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Buckling of Steel Conical Panels Reinforced with Stiffeners

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Abstract. Conical shells and their panels are important elements of building structures, but have not been studied sufficiently. This paper explores buckling of truncated steel conical panels reinforced with an orthogonal grid of stiffener plates. The panels are simply supported and are subjected to external uniformly distributed transverse load acting normal to the surface. A geometrically nonlinear mathematical model that takes into account lateral shearing is used. Two options of describing the effect of stiffener plates are considered: the refined discrete method and the method of structural anisotropy (the stiffness of the plates is "smeared"). The computational algorithm is based on the Ritz method and the method of continuing the solution using the best parameter. The algorithm is implemented using Maple analytical computing software. The values of critical buckling loads were obtained for two cases of conical panels with different stiffener options. The load-deflection curves are presented. The convergence of the methods for describing the effect of stiffeners with the increase in their number is discussed. It was found that for conical panels, when choosing a small number of unknown coefficients in the approximation, the value of the critical load may be "overshot", and it is necessary to select a larger number of unknowns compared to cylindrical panels or flat shells of double curvature.

Keywords: shells, conical panels, buckling, stiffeners, Ritz method, refined discrete method, structural anisotropy method

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Устойчивость стальных конических панелей, усиленных ребрами жесткости

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Аннотация. Конические оболочки и их панели являются важными элементами строительных конструкций, однако изучены еще недостаточно. В работе представлено исследование устойчивости стальных усеченных конических панелей, подкрепленных ортогональной сеткой ребер жесткости. Конструкции закреплены шарнирно-неподвижно и находятся под действием внешней равномерно распределенной поперечной нагрузки, действующей по нормали к поверхности. Используется геометрически нелинейная математическая модель, учитывающая поперечные сдвиги. Учет ребер жесткости рассматривается в двух вариантах: по уточненному дискретному методу и методу конструктивной анизотропии (жесткость ребер «размазывается»). Расчетный алгоритм основан на методе Ритца и методе продолжения решения по наилучшему параметру. Программная реализация выполнена в среде аналитических вычислений Maple. Для двух вариантов конических панелей получены значения критических нагрузок потери устойчивости при разных вариантах подкрепления ребрами жесткости. Показаны графики зависимостей «нагрузка — прогиб». Сделаны выводы о сходимости методов учета ребер жесткости при увеличении числа подкрепляющих элементов. Выявлено, что для конических панелей при выборе в аппроксимации малого числа неизвестных коэффициентов возможно «проскакивание» значения критической нагрузки и необходимо выбирать большее число неизвестных по сравнению с цилиндрическими панелями или пологими оболочками двоякой кривизны.

Ключевые слова: оболочки, конические панели, устойчивость, ребра жесткости, метод Ритца, уточненный дискретный метод, метод конструктивной анизотропии

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1. Introduction

The analysis of deformation of thin-walled elements in structures is essential in various technical fields, including structural engineering. Thin-walled shell structures are used in constructing hangars, petroleum tanks, industrial reservoirs, as well as large span public facilities [1–5]. In analyzing such structures, special attention has to be given to buckling [6–9] and determining the critical loads. For example, H.M. Waqas et. al. [8] model the buckling process of shells with different dimensions and thickness using the first-order shear deformation

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theory (FSDT). The program was developed in MATLAB environment. The results were compared with previous studies and FEM analysis.

Conical shells and their panels are common in building structures. However, they are more complex in terms of calculations compared to cylindrical or flat shells of double curvature. High-accuracy non-linear analysis of deformation of conical panels non-linearity is a topical problem [10–14].

S.N. Krivoshapko [1] points out that thin-walled structures are employed in construction in just about all countries, the difference generally being in the selected materials.

A. Sofiyev [12] presents an extensive review of studies on vibration and buckling of conical shells made of functionally graded materials (FGM). Therein, various design problems, for example, linear and non-linear vibrations and buckling due to different loadings and environments are discussed. The author highlights the potential of such structures in nuclear, space and marine engineering, electronics and biomedicine fields.

S.-R. Cho et. al. [14] conduct experimental and numerical investigations of the strength of steel conical shells under external hydrostatic pressure. Initial imperfections and other geometrical parameters were taken into account. The numerical analysis was performed using ABAQUS software.

Papers [12; 14–19] are devoted to bucking of conical panels. Article [15] contains a buckling analysis of truncated conical shells subjected to axial compression and uniform external pressure. The shells are reinforced by orthogonal stiffeners. The obtained equations are solved using the Galerkin method. The effect of material properties, dimensional parameters and stiffener plates on the bucking behavior of the shell is discussed.

A.K. Gupta et. al. [17] investigate progressive failure of multilayer conical panels under compression taking into account geometric non-linearity and damage propagation in the material. Non-linear equations are solved using the Newton–Raphson iterative method.

Using stiffener plates substantially increases the performance characteristics of shells. Such structures were studied, for example, in papers [20–23].

A.A. Dudchenko and V.N. Sergeev [24] present a mathematical model of the deformation of a stiffened conical shell. Nonlinear equilibrium equations for a shell stiffened by a discrete set of frames are derived using vector analysis. The shell and frame constitutive stiffness relationships are worked out.

The purpose of this study is to employ the previously developed mathematical models, analysis algorithm and the refined discrete method of accounting for the effect of stiffeners to evaluate buckling of conical shells and discuss their practical potential.

2. Methods

Let us consider a non-closed conical panel with segment angle b (Figure 1). The panel is simply supported along the contour and is subjected to uniform load q_0, which acts normal to the surface. Besides that, dead load is also taken into account. Thus, the total load along each axis of the local coordinate system is given by $q = q_0 + q_{sv}$, $P_x = P_{xsv}$, $P_y = P_{ysv}$.

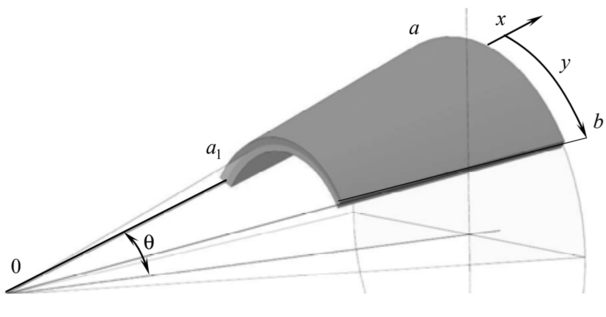

Figure 1. Conical panel in the local coordinate system
Source: made by the authors

Let us use the geometrically non-linear mathematical model obtained in paper [25]. The model is based on the Timoshenko–Reissner hypotheses, considers lateral shearing and material orthotropy and allows to analyze shells of various shapes. Moreover, the model accounts for stiffening elements according to the refined discrete method (the contact of the shell and the stiffener occurs “along the strip”) and the structural anisotropy method (smearing the stiffness of the stiffeners across the entire structure). Each of the many relationships of this model are not presented here, instead, let us only show the non-linear geometric relations and the full deformational potential energy functional:

$$E_s = E_s^0 + E_p^R, \tag{1}$$

where E_s^0 is the part of the functional, which corresponds to the shell and the external force work:

$$E_s^0 = \frac{1}{2}\int_{a_1}^{a}\int_{0}^{b}\Big[N_x^0\varepsilon_x + N_y^0\varepsilon_y + \frac{1}{2}\left(N_{xy}^0 + N_{yx}^0\right)\gamma_{xy} + M_x^0\chi_1 + M_y^0\chi_2 + \left(M_{xy}^0 + M_{yx}^0\right)\chi_{12} +$$

$$+Q_x^0\left(\Psi_x - \theta_1\right) + Q_y^0\left(\Psi_y - \theta_2\right) - 2\left(P_xU + P_yV + qW\right)\Big]x\sin\theta dxdy, \tag{2}$$

and E_p^R is the part of the functional, which corresponds to the stiffeners:

$$E_p^R = \frac{1}{2}\int_{a_1}^{a}\int_{0}^{b}\Big[N_x^R\varepsilon_x + N_y^R\varepsilon_y + \frac{1}{2}\left(N_{xy}^R + N_{yx}^R\right)\gamma_{xy} + M_x^R\chi_1 + M_y^R\chi_2 + \left(M_{xy}^R + M_{yx}^R\right)\chi_{12} +$$

$$+Q_x^R\left(\Psi_x - \theta_1\right) + Q_y^R\left(\Psi_y - \theta_2\right)\Big]x\sin\theta dxdy. \tag{3}$$

The non-linear geometric relationships for conical panels have the following form:

$$\begin{aligned}
\varepsilon_x &= \frac{\partial U}{\partial x} + \frac{1}{2}\theta_1{}^2,\\
\varepsilon_y &= \frac{1}{x\sin\theta}\frac{\partial V}{\partial y} + \frac{U}{x} - \frac{\text{ctg}\theta}{x}W + \frac{1}{2}\theta_2{}^2,\\
\gamma_{xy} &= \frac{\partial V}{\partial x} + \frac{1}{x\sin\theta}\frac{\partial U}{\partial y} - \frac{V}{x} + \theta_1\theta_2,
\end{aligned} \tag{4}$$

$$\theta_1 = -\left(\frac{\partial W}{\partial x}\right),\quad \theta_2 = -\left(\frac{1}{x\sin\theta}\frac{\partial W}{\partial y} + \frac{ctg\theta}{x}V\right),$$

$$\chi_1 = \frac{\partial\Psi_x}{\partial x},\quad \chi_2 = \frac{1}{x\sin\theta}\frac{\partial\Psi_y}{\partial y} + \frac{\Psi_x}{x},\quad \chi_{12} = \frac{1}{2}\left[\frac{\partial\Psi_y}{\partial x} + \frac{1}{x\sin\theta}\frac{\partial\Psi_x}{\partial y} - \frac{\Psi_y}{x}\right],$$

where $\varepsilon_x, \varepsilon_y$ are the strains along axes x, y of the middle surface; γ_{xy} is the shearing strain in plane $x\text{O}y$.

Functional (1) represents the difference of the potential energy of the system and the work of external loads P_x, P_y, q, and it is a function of five unknown displacement functions U, V, W, Ψ_x, Ψ_y.

According to the Lagrange variational principle, the equilibrium is reached when the energy is at minimum. For minimizing functional (1), let us use the Ritz method. For this purpose, the unknown functions are represented as a sum of products of the unknown numeric parameters and the basis functions:

$$U(x,y) = \sum_{k=1}^{\sqrt{N}}\sum_{l=1}^{\sqrt{N}} U_{kl}X_1^kY_1^l,\quad V(x,y) = \sum_{k=1}^{\sqrt{N}}\sum_{l=1}^{\sqrt{N}} V_{kl}X_2^kY_2^l,\quad W(x,y) = \sum_{k=1}^{\sqrt{N}}\sum_{l=1}^{\sqrt{N}} W_{kl}X_3^kY_3^l, \tag{5}$$

$$\Psi_x(x,y) = \sum_{k=1}^{\sqrt{N}}\sum_{l=1}^{\sqrt{N}} \Psi_{xkl}X_4^kY_4^l,\quad \Psi_y(x,y) = \sum_{k=1}^{\sqrt{N}}\sum_{l=1}^{\sqrt{N}} \Psi_{ykl}X_5^kY_5^l,$$

where $U_{kl}, V_{kl}, W_{kl}, \Psi_{xkl}, \Psi_{ykl}$ are the unknown coefficients; $X_1^k - X_5^k$, $Y_1^l - Y_5^l$ are the known approximating functions of x and y, which satisfy boundary conditions at the contour of the shell; $N = 1, 4, 9, 16, ...$ is the number of decomposition terms.

With greater number of terms N, the solution accuracy increases, but the computational cost also rises.

By substituting expression (5) into functional (1), the functional is converted into a function, which has $5N$ unknown parameters. Thus, we arrive at a function minimization problem: the derivatives with respect to all the unknown parameters need to be set to zero. As a result, we obtain a system of non-linear algebraic equations, the solution of which requires special techniques. The method of continuing the solution using the best parameter (the arc length of equilibrium states in multidimensional space) is recommended. A detailed description of this method for solving such class of problems is presented in [26].

The mathematical model and the algorithm are implemented using the Maple analytical computation software. All the relations in the program are defined with dimensionless parameters when performing the calculations.

3. Results and Discussion

Let us first consider truncated conical panels without stiffeners. Their geometric parameters, along with surface area S^0 and volume V^0 are given in Table 1.

Table 1

Characteristics of the conical panels under consideration

Case	Input data					Parameter	
	***h*, m**	***a*, m**	***b*, rad**	**a_1, m**	**θ, rad**	**S^0, m²**	**V^0, m³**
1	0.02	8	2.574	4	0.2618	16	0.32
2	0.01	25	π	5	0.78	663	6.63

Table 2 presents the critical buckling load for cases 1 and 2.

The panel material is S345 steel ($E = 2.1 \cdot 10^5$ MPa, $\mu = 0.3$, $\rho = 7800$ kg/m³).

Table 2

Critical buckling loads for steel conical panels (without stiffeners)

Case	Material	*N*	Critical buckling load q_{cr}, MPa
1	Steel S345	9	+
		16	5.9667
2	Steel S345	4	+
		9	+
		16	1.0561
		25	0.2721

The convergence analysis of the Ritz method was performed by calculating the case 2 panel at different N. It showed that at small N the critical load cannot be traced, and it is necessary to retain at least 16 unknown variables for each unknown function.

Figure 2 demonstrates the load-deflection curves for the case 2 conical panel. As seen from the graph, the number of terms in the Ritz method has a significant effect on determining the critical load value. Moreover, it sometimes allows to find a point of local buckling before the global buckling.

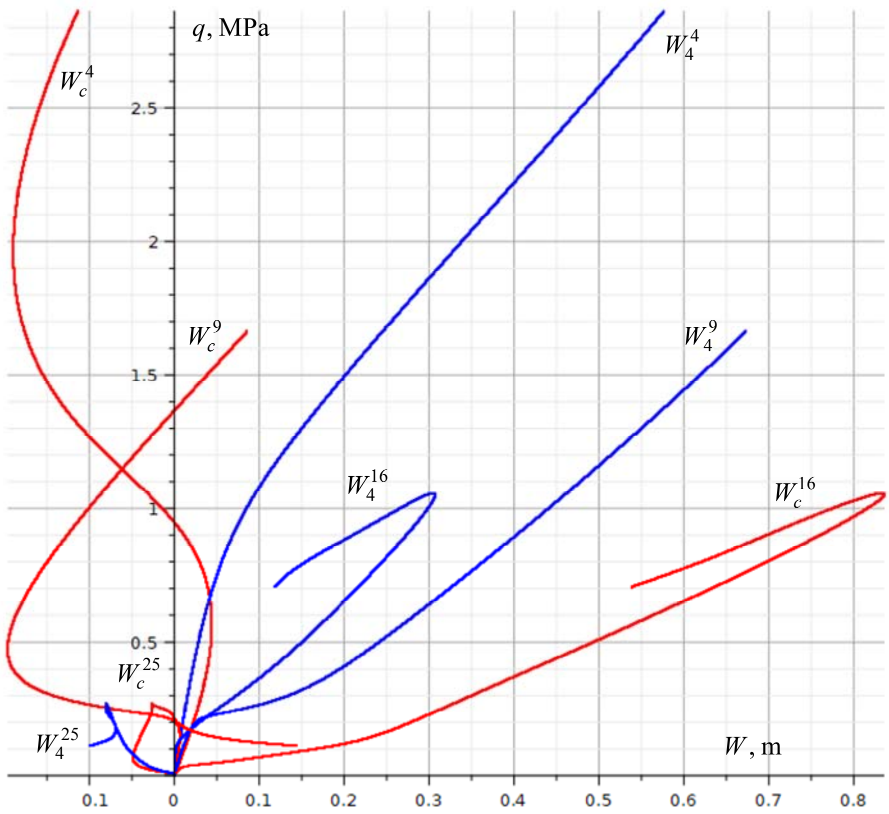

Figure 2. Load-deflection curves for steel conical panel (case 2)
S o u r c e : made by the authors

Let us now consider conical panels with stiffeners. The orthogonal stiffener plate grid is located on the inside of the shell. The height and width of the plates are selected as $h^i = h^j = 3h,\ r_i = r_j = 2h$ respectively. The number of stiffeners is the same in both directions, increasing it by a factor of 2 or 4 for each new grid case.

Table 3 shows the critical buckling loads for the considered steel conical panels. The distance between the stiffeners is denoted as x_r. The difference in the critical load values for the structural anisotropy method and the refined discrete method is given in percentage. These values are represented graphically in Figure 3.

Table 3

Values of critical buckling loads q_{cr} for conical panels

Method	***N***	q_{cr}, MPa							
		0×0	**2×2**	**4×4**	**6×6**	**8×8**	**12×12**	**16×16**	**20×20**
		Case 1							
x_r, m	–	–	1.96	0.98	0.65	0.49	0.32	0.24	0.19
Refined Discrete Method	9	+	+	15.43	17.01	18.59	21.50	24.10	26.47
	16	5.96	8.7915	12.84	15.16	16.88	19.79	22.23	24.40
Structural Anisotropy Method	9	+	+	15.42	17.02	18.60	21.51	24.11	26.48
	16	5.96	9.78	13.17	15.16	16.90	19.80	22.24	24.41
$\lvert\Delta\rvert$, %	9	+	+	0.09	0.02	0.06	0.05	0.03	0.03
	16	+	11.3	2.50	0	0.14	0.04	0.03	0.03

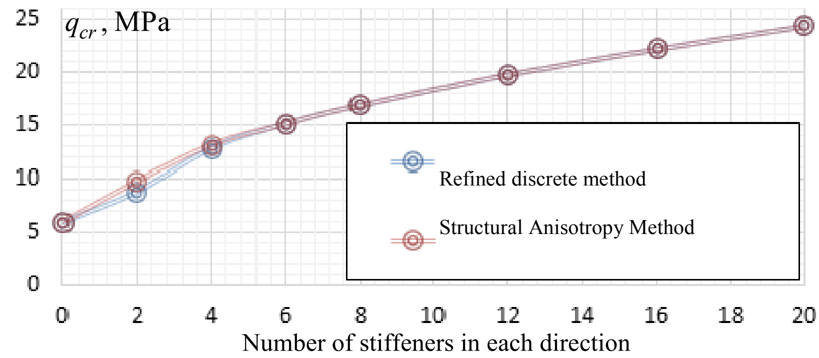

Figure 3. Comparison of methods for taking into account stiffeners for case 1 conical panel
Source: made by the authors

Figure 4 shows the load q — deflection W relationship at different number of stiffeners for the case 1 panel. The curves for both the structural anisotropy method and the refined discrete method are given.

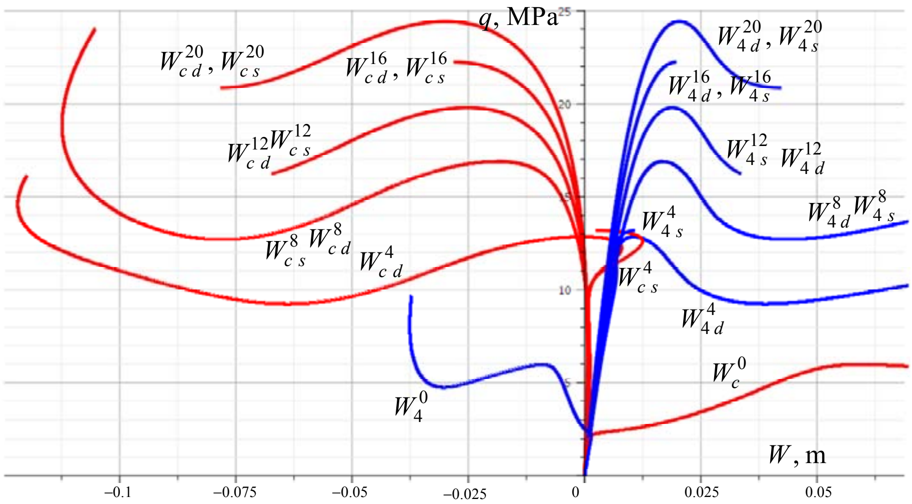

Figure 4. Load q — deflection W relationship at different numbers of stiffeners for the case 1 conical panel (N = 16)
Source: made by the authors

4. Conclusion

The object of this study is truncated conical panels reinforced with stiffeners.

The analysis methodology is based on the Ritz method, the method of continuing the solution using the best parameter, and the refined discrete method proposed earlier.

In the study, the mathematical model and analysis algorithm were adapted for a new class of problems. A corresponding program was implemented and computational experiments were carried out. As a result, the following conclusions can be drawn:

1. The presented approach can be applied to analyze conical plates with stiffeners and allows to study their buckling.

2. It can be seen from the obtained data for the case 1 panel, that the structural anisotropy method converges with the refined discrete method later than for the cylindrical panels studied previously by the authors. Possible

reasons may be that the conical panel has a more complex geometrical shape and is non-symmetrical, which requires to account for non-symmetrical terms in the approximation.

3. Increasing the number of terms in the approximation to $N = 25$ leads to substantially greater computational cost (with the refined discrete method). However, it is reasonable to perform such analysis with the structural anisotropy method.

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