2023. 19(4). 362–371

СТРОИТЕЛЬНАЯ МЕХАНИКА ИНЖЕНЕРНЫХ КОНСТРУКЦИЙ И СООРУЖЕНИЙ

STRUCTURAL MECHANICS OF ENGINEERING CONSTRUCTIONS AND BUILDINGS

ISSN 1815-5235 (Print), 2587-8700 (Online)

HTTP://JOURNALS.RUDN.RU/STRUCTURAL-MECHANICS

ДИНАМИКА КОНСТРУКЦИЙ И СООРУЖЕНИЙ
DYNAMICS OF STRUCTURES AND BUILDINGS

DOI: 10.22363/1815-5235-2023-19-4-362-371
EDN: WCETZI
UDC 624.04:531.391.3

RESEARCH ARTICLE / НАУЧНАЯ СТАТЬЯ

The formula for the first natural frequency and the frequency spectrum of a spatial regular truss

Mikhail N. Kirsanov

National Research University “MPEI”, *Moscow, Russian Federation*
c216@ya

Article history
Received: May 16, 2023
Revised: August 14, 2023
Accepted: August 20, 2023

Acknowledgements
This work was financially supported by RSF 22-21-00473.

For citation
Kirsanov M.N. The formula for the first natural frequency and the frequency spectrum of a spatial regular truss. *Structural Mechanics of Engineering Constructions and Buildings*.2023;19(4):362–371. http://doi.org/10.22363/1815-5235-2023-19-4-362-371

Abstract. A scheme of a statically determinate spatial truss is proposed. The gable cover of the structure is formed by isosceles rod triangles with supports in the form of racks on the sides. A formula is derived for the lower boundary of the structure’s first natural frequency under the assumption that its mass is concentrated in the nodes. To calculate the stiffness of the truss according to the Maxwell — Mohr formula, the forces in the rods are found by cutting out the nodes in an analytical form. The lower limit of the fundamental frequency is calculated using the Dunkerley partial frequency method. A series of solutions obtained for trusses with a different number of panels is generalized to an arbitrary order of a regular truss by induction using Maple symbolic mathematics operators. Comparison of the analytical solution with the numerical value of the first frequency of the spectrum shows good agreement between the results. The spectra of a series of regular trusses of various orders are analyzed. Two spectral constants of the problem are found, one of which is the highest frequency of truss vibrations, which does not depend on their order.

Keywords: spatial truss, Dunkerley method, fundamental frequency, analytical solution, natural oscillations, regular construction, spectrum, spectral constant, induction method, Maxwell — Mohr formula

Mikhail N. Kirsanov, Doctor of Physical and Mathematical Sciences, Professor of the Department of Robotics, Mechatronics, Dynamics and Strength of Machines, National Research University “MPEI”, Moscow, Russian Federation; ORCID: 0000-0002-8588-3871; E-mail: c216@ya

Формула для первой частоты собственных колебаний и спектр частот пространственной регулярной фермы

М.Н. Кирсанов

Национальный исследовательский университет «МЭИ», *Москва, Российская Федерация*
✉ c216@ya

История статьи
Поступила в редакцию: 16 мая 2023 г.
Доработана: 14 августа 2023 г.
Принята к публикации: 20 августа 2023 г.

Благодарности
Работа выполнена при финансовой поддержке РНФ 22-21-00473.

Для цитирования
Kirsanov M.N. The formula for the first natural frequency and the frequency spectrum of a spatial regular truss // Строительная механика инженерных конструкций и сооружений. 2023. Т. 19. № 4. С. 362–371. http://doi.org/10.22363/1815-5235-2023-19-4-362-371

Аннотация. Предложена схема статически определимой пространственной фермы. Двускатное покрытие конструкции образовано равнобедренными стержневыми треугольниками с опорами в виде стоек по боковым сторонам. Выводится формула для нижней границы первой собственной частоты сооружения в предположении, что его масса сконцентрирована в узлах. Для расчета жесткости фермы по формуле Максвелла — Мора усилия в стержнях находятся методом вырезания узлов в аналитической форме. Нижняя граница основной частоты рассчитывается методом парциальных частот Донкерлея. Серия решений, полученных для ферм с различным числом панелей, обобщается на произвольный порядок регулярной фермы методом индукции с привлечением операторов символьной математики Maple. Сравнение аналитического решения с численным значением первой частоты спектра показывает хорошее совпадение результатов. Анализируются спектры серии регулярных ферм различного порядка. Обнаружены две спектральные константы задачи, одна из которых — высшая частота колебаний ферм, не зависящая от их порядка.

Ключевые слова: пространственная ферма, метод Донкерлея, основная частота, аналитическое решение, собственные колебания, регулярная конструкция, спектр, спектральная константа, метод индукции, формула Максвелла — Мора

1. Introduction

The calculation of natural frequencies of structures in practice is carried out by the finite element method [1–3]. In this way, it is possible to calculate the entire spectrum of natural frequencies of rather complex, including statically indeterminate, structures, taking into account various types of fastening, material inhomogeneities, errors in mounting and manufacturing of structural elements, etc. For simple statically determinate constructions, analytical solutions for the lower and upper bounds of the first frequency are also possible. Such solutions are of particular value for regular constructions with a periodic structure. This is achieved by using the induction method [4–6]. Solutions to deformation problems for planar regular trusses with an arbitrary number of panels using the Maple computer mathematics system were obtained in [7; 8]. Formulas for deflections of some spatial rod systems are derived in [9; 10]. For the first time, the question of the existence of schemes of bar statically determinate structures was raised by Hutchinson R.G. and Fleck N.A. [11; 12], Zok F.W., Latture R.M., and Begley M.R. [13]. The optimization and classification of regular trusses was carried out by Kaveh A. [14; 15]. Analytical methods for calculating elements of building structures using the Maple system are considered in [16; 17]. The handbook [18] contains diagrams of various planar regular trusses and formulas for calculating their deflections, displacements of movable supports, and forces in characteristic rods. An analytical calculation of the deflection of a planar externally statically indeterminate truss with an arbitrary number of panels was performed using the computer mathematics system in [19]. The formula for the dependence of the deflection of a planar truss on the number of panels was derived by induction in [20]. In [21], a lower bound for the first natural oscillation frequency of a flat truss was obtained using the Dunkerley method. The frequency spectrum of a family of regular trusses is also numerically analyzed here. A simplified Dunkerley method for estimating the first natural frequency of a planar truss was proposed in [22]. When simplifying the desired calculation formula for the first frequency, it is proposed here to calculate the sum of the members of the sequence by the average value of its element. The analytical dependence of the deflection of the spatial console of a triangular profile on

Кирсанов Михаил Николаевич, доктор физико-математических наук, профессор кафедры робототехники, мехатроники, динамики и прочности машин, Национальный исследовательский университет «МЭИ», Москва, Российская Федерация; ORCID: 0000-0002-8588-3871; E-mail: c216@ya

the number of panels was derived in [23]. The formula for the deflection of a cantilever truss with a cruciform lattice, depending on the redistribution of the cross-sectional areas of the rods and the number of panels, was obtained by induction in [24]. In [25], some exact solutions were found for the problems of deflection of arch-type planar trusses.

The number of schemes of rod statically determinate regular constructions that are attractive from the point of view of the possibility of obtaining analytical solutions is very limited. In [11], the problem of finding such constructions was even called “hunting”. There are especially few schemes of regular statically determinate spatial trusses. In this paper, we propose a scheme of a spatial truss and derive a formula for the lower limit of its first natural frequency.

2. Construction

The truss consists of *n* panels of length 2*a* and height *h* (Fig. 1) of six rods each. The sides of the panel are isosceles rod triangles connected at the bottom by a horizontal rod of length 2*b*. The side supports of the structure are racks with a height *d* on one side and buildings with additional horizontal connections on the other. The construction is asymmetrical. In hinge *A*, it is fixed on a spherical support.

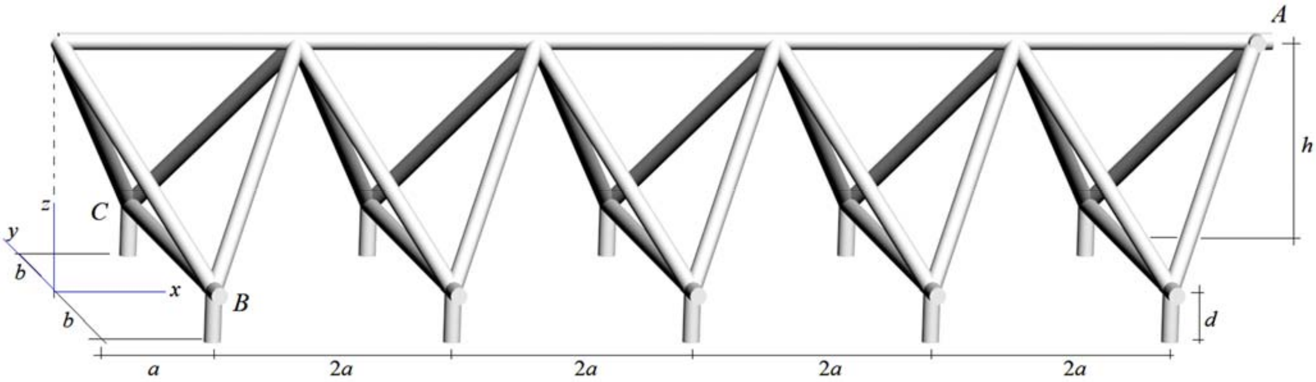

Figure 1. Truss scheme, *n* = 5

A truss of *n* panels $\eta = 9n + 3$ contains rods, of which *n* rods of length 2*a* in the upper chord, 4*n* inclined side rods of length $c = \sqrt{a^2 + b^2 + h^2}$, 2*n* support posts of height *d*, *n* horizontal external support braces, *n* horizontal braces of length 2*b*, and three rods simulating the spherical hinge *A* in upper belt. The number of internal nodes endowed with masses is equal to *K* = 3*n*+1. The inertial properties of the structure are modeled by concentrated masses in the nodes, oscillating along the vertical *z* axis.

3. Methods

To calculate the forces in the elements, the coordinates of the truss hinges are entered into the Maple system program:

$$x_i = x_{i+n} = a(2i-1),\ y_i = -y_{i+n} = -b,\ z_i = z_{i+n} = d,\ i = 1,..,n.$$

The coordinates of the hinges of the supports on the base:

$$\begin{aligned} &x_{i+3n+1} = x_{i+4n+1} = x_{i+5n+1} = x_i, \\ &y_{i+3n+1} = -y_{i+4n+1} = -b,\ z_{i+3n+1} = z_{i+4n+1} = 0, \\ &y_{i+5n+1} = -s,\ z_{i+5n+1} = d. \end{aligned}$$

Here *s* is the length of the lateral horizontal support links. This value will not be included in the calculation, since for the selected type of mass oscillations in the nodes (vertically), the force in these rods will be zero. The structure of the lattice, determined by the order of connection of the rods into nodes (hinges), is programmed by lists of the numbers of the ends of the rods:

$$N_i = [i, i+2n],\ N_{i+n} = [i, i+2n+1],$$
$$N_{i+2n} = [i+n, i+2n],\ N_{i+3n} = [i+n, i+2n+1],$$
$$N_{i+4n} = [i+2n, i+2n+1], N_{i+5n} = [i, i+n],$$
$$N_{i+6n} = [i, i+3n+1],\ N_{i+7n} = [i+n, i+4n+1],$$
$$N_{i+8n} = [i, i+5n+1],\ i = 1,..,n.$$

The numbering order of the nodes and the choice of the beginning of the bar and its end does not affect either the force or its sign.

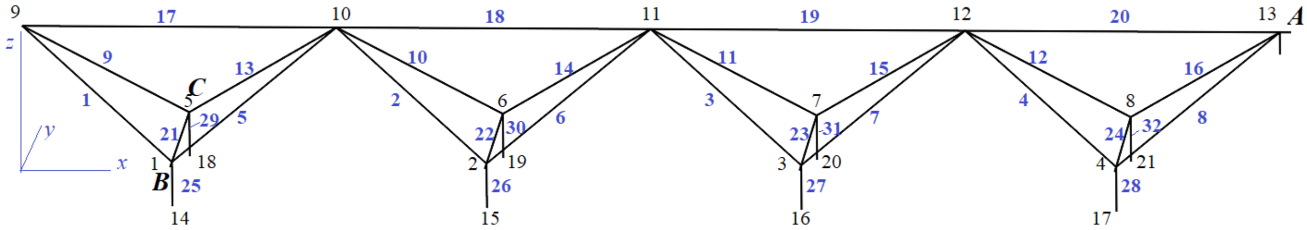

Figure 2. Numbering of knots and elements, $n = 4$

Based on the data on the coordinates and structure of the lattice, a matrix of a system of linear equations is compiled to determine the forces in the rods by cutting out nodes.

The system of linear equations for the equilibrium of nodes in projections onto the coordinate axes *x, y, z* is written in matrix form $\mathbf{GS} = \mathbf{B}$. Here $\mathbf{G}$ — is the matrix of direction cosines in the projection equations, $\mathbf{S}$ — is the force vector in the rods, including the support reactions, $\mathbf{B}$ is the load vector. The projections of the conditional vectors of the rods look like $l_{x,i} = x_{N_{i,1}} - x_{N_{i,2}}$, $l_{y,i} = y_{N_{i,1}} - y_{N_{i,2}}$, $l_{z,i} = z_{N_{i,1}} - z_{N_{i,2}}$. Since the forces are applied to one end of the rod and the other in opposite directions, the direction cosines have different signs:

$$G_{3N_{i,1}-2,i} = l_{x,i} / l_i,\ G_{3N_{i,1}-1,i} = l_{y,i} / l_i,\ \ G_{3N_{i,1},i} = l_{z,i} / l_i,$$
$$G_{3N_{i,2}-2,i} = -l_{x,i} / l_i, G_{3N_{i,2}-1,i} = -l_{y,i} / l_i,\ G_{3N_{i,2},i} = -l_{z,i} / l_i.$$

where $l_i = \sqrt{l_{x,i}^2 + l_{y,i}^2 + l_{z,i}^2}$ — is the length of the rod. Rows of the matrix $\mathbf{G}$ with numbers $3j - 2$, $j = 1,..,K$ contain the coefficients of the projection equation on the horizontal *x* axis, rows with numbers $3j - 1$ correspond to the projection equation on the *y* axis, and those with numbers $3j$ correspond to the projection equation on the vertical *z* axis.

The solution of the system of equations — the forces in the elements, is searched for in symbolic form using the Maple system.

Calculation of the natural vibration frequencies of the structure is carried out according to a simplified, but widespread truss model, in which the mass is evenly distributed over all internal nodes. If we assume only vertical motions of masses along the *z* axis, then the number of degrees of freedom of the considered structure is equal to *K*. In an analytical form for such a system, one can obtain a lower estimate of the first frequency using the Donkerley method. The Donkerley formula [26; 27] for the lower frequency limit has the form

$$\omega_D^{-2} = \sum_{p=1}^{K} \omega_p^{-2}. \tag{1}$$

where ω_p — are partial frequencies. Here, in fact, the problem of the eigenvalues of a matrix is replaced by the calculation of its trace. The equation of vertical oscillations of a separate mass *m*, has the form

$$m\ddot{z}_p + D_p z_p = 0,\ p = 1,..., K, \tag{2}$$

where D_p — is stiffness, the reciprocal of compliance $\delta_p = 1/D_p$. Compliance (linear displacement along the *z*-axis) is determined by the Maxwell–Mohr formula. Assuming that the stiffnesses of all rods *EF* are the same, we have the expression:

$$\delta_p = 1/D_p = \sum_{j=1}^{\eta} \left(S_j^{(p)}\right)^2 l_j / (EF), \tag{3}$$

where $S_j^{(p)}$ — is the force in the rod with number *j* from the action of a vertical unit force applied to the node *p*, in which the mass is located. The value of the stiffness coefficient and the partial frequency are affected by the location of the mass. For harmonic oscillations: $z_p = U_p \sin(\omega t + \varphi)$, formula $\omega_p = \sqrt{D_p / m}$ follows from (2). Substitution of this expression in (1) gives a formula for calculating the partial frequency:

$$\omega_D^{-2} = m \sum_{p=1}^{K} \delta_p = m\Delta(n). \tag{4}$$

4. Results and discussion

4.1. The Dunkerley's method. Calculations of the frequencies of trusses with a different number of panels show that the coefficient $\Delta(n)$ in (4) has the form

$$\Delta(n) = \frac{C_1 a^3 + C_2 b^3 + C_3 c^3 + C_4 h^2 d}{EFh^2}. \tag{5}$$

Only the coefficients in the numerator depend on the order of the truss *n* in this expression. Sequential calculation of trusses with a different number of panels gives the following expressions:

$$\Delta(1) = (a^3 + 2b^3 + c^3 + 6h^2 d) / (EFh^2),$$
$$\Delta(2) = (6a^3 + 6b^3 + 3c^3 + 13h^2 d) / (EFh^2),$$
$$\Delta(3) = (12a^3 + 12b^3 + 6c^3 + 22h^2 d) / (EFh^2),$$
$$\Delta(4) = (20a^3 + 20b^3 + 10c^3 + 33h^2 d) / (EFh^2), \ldots$$

To identify patterns in the formation of coefficients in these expressions, the sequence should be extended to eight. In this case, the operators of the Maple system can derive formulas for the common members: $C_1 = C_2 = n(n+1),\ C_3 = n(n+1)/2,\ C_4 = n^2 + 4n + 1$.

Thus, the dependence of the lower limit of the first frequency on the number of panels in the truss has the form:

$$\omega_D = h\sqrt{\frac{EF}{m(n(1+n)(a^3 + b^3 + c^3/2) + (n^2 + 4n + 1)dh^2)}}. \tag{6}$$

The solution can be checked against a numerical one in the Maple system using the Eigenvalues operator from the Linear Algebra package of linear algebra. In this case, the same program is used, according to which formula (6) was derived.

For example, the following parameters of a steel structure with masses $m = 200$ kg at the nodes were chosen: modulus of elasticity $E = 2.1 \cdot 10^5$ MPa, cross-sectional area of the rods 9cm^2, dimensions $a =$ 3m, $h =$ 4m, $b =$ 8m, $d =$ 0.5m. Figure 3 shows the dependence curves of the first frequency on the number of panels, obtained numerically and analytically by formula (6).

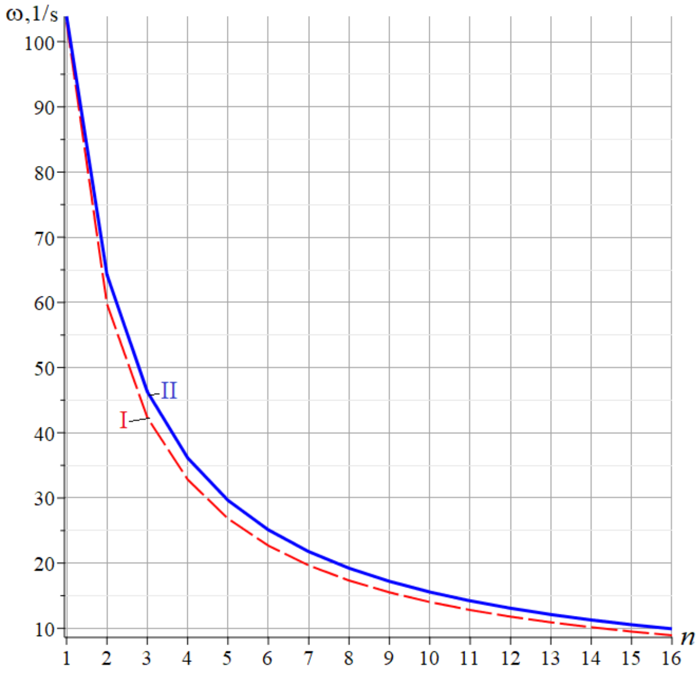

Figure 3. Dependence of the fundamental oscillation frequency on the number of panels:
I — solution (6); II — numerical solution

The frequency values obtained by the Dunkerley method, as expected, are less than the first frequency of the entire spectrum of natural frequencies calculated numerically.

The error of the proposed solution (6) can be refined by introducing the value of the relative error $\varepsilon = (\omega_1 - \omega_D)/\omega_1$ (Figure 4). For trusses with a higher height *h*, the accuracy of the approximate analytical solution is less. With an increase in the number of panels, the error slightly increases, asymptotically approaching an acceptable value of 10% for the chosen dimensions.

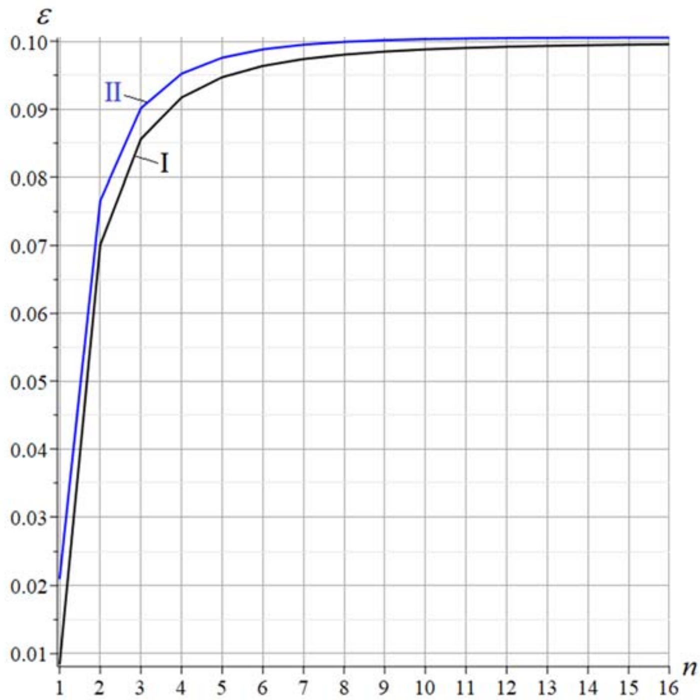

Figure 4. Dependence of the relative error on the number of panels:
I — *h* = 4 m; II — *h* = 8 m

4.2. Natural frequency spectrum of a family of regular trusses. If the frequency spectra of a series of trusses with a different number of panels are placed in the same coordinate axes, then certain regularities are found in the resulting picture. In Figure 5, dots indicate frequencies. Spectra of trusses of different orders are conditionally united by curves of the same color. Curve 1 corresponds to all four truss frequencies with one panel. The abscissa shows the numbers of frequencies in the ordered spectra. A characteristic feature is obvious: a frequency jump in all spectra after a frequency with a number equal to the truss order. Another feature is the presence of the upper boundary of the spectra. Regardless of the construction order, the highest frequency remains the same. This is the spectral constant of the construction. Another spectral constant is a straight line approached by frequencies with numbers n in the spectrum, where *n* is the order of the truss (Figure 6). The corresponding points lie on a curve of almost hyperbolic shape (spectral isoline), the asymptote of which, for the chosen dimensions of the truss, the rigidity of the rods, and the magnitude of the masses at the nodes, is equal to $143s^{-1}$.

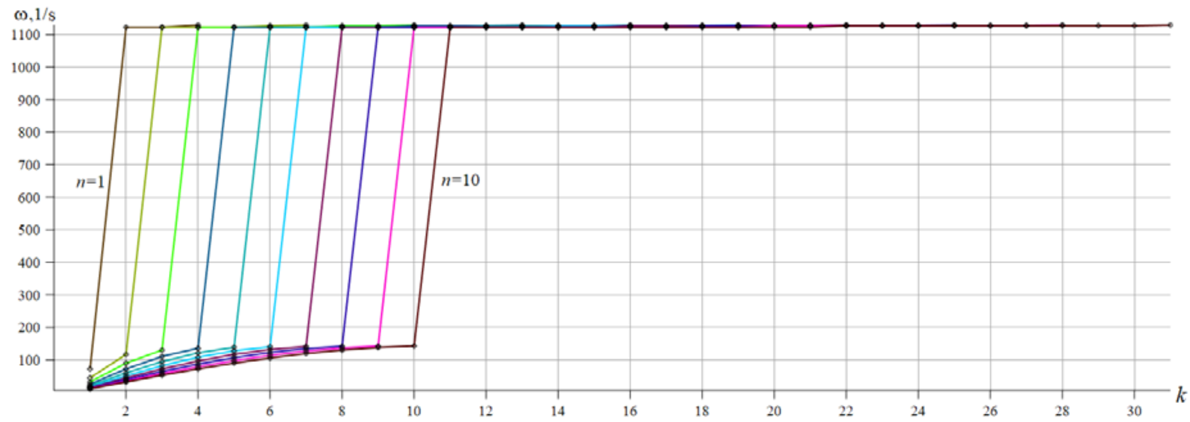

Figure 5. Spectra of Regular Trusses

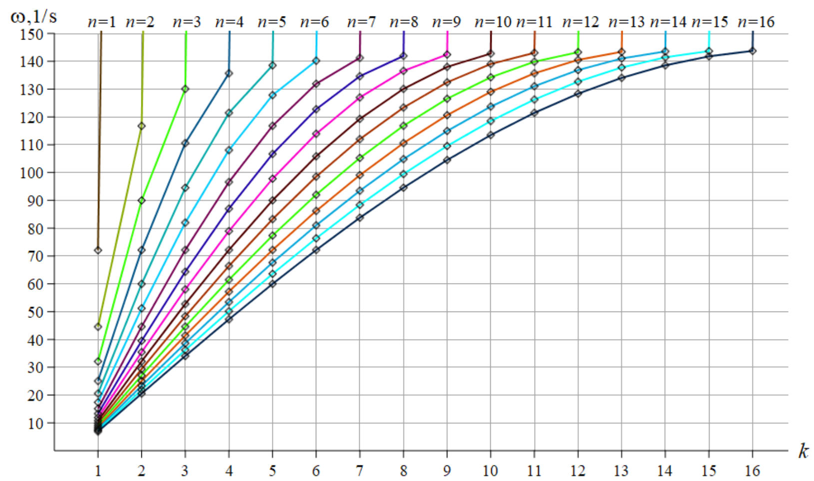

Figure 6. Spectra of Regular Trusses. First *n* frequencies

The frequency jump from frequency ω_n to frequency ω_{n+1}, where *n* is the order of the truss, is large enough. The frequency interval $[\omega_n, \omega_{n+1}]$ forms a certain resonant safety zone in the sense of ensuring the absence of resonance in the event of external periodic disturbances. In particular, if an engine with a natural frequency from this interval is installed on the truss, then, regardless of the number of panels, resonance phenomena in the truss-engine system can be avoided.

When changing the geometric parameters of the structure, the constants change in different ways, depending on the variable size. So, if you change the longitudinal dimension a from 3m to 16m, the upper frequency limit decreases by only 0.5%. Similarly, changing the size of b within the same limits changes this frequency by 3.8%. When the truss height is changed from 4m to 17m, the highest frequency increases by 2%.

Only two spectral constants and one clearly defined spectral isoline are found here. For comparison, in [28], up to eight isolines were found in the spectrum of a family of planar regular trusses of the same type. A distinctive feature of the dependence of the first frequency on the number of panels found here is that the accuracy of the analytical estimate does not increase with increasing construction order, but decreases. However, a positive factor is the limited growth of the error and the relatively high accuracy, which is not typical for the known solutions [29; 30] according to Dunkerley. In the Dunkerley frequency calculation of a spatial L-shaped truss [31], where an analytical solution was also obtained, the accuracy varies from 28% for a small number of panels to several percent for a large number. In the resulting solution, on the contrary — from 1% to 10% with a large number of panels.

5. Conclusion

A model of a spatial regular truss is constructed and a formula for the dependence of the first frequency on the number of panels is derived. The statement of the problem of dynamics is simplified to one degree of freedom for each mass in the node. Accounting for the remaining degrees of freedom practically does not change the algorithm used, nor does it change the qualitative side of the results, but somewhat complicates the form of the solution and is not presented here. Compared with the known similar solutions, the resulting formula is much simpler and noticeably more accurate when compared with the numerical solution obtained without simplification by the Dunkerley method. The spectrum of a family of regular trusses is also simpler than in other problems.

The main results of the work:

1. A model of a spatial regular truss is proposed and, using well-known algorithms, a formula for the dependence of the first frequency on the number of panels is derived.
2. In the frequency spectrum of the truss family, two clearly defined spectral constants are found, one of which is the upper boundary of the spectrum.
3. Resonant safety zone of a family of regular trusses is found.

References

1. Han Q.H., Xu Y., Lu Y., Xu J., Zhao Q.H. Failure Mechanism of Steel Arch Trusses: Shaking Table Testing and FEM Analysis. *Engineering Structures.* 2015;82:186–198. https://doi.org/10.1016/J.ENGSTRUCT.2014.10.013
2. Colajanni P., La Mendola L., Latour M., Monaco A., Rizzano G. FEM Analysis of Push-out Test Response of Hybrid Steel Trussed Concrete Beams (HSTCBs). *Journal of Constructional Steel Research.* 2015;111:88–102. https://doi.org/10.1016/j.jcsr.2015.04.011
3. Macareno L.M., Agirrebeitia J., Angulo C., Avilés R. FEM Subsystem Replacement Techniques for Strength Problems in Variable Geometry Trusses. *Finite Elements in Analysis and Design.* 2008;44:346–357. https://doi.org/10.1016/j.finel.2007.12.003
4. Belyankin N.A., Boyko A.Yu. Formulas for the deflection of a beam truss with an arbitrary number of panels under uniform loading. *Structural mechanics and structures*. 2019;1(20):21–29. (In Russ.) EDN: YZOZGH
5. Tkachuk G.N. The formula for the dependence of the deflection of an asymmetrically loaded flat truss with reinforced braces on the number of panels. *Structural mechanics and structures*. 2019;2(21):32–39. (In Russ.) EDN: JKKMFY
6. Boyko A.Yu., Tkachuk G.N. Derivation of formulas for the dependence of the deflection of a flat articulated-rod frame on the number of panels in the Maple system. *Structural mechanics and structures*. 2019;4(23):15–25. (In Russ.) EDN: ZJDBGW
7. Kirsanov M.N. Stress state and deformation of a rectangular spatial rod coating. *Scientific Bulletin of the Voronezh State University of Architecture and Civil Engineering. Construction and architecture*. 2016;1(41):93–100. (In Russ.) EDN: VNXUON
8. Buka-Vaivade K., Kirsanov M.N., Serdjuks D.O. Calculation of deformations of a cantilever-frame planar truss model with an arbitrary number of panels. *Vestnik MGSU [Monthly Journal on Construction and Architecture]*. 2020;15(4):510–517. https://doi.org/10.22227/1997-0935.2020.4.510-517
9. Kirsanov M.N. Evaluation of the deflection and stability of a spatial beam truss. *Structural Mechanics and Analysis of Constructions*. 2016;5(268):19–22. (In Russ.)
10. Larichev S.A. Inductive analysis of the effect of building lift on the rigidity of a spatial beam truss. *Collection of scientific and methodological articles* "*Trends in Applied Mechanics and Mechatronics"*. Moscow: Infra-M Publ.; 2015;(1):4–8. (In Russ.) EDN: AZKRYX
11. Hutchinson R.G., Fleck N.A. Microarchitectured cellular solids — the hunt for statically determinate periodic trusses. *Journal of Applied Mathematics and Mechanics*. 2005;85(9):607–617. https://doi.org/10.1002/zamm.200410208

12. Hutchinson R.G., Fleck N.A. The structural performance of the periodic truss. *Journal of the Mechanics and Physics of Solids*. 2006;54(4):756–782. https://doi.org/10.1016/j.jmps.2005.10.008

13. Zok F.W., Latture R.M., Begley M.R. Periodic truss structures. *Journal of the Mechanics and Physics of Solids*. 2016;96:184–203. https://doi.org/10.1016/j.jmps.2016.07.007

14. Kaveh A., Rahami H., Shojaei I. *Swift Analysis of Civil Engineering Structures Using Graph Theory Methods.* Springer Cham Publ.; 2020, 290 p. https://doi.org/10.1007/978-3-030-45549-1

15. Kaveh A., Hosseini S.M., Zaerreza A. Size, Layout, and Topology Optimization of Skeletal Structures Using Plasma Generation Optimization. *Iranian Journal of Science and Technology. Transactions of Civil Engineering.* 2020;45(2):513–543. https://doi.org/10.1007/S40996-020-00527-1

16. Goloskokov D.P., Matrosov A.V. A Superposition Method in the Analysis of an Isotropic Rectangle. *Applied Mathematical Sciences.* 2016;10(54):2647–2660. https://doi.org/10.12988/ams.2016.67211

17. Goloskokov D.P., Matrosov A.V. Comparison of two analytical approaches to the analysis of grillages. *2015 International Conference on "Stability and Control Processes" in Memory of V.I. Zubov, SCP 2015. Proceedings*. 2015: 382–385. https://doi.org/10.1109/SCP.2015.7342169

18. Kirsanov M.N. *Planar Trusses: Schemes and Formulas.* Cambridge Scholars Publishing. Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK; 2019. ISBN: (13):978-1-5275-3531-2

19. Komerzan E.V., Sviridenko O.V. Analytical calculation of the deflection of a flat externally statically indeterminate truss with an arbitrary number of panels. *Structural mechanics and structures*. 2021;2(29):29–37. (In Russ.) EDN: LJWUOW

20. Dai Q. Analytical Dependence of Planar Truss Deformations on the Number of Panels. *AlfaBuild*. 2021;17:1701. https://doi.org/10.34910/ALF.17.1

21. Kirsanov M., Luong C.L. Deformations and natural frequency spectrum of a planar truss with an arbitrary number of panels. *AlfaBuild*. 2022;25:2507. https://doi.org/10.57728/ALF.25.7

22. Kirsanov M. Simplified Dunkerley method for estimating the first oscillation frequency of a regular truss. *Construction of Unique Buildings and Structures*. 2023;108:10801. https://doi.org/10.4123/CUBS.108.1

23. Domanov E.V. Analytical dependence of the deflection of a spatial cantilever of a triangular profile on the number of panels. *Science Almanac*. 2016;6-2(19):214–217. (In Russ.) https://doi.org/10.17117/na.2016.06.02.214

24. Petrichenko E.A. On the deflection of a cantilever truss with a cross-shaped lattice depending on the redistribution of the areas of the rods and the number of panels. *Science Almanac*. 2016;6-2(19):279–266. (In Russ.) https://doi.org/ 10.17117/na.2016.06.02.279

25. Osadchenko N.V. Analytical solutions to the problems of deflection of flat arch-type trusses. *Structural mechanics and structures*. 2018;1(16):12–33. (In Russ.) EDN: YSLUGL

26. Rutenberg A. A Lower Bound for Dunkerley's Formula in Continuous Elastic Systems. *Journal of Sound and Vibration.* 1976;45(2):249–252. https://doi.org/10.1016/0022-460X(76)90599-X

27. Low K.H. A Modified Dunkerley Formula for Eigenfrequencies of Beams Carrying Concentrated Masses. *International Journal of Mechanical Sciences.* 2000;42(7):1287–1305. https://doi.org/10.1016/S0020-7403(99)00049-1

28. Komerzan E.V., Maslov A.N. Analytical estimation of the fundamental frequency of natural vibrations of a regular truss. *Structural mechanics and structures*. 2023;2(37):17–26. (In Russ.) https://doi.org/10.36622/VSTU.2023.37.2.002

29. Shchigol E.D. Formula for the Lower Estimation of the Natural Oscillations of a Planar Regular Beam Truss with a Rectilinear Upper Chord. *Structural mechanics and structures*. 2023;2(37):46–53. (In Russ.) https://doi.org/10.36622/VSTU.2023.37.2.005

30. Manukalo A.S. Analysis of the value of the first frequency of natural vibrations of a flat trussed truss. *Structural mechanics and structures*. 2023;2(37):54–60. (In Russ.) https://doi.org/10.36622/VSTU.2023.37.2.006

31. Komerzan E.V., Maslov A.N. Estimation of the fundamental oscillation frequency of an L-shaped spatial truss. *Structural mechanics and structures*. 2023;2(37):35–45. (In Russ.) https://doi.org/10.36622/VSTU.2023.37.2.004

Список литературы

1. *Han Q.H., Xu Y., Lu Y., Xu J., Zhao Q.H.* Failure Mechanism of Steel Arch Trusses: Shaking Table Testing and FEM Analysis // Engineering Structures. 2015. Vol. 82. P. 186–198. https://doi.org/10.1016/J.ENGSTRUCT.2014.10.013

2. *Colajanni P., La Mendola L., Latour M., Monaco A., Rizzano G.* FEM Analysis of Push-out Test Response of Hybrid Steel Trussed Concrete Beams (HSTCBs) // Journal of Constructional Steel Research. 2015. Vol. 111. P. 88–102. https://doi.org/10.1016/j.jcsr.2015.04.011

3. *Macareno L.M., Agirrebeitia J., Angulo C., Avilés R.* FEM Subsystem Replacement Techniques for Strength Problems in Variable Geometry Trusses // Finite Elements in Analysis and Design. 2008. Vol. 44. P. 346–357. https://doi.org/10.1016/j.finel.2007.12.003

4. *Белянкин Н.А., Бойко А.Ю.* Формулы для прогиба балочной фермы с произвольным числом панелей при равномерном загружении // Строительная механика и конструкции. 2019. № 1 (20). С. 21–29. EDN: YZOZGH

5. *Ткачук Г.Н.* Формула зависимости прогиба несимметрично нагруженной плоской фермы с усиленными раскосами от числа панелей // Строительная механика и конструкции. 2019. № 2 (21). С. 32–39. EDN: JKKMFY

6. *Бойко А.Ю., Ткачук Г.Н.* Вывод формул зависимости прогиба плоской шарнирно-стержневой рамы от числа панелей в системе Maple // Строительная механика и конструкции. 2019. № 4 (23). С. 15–25. EDN: ZJDBGW

7. *Кирсанов М.Н.* Напряженное состояние и деформации прямоугольного пространственного стержневого покрытия // Научный вестник Воронежского государственного архитектурно-строительного университета. Строительство и архитектура. 2016. № 1 (41). С. 93–100. EDN: VNXUON

8. *Buka-Vaivade K., Kirsanov M.N., Serdjuks D.O.* Calculation of deformations of a cantilever-frame planar truss model with an arbitrary number of panels // Вестник МГСУ. 2020. Т. 15. Вып. 4. С. 510–517. https://doi.org/10.22227/1997-0935.2020.4.510-517

9. *Кирсанов М.Н.* Оценка прогиба и устойчивости пространственной балочной фермы // Строительная механика и расчет сооружений. 2016. № 5 (268). С. 19–22.

10. *Ларичев С.А.* Индуктивный анализ влияния строительного подъема на жесткость пространственной балочной фермы // Сборник научно-методических статей “Trends in Applied Mechanics and Mechatronics”. М.: Инфра-М., 2015. Т. 1. С. 4–8. EDN: AZKRYX

11. Hutchinson R.G., Fleck N.A. Microarchitectured cellular solids — the hunt for statically determinate periodic trusses // Journal of Applied Mathematics and Mechanics. 2005. Vol. 85. Is. 9. P. 607–617. https://doi.org/10.1002/zamm.200410208

12. *Hutchinson R.G., Fleck N.A.* The structural performance of the periodic truss // Journal of the Mechanics and Physics of Solids. 2006. Vol. 54. № 4. P. 756–782. https://doi.org/10.1016/j.jmps.2005.10.008

13. *Zok F.W., Latture R.M., Begley M.R.* Periodic truss structures // Journal of the Mechanics and Physics of Solids. 2016. Vol. 96. P. 184–203. https://doi.org/10.1016/j.jmps.2016.07.007

14. *Kaveh A., Rahami H., Shojaei I.* Swift Analysis of Civil Engineering Structures Using Graph Theory Methods. Springer Cham Publ.; 2020. 290 p. https://doi.org/10.1007/978-3-030-45549-1

15. *Kaveh A., Hosseini S.M., Zaerreza A.* Size, Layout, and Topology Optimization of Skeletal Structures Using Plasma Generation Optimization // Iranian Journal of Science and Technology. Transactions of Civil Engineering. 2020. No. 45 (2). P. 513–543. https://doi.org/10.1007/S40996-020-00527-1

16. *Goloskokov D.P., Matrosov A.V.* A Superposition Method in the Analysis of an Isotropic Rectangle // Applied Mathematical Sciences. 2016. No. 10 (54). P. 2647–2660 https://doi.org/10.12988/ams.2016.67211

17. *Goloskokov D.P., Matrosov A.V.* Comparison of two analytical approaches to the analysis of grillages // 2015 International Conference on “Stability and Control Processes” in Memory of V.I. Zubov, SCP 2015. Proceedings. 2015. P. 382–385. https://doi.org/10.1109/SCP.2015.7342169

18. *Kirsanov M.N.* Planar Trusses: Schemes and Formulas. Cambridge Scholars Publishing. Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK; 2019. 198 p. ISBN (13): 978-1-5275-3531-2

19. *Комерзан Е.В., Свириденко О.В.* Аналитический расчет прогиба плоской внешне статически неопределимой фермы с произвольным числом панелей // Строительная механика и конструкции. 2021. № 2 (29). С. 29–37. EDN: LJWUOW

20. *Dai Q.* Analytical Dependence of Planar Truss Deformations on the Number of Panels // AlfaBuild. 2021. No. 17. Article 1701. https://doi.org/10.34910/ALF.17.1

21. *Kirsanov M., Luong C.L.* Deformations and natural frequency spectrum of a planar truss with an arbitrary number of panels // AlfaBuild. 2022. No. 25. Article 2507. https://doi.org/10.57728/ALF.25.7

22. *Kirsanov M.* Simplified Dunkerley method for estimating the first oscillation frequency of a regular truss // Construction of Unique Buildings and Structures. 2023. No. 108. Article 10801. https://doi.org/10.4123/CUBS.108.1

23. *Доманов Е.В.* Аналитическая зависимость прогиба пространственной консоли треугольного профиля от числа панелей // Научный альманах. 2016. № 6–2(19). С. 214–217. https://doi.org/10.17117/na.2016.06.02.214

24. *Петриченко Е.А*. О прогибе консольной фермы с крестообразной решеткой в зависимости от перераспределения площадей стержней и числа панелей // Научный альманах. 2016. № 6–2 (19). С. 279–266. https://doi.org/10.17117/na.2016.06.02.279

25. *Осадченко Н.В.* Аналитические решения задач о прогибе плоских ферм арочного типа // Строительная механика и конструкции. 2018. № 1 (16). С. 12–33. EDN: YSLUGL

26. *Rutenberg* A. A Lower Bound for Dunkerley's Formula in Continuous Elastic Systems // Journal of Sound and Vibration. 1976. Vol. 45. Is. 2. P. 249–252. https://doi.org/10.1016/0022-460X(76)90599-X

27. *Low K.H.* A Modified Dunkerley Formula for Eigenfrequencies of Beams Carrying Concentrated Masses // International Journal of Mechanical Sciences. 2000. Vol. 42. Is. 7. P. 1287–1305. https://doi.org/10.1016/S0020-7403 (99)00049-1

28. *Комерзан Е.В., Маслов А.Н.* Аналитическая оценка основной частоты собственных колебаний регулярной фермы // Строительная механика и конструкции. 2023. № 2 (37). С. 17–26. https://doi.org/10.36622/VSTU.2023.37.2.002

29. *Щиголь Е.Д.* Формула для нижней оценки собственных колебаний плоской регулярной балочной фермы с прямолинейным верхним поясом // Строительная механика и конструкции. 2023. № 2 (37). С. 46–53. https://doi.org/10.36622/VSTU.2023.37.2.005

30. *Манукало А.С.* Анализ значения первой частоты собственных колебаний плоской шпренгельной фермы // Строительная механика и конструкции. 2023. № 2 (37). С. 54–60. https://doi.org/10.36622/VSTU.2023.37.2.006

31. *Комерзан Е.В., Маслов А.Н.* Оценка основной частоты колебаний Г-образной пространственной фермы // Строительная механика и конструкции. 2023. № 2 (37). С. 35–45. https://doi.org/10.36622/VSTU.2023.37.2.004