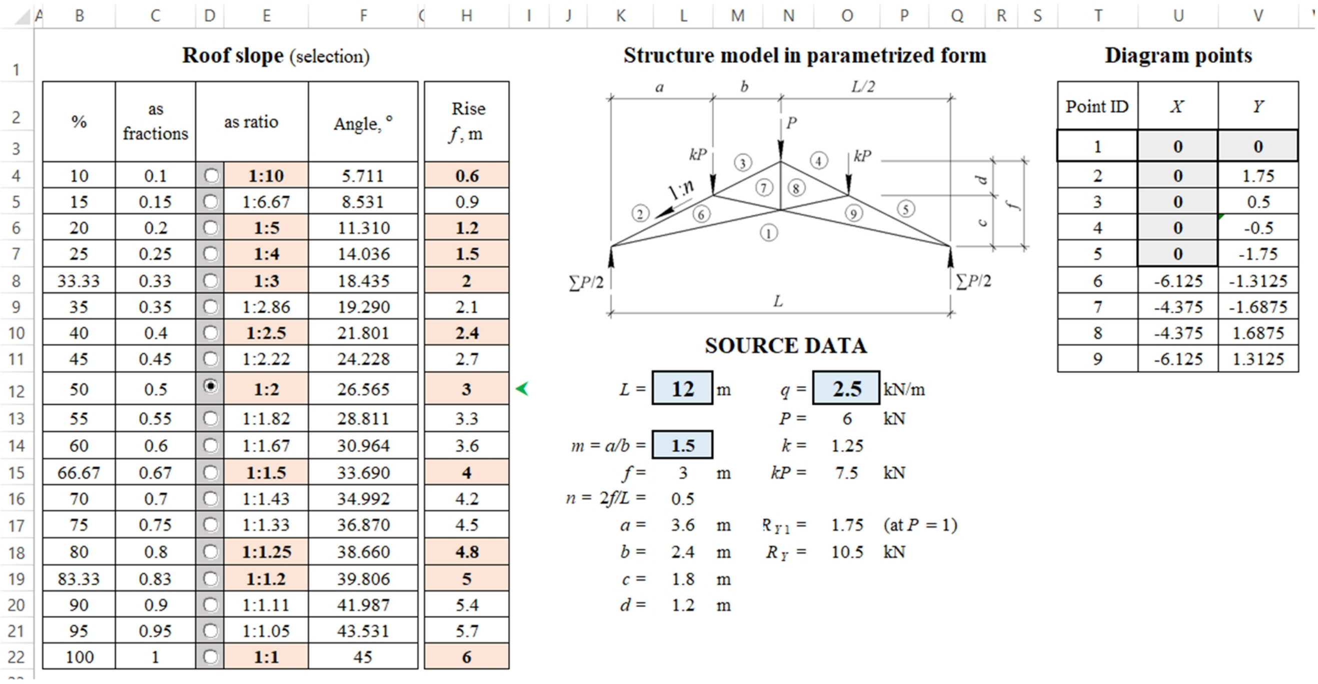


Figure 4. The process of parameterization of the Maxwell — Cremona force diagram
S o u r c e: made by V.A. Repin

The obtained results are entered in the cells of the spreadsheet in Figure 5. The computations show that the values of forces obtained from the diagram are directly proportional to the value of load *P*. Therefore, it is reasonable to use a unit load $P = 1$ in the formulas for determining the forces to ensure proper control over the calculation process. The design values of member forces of the investigated structure are calculated by multiplying the forces from the unit load by the design value of *P*, determined via the design value of load *q*.

In addition, the determination and selection of the value of parameter *n* (slope of the upper chord), can be arranged in an informative tabular form (see Figure 5).



Roof slope (selection)

%	as fractions	as ratio	Angle, °	Rise *f*, m
10	0.1	1:10	5.711	0.6
15	0.15	1:6.67	8.531	0.9
20	0.2	1:5	11.310	1.2
25	0.25	1:4	14.036	1.5
33.33	0.33	1:3	18.435	2
35	0.35	1:2.86	19.290	2.1
40	0.4	1:2.5	21.801	2.4
45	0.45	1:2.22	24.228	2.7
50	0.5	1:2	26.565	3
55	0.55	1:1.82	28.811	3.3
60	0.6	1:1.67	30.964	3.6
66.67	0.67	1:1.5	33.690	4
70	0.7	1:1.43	34.992	4.2
75	0.75	1:1.33	36.870	4.5
80	0.8	1:1.25	38.660	4.8
83.33	0.83	1:1.2	39.806	5
90	0.9	1:1.11	41.987	5.4
95	0.95	1:1.05	43.531	5.7
100	1	1:1	45	6

Diagram points

Point ID	*X*	*Y*
1	0	0
2	0	1.75
3	0	0.5
4	0	-0.5
5	0	-1.75
6	-6.125	-1.3125
7	-4.375	-1.6875
8	-4.375	1.6875
9	-6.125	1.3125

Figure 5. Calculation of the nodal points of the force diagram (for $n = ½$; $m = 1.5$)

S o u r c e: made by V.A. Repin

The calculated values of nodal coordinates fully coincide with the ones obtained graphically (see Figure 5 and Figure 2, *b*).

On the basis of these data, the force diagram can be easily constructed in the *MS Excel* environment (Figure 6).

The values of forces from the design load, $q = 2.5$ kN/m ($P = 6.0$ kN) in this case, fully coincide with the design values obtained using finite element software (Figure 7).

Thus, a software tool for analysing scissors trusses was developed. It allows to effectively and illustratively calculate the forces depending on the following parameters:

- geometric: span *L*; rise *f* (roof slope *n*); position of the connections of the lower chords to the upper chords — *m*;
- external load *q* (or *P*).

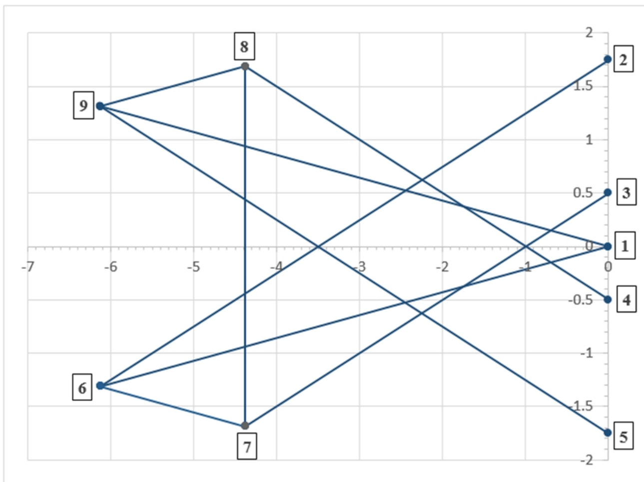
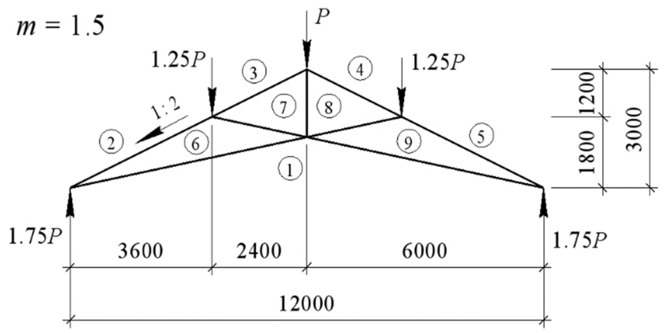


Figure 6. The *M—C* force diagram, constructed using the "Chart" function in *MS Excel*

S o u r c e: made by V.A. Repin

Parameterization of the topology of the truss web provides a rational configuration of the structure, which allows to minimize the values of forces in its elements. This, in turn, contributes to the achieving higher economic efficiency of the truss structure [17; 18].

Graphical methods do not lose their popularity in structural design owing to their computational efficiency [19; 20], which is due to the simplicity of determining forces in the truss elements, as well as the clear reflection of the relationship between their values. These results also demonstrate the possibilities and relevance of *M–C* diagram application in present-day conditions.



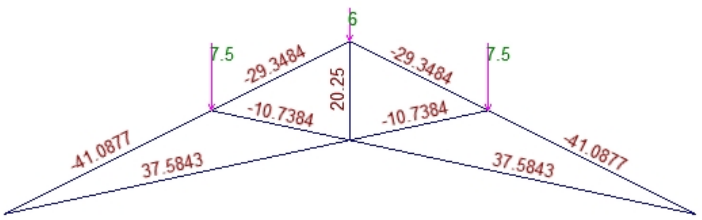
a

X	Y	Z	AA	AB	AC	AD

Diagram construction and calculation of forces

Element ID	X1 / X2	Y1 / Y2	Length	Force, kN
1 - 9	0	0	6.2640	37.584
	-6.125	1.3125		
1 - 6	0	0	6.2640	37.584
	-6.125	-1.3125		
2 - 6	0	1.75	6.8480	41.088
	-6.125	-1.3125		
3 - 7	0	0.5	4.8914	29.348
	-4.375	-1.6875		
4 - 8	0	-0.5	4.8914	29.348
	-4.375	1.6875		
5 - 8	0	-1.75	6.8480	41.088
	-6.125	1.3125		
6 - 7	-6.125	-1.3125	1.7897	10.738
	-4.375	-1.6875		
7 - 8	-4.375	-1.6875	3.3750	20.250
	-4.375	1.6875		
8 - 9	-4.375	1.6875	1.7897	10.738
	-6.125	1.3125		

P = 6 kN



c

b

Figure 7. Analysis of the results of calculating the element forces in the investigated truss in two ways:
a — model; *b* — parametrically according to the *M–C* diagram; *c* — using finite element method
S o u r c e: made by V.A. Repin

4. Conclusion

1. The Maxwell — Cremona diagram allows to quickly and illustratively determine the forces in the truss elements. It also allows to find the relationships between the forces in the elements of a scissors truss and the position of connections of the lower chords to the upper chords, governed by the ratio *a*/*b*, and the roof slope.

2. The graphical method of obtaining the values of element forces fully characterizes the behavior of the scissors truss.

3. The use of a spreadsheet editor allows to efficiently and informatively implement the construction of *M–C* diagram graphs based on the parametric algorithms presented above. It is possible to analyze the stress-strain state of the structure depending on the rise, roof slope and other factors.

4. In addition, due to the computational potential of the spreadsheet editor, it is possible to extend the functionality of this software tool to solve a number of additional tasks, for example: selection of element cross-sections, determination of the rational configuration of the structure in terms of reducing material consumption, installation weight, etc.

5. The proposed approach to the analysis of scissor trusses can serve as a basis for the parametrization of trusses with other web types as well.

References

1. Garifullin M.R., Naumova E.A., Zhuvak O.V., Barabash A.V. Surrogate modeling in construction. *Construction of unique buildings and structures*. 2016;2(41):118–132. (In Russ.) EDN VPWHRR

2. Chibrikin D.A., Lukin M.V., Lukina A.V., Tyurikova T.V., Roshchina S.I. Numerical investigation of the stress-strain state of a modified wooden beam. *Izvestia of higher educational institutions. Forest Journal.* 2022;3(387):167–178. (In Russ.) https://doi.org/10.37482/0536-1036-2022-3-167-178

3. Schulze B., Millar *C.* Graphic statics and symmetry. *International Journal of Solidsand Structures.* 2023, 112492. http://doi.org/10.1016/j.ijsolstr.2023.112492

4. Markou A.A., Ruan G. Graphic statics: projective funicular polygon. *Structures.* 2022;41:1390–1396. http:// doi.org/10.1016/j.istruc.2022.05.049

5. Lu Y., Hablicsek M., Akbarzadeh M. Algebraic 3D Graphic Statics with Edge and Vertex Constraints: A Comprehensive Approach to Extend the Solution Space for Polyhedral Form-Finding. *Computer-Aided Design.* 2024;166:103620. http:// doi.org/10.1016/j.cad.2023.103620

6. Radhi A., Iacobellis V., Behdinan K. Manipulation of topologically optimized structures using graphic statics. *Materialsand Design.* 2021;198:109286. http://doi.org/10.1016/j.matdes. 2020.109286

7. Sergeev M., Rimshin V., Lukin M., Zdralovic N. Multi-span composite beam. *IOP Conference Series: Materials Science and Engineering.* 2020;896:012058. https://doi.org/10.1088/1757-899X/896/1/012058

8. Xu Z., Cui Y., Li B. Truss Structure Optimization Design Based on FE-PSO-SQP Algorithm. In: Kountchev R., Mironov R., Nakamatsu K. (eds). *New Approaches for Multidimensional Signal Processing. NAME SP2022.* Smart Innovation, Systems and Technologies. Singapore: Springer; 2022;332:151–158. https://doi.org/10.1007/978-981-19-7842-5_14

9. Zhidkov K.E., Zverev V.V., Kapyrin N.V. Experimental field studies of wooden trusses on metal toothed plates. *Construction mechanics and structures*. 2021;4(31):90–98. (In Russ.) https://doi.org/10.36622/VSTU.2021.31.4.008

10. Larsen S.D., Sigmund O., Groen J. Optimal truss and frame design from projected homogenization-based topology optimization. *Structural and Multidisciplinary Optimization*. 2018;57(4):1461–1474. https://doi.org/10.1007/ s00158-018-1948-9

11. Lukin M.V., Chibrikin D.A., Roshchina S.I. Numerical studies of modified composite beams taking into account the physical nonlinearity of wood. *News of higher educational institutions. Construction.* 2023;5(773):5–19. (In Russ.) https://doi.org /10.32683/0536-1052-2023-773-5-5-19

12. Cai S., Zhang H., Zhang W. An integrated design approach for simultaneous shape and topology optimization of shell structures. *Computer Methods in Applied Mechanics and Engineering*. 2023;415:116218. https://doi.org/10.1016/ j.cma.2023.116218

13. Khokhlov A.V. Properties of a family of constant-velocity loading curves generated by a nonlinear Maxwell-type viscoelastic plasticity model. *Mechanical engineering and engineering education*. 2017;1(50):57–71. EDN YLIVPZ

14. Buzurukov Zh.I., Segaev I.N. Farms. *Pridneprovsky scientific bulletin*. 2019;5(5):12–15. (In Russ.) EDN DNWNVJ

15. Massafra A., Prati D., Predari G., Gulli R. Wooden truss analysis, preservation strategies, and digital documentation through parametric 3D modeling and HBIM workflow. *Sustainability.* 2020;12(12):4975. https://doi.org/ 10.3390/su12124975

16. Repin V.A., Lukina A.V., Usov A.S. Rational constructive solutions of triangular farms. *Construction mechanics of engineering structures and structures*. 2023;19(2):199–209. (In Russ.) http://doi.org/10.22363/1815-5235-202319-2-199-209

17. Marutyan A.S. Steel lattice structures made of square pipes with upper belts reinforced with channels, and their calculation. *Construction mechanics and calculation of structures*. 2015;5(262):62–69. (In Russ.) EDN UIXKZX

18. Khudyakov V.A., Pastushkov V.G. Designing farms using main voltage lines. *Transport. Transport facilities. Ecology*. 2015;1:131–147. EDN TXOYRR

19. Bolhassani M., Akbarzadeh M., Mahnia M., Taherian R. On structural behavior of a funicular concrete polyhedral frame designed by 3D graphic statics. *Structures.* 2018;(14):56–68. https://doi.org/10.1016/j.istruc.2018.02.002

20. Kholodar B.G. Determination of the stress-strain state of a farm using the Maxwell — Cremona diagram. *Bulletin of the Brest State Technical University. Construction and architecture*. 2016;1(97):39–42. (In Russ.) EDN YWUOCL

21. Shishov I.I., Lisyatnikov M.S., Roshchina S.I., Lukina A.V. Covering of a single-storey industrial building with wide beams of box-shaped cross-section of stepwise variable height. *Bulletin of the South Ural State University. Series: Construction and Architecture*. 2021;21(1):22–29. (In Russ.) https://doi.org/10.14529/build210103

22. van Sosin B., Rodin D., Sliusarenko H., Bartoň M., Elber G. The Construction of Conforming-to-shape Truss Lattice Structures via 3D Sphere Packing. *CAD Computer-Aided Design*. 2021;132:102962. https://doi.org/10.1016/ j.cad.2020.102962

Список литературы

1. *Гарифуллин М.Р., Наумова Е.А., Жувак О.В., Барабаш А.В.* Суррогатное моделирование в строительстве // Строительство уникальных зданий и сооружений. 2016. № 2(41). С. 118–132. EDN VPWHRR
2. *Чибрикин Д.А., Лукин М.В., Лукина А.В., Тюрикова Т.В., Рощина С.И.* Численное исследование напряженно-деформированного состояния модифицированной деревянной балки // Известия высших учебных заведений. Лесной журнал. 2022. № 3(387). С. 167–178. http://doi.org/10.37482/0536-1036-2022-3-167-178
3. *Schulze B., Millar C.* Graphic statics and symmetry. *International Journal of Solidsand Structures.* 2023. Vol. 283. http://doi.org/10.1016/j.ijsolstr.2023.112492
4. *Markou A.A., Ruan G.* Graphic statics: projective funicular polygon// Structures. 2022. Vol. 41. P.1390–1396. http://doi.org/10.1016/j.istruc.2022.05.049
5. *Lu Y., Hablicsek M., Akbarzadeh M.* Algebraic 3D Graphic Statics with Edge and Vertex Constraints: A Comprehensive Approach to Extend the Solution Space for Polyhedral Form-Finding // Computer-Aided Design. 2024. Vol. 166. http://doi.org/10.1016/j.cad.2023.103620
6. *Radhi A., Iacobellis V., Behdinan K.* Manipulation of topologically optimized structures using graphic statics // Materialsand Design. 2021. Vol. 198. http://doi.org/10.1016/j.matdes.2020.109286
7. *Sergeev M., Rimshin V., Lukin M., Zdralovic N.* Multi-span composite beam // IOP Conference Series: Materials Science and Engineering. 2020. Vol. 896. https://doi.org/10.1088/1757-899X/896/1/012058
8. *Xu Z., Cui Y., Li B.* Truss Structure Optimization Design Based on FE-PSO-SQP Algorithm. In: Kountchev R., Mironov R., Nakamatsu K. (eds). New Approaches for Multidimensional Signal Processing. NAME SP2022. Smart Innovation, Systems and Technologies. Singapore: Springer. 2022. Vol. 332. P. 151–158. https://doi.org/10.1007/978-981-19-7842-5_14
9. *Жидков К.Е., Зверев В.В., Капырин Н.В.* Экспериментальные натурные исследования деревянных ферм на металлических зубчатых пластинах // Строительная механика и конструкции. 2021. № 4 (31). С. 90–98. https://doi.org/10.36622/VSTU.2021.31.4.008
10. *Larsen S.D., Sigmund O., Groen J.,* Optimal truss and frame design from projected homogenization-based topology optimization // Structural and Multidisciplinary Optimization. 2018. Vol. 57(4). P. 1461–1474. https://doi.org/10.1007/s00158-018-1948-9
11. *Лукин М.В., Чибрикин Д.А., Рощина С.И.* Численные исследования модифицированных композитных балок с учетом физической нелинейности древесины // Известия высших учебных заведений. Строительство. 2023. № 5(773). С. 5–19. https://doi.org/10.32683/0536-1052-2023-773-5-5-19
12. *Cai S., Zhang H., Zhang W.* An integrated design approach for simultaneous shape and topology optimization of shell structures // Computer Methods in Applied Mechanics and Engineering. 2023. Vol. 415. https://doi.org/10.1016/j.cma.2023.116218
13. *Хохлов А.В.* Свойства семейства кривых нагружения с постоянной скоростью, порождаемых нелинейной моделью вязкоупругопластичности типа Максвелла // Машиностроение и инженерное образование. 2017. № 1 (50). С. 57–71. EDN YLIVPZ
14. *Бузуруков Ж.И., Сегаев И.Н.* Фермы // Приднепровский научный вестник. 2019. Т. 5. № 5. С. 12–15. EDN DNWNVJ
15. *Massafra A., Prati D., Predari G., Gulli R.* Wooden truss analysis, preservation strategies, and digital documentation through parametric 3D modeling and HBIM workflow // Sustainability. 2020. Vol. 12(12). https://doi.org/10.3390/su12124975
16. *Репин В.А., Лукина А.В., Усов А.С.* Рациональные конструктивные решения треугольных ферм // Строительная механика инженерных конструкций и сооружений. 2023. Т. 19. № 2. С. 199–209. http://doi.org/10.22363/1815-5235-202319-2-199-209
17. *Марутян А.С.* Стальные решетчатые конструкции из квадратных труб с верхними поясами, усиленными швеллерами, и их расчет // Строительная механика и расчет сооружений. 2015. № 5(262). С. 62–69. EDN UIXKZX
18. *Ходяков В.А., Пастушков В.Г.* Проектирование ферм с использованием линий главных напряжений // Транспорт. Транспортные сооружения. Экология. 2015. № 1. С. 131–147. EDN TXOYRR
19. *Bolhassani M., Akbarzadeh M., Mahnia M., Taherian R.* On structural behavior of a funicular concrete polyhedral frame designed by 3D graphic statics // Structures. 2018. Vol. 14. P 56–68. https://doi.org/10.1016/j.istruc.2018.02.002
20. *Холодарь Б.Г.* Определение напряженно-деформированного состояния фермы с использованием диаграммы Максвелла-Кремоны // Вестник Брестского государственного технического университета. Строительство и архитектура. 2016. № 1(97). С. 39–42. EDN YWUOCL
21. *Шишов И.И., Лисятников М.С., Рощина С.И., Лукина А.В.* Покрытие одноэтажного промышленного здания широкими балками коробчатого поперечного сечения ступенчато переменной высоты // Вестник Южно-Уральского государственного университета. Серия: Строительство и архитектура. 2021. Т. 21. № 1. С. 22–29. https://doi.org/10.14529/build210103
22. *van Sosin B., Rodin D., Sliusarenko H., Bartoй M., Elber G.* The Construction of Conforming-to-shape Truss Lattice Structures via 3D Sphere Packing // CAD Computer-Aided Design. 2021. Vol. 132. https://doi.org/10.1016/j.cad.2020.102962