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Methodology for determining progressing ultimate states based on the displacement method

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Abstract. Solving of calculation problems for building structures is currently based on the principle of minimum total energy of structures deformation. However, it is not possible to determine the remaining bearing capacity of the structure using this principle. In the study it is proposed to use the criterion of critical levels of deformation energy to solve this problem. As a result, the ultimate state conditions of a design are formulated on the basis of extreme values of generalized parameters of designing over the whole area of their admissible values, including the boundary. The task is solved as a problem of eigenvalues for the stiffness matrix of the system. The extreme values of design parameters that correspond to critical energy levels are found, which are used to find the maximum possible value of the energy of deformation for the considered structure. The residual bearing capacity is calculated by the value of residual potential energy, which, in turn, is equal to the difference between the maximum possible value of the deformation energy of the structure and the work of external forces. A gradual methodology for investigating the progressive ultimate limit state is proposed, which is based on the sequential exclusion of those elements where the onset of the ultimate limit state is expected firstly. An example of the practical use of the proposed methods is given on the example of calculating a simple but visual design – a statically indeterminate truss.

Keywords: rod systems, matrix methods of calculation, self-stress, deformation energy, ultimate state, critical levels

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Методика определения прогрессирующих предельных состояний на основе метода перемещений

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Аннотация. Решение задач расчета строительных конструкций в настоящее время основывается на принципе минимума полной энергии деформации конструкций. Однако определить остаточную несущую способность конструкции, используя этот принцип, не представляется возможным. В исследовании предлагается использовать для решения этой задачи критерий критических уровней энергии деформации. Условия предельного состояния конструкции в результате формулируются на основе экстремальных значений обобщенных параметров проектирования на всей области их допустимых значений, включая границу. Задача решается как проблема собственных значений для матрицы жесткости системы. Отыскиваются экстремальные значения параметров проектирования, соответствующие критическим уровням энергии, по которым находится максимально возможная величина энергии деформации рассматриваемой конструкции. Остаточная несущая способность вычисляется по значению остаточной потенциальной энергии, которая в свою очередь равна разнице максимально возможной величины энергии деформации конструкции и работы внешних сил. Предложена пошаговая методика исследования прогрессирующего предельного состояния, основанная на последовательном исключении тех элементов, в которых в первую очередь ожидается наступление предельного состояния. Приводится пример практического использования предлагаемых методик на примере расчета простой, но наглядной конструкции – статически неопределимой фермы.

Ключевые слова: стержневые системы, матричные методы расчета, самонапряжение, энергия деформации, критические уровни, предельное состояние

1. Introduction

Nowadays, almost all calculations associated with the estimation of ultimate states reached by a structure during designing of structures are performed in the Lagrangian form [1–6]. This formulation of the problem allows us to obtain the design of load-bearing structures of a construction only for specified values of loads. Considering that the geometrical parameters of the unsafe section are used to design similar structures of the load-bearing structural system, the building practically always has a significant safety reserve. This is due both to the unification of elements carried out by the designer, and to the imperfection of the calculation methods used, which do not allow taking into account all the features of the behavior of a real structure under the influence of loads.

When choosing a methodology for calculating a structure, the designer is faced with problems beyond his scope of competency. For example, when determining the rational cross-section of a bending beam, under the assumption of elastic deformation of the material, it should choose the shape of the cross-section in the form of an I-beam, where the material is mainly concentrated in the fibers that are the most distant from the beam axis. At the same time, a similar calculation in the elastic–plastic stage of deformation, leads it to the shape of

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a cross-section in the form of a rhombus. In this case, the material is mainly concentrated near the neutral axis of the element. The desire to obtain a rational distribution of material in the structure comes into conflict with the understanding of efficiency from an economic point of view and the construction technology.

Attempts to use the theory of optimal structural design to obtain more efficient projects [7–10], as it was shown in studies [11], do not provide significant results.

This problem is aggravated by the fact that it is currently impossible to accurately determine the available reserve of the designed structures using generally accepted calculation methods. No methods are known to us that allow to determine the full (maximum) load-bearing capacity of designed or already built structures, and this is the primary task during inspection of their technical condition [12–21]. If trying to find the remaining load-bearing capacity of a structure after application of the project load, on the basis of the classical Lagrange approach, then consideration of all possible variants of loading leads to the necessity to solve an endless chain of tasks, which is impossible to fulfill even with the use of the most modern computational techniques.

To resolve the mentioned difficulties, it is proposed to use the criterion of critical levels of internal energy of deformation [22]. It can be used to find the maximum possible deformation energy of a structure and based on it evaluate the remaining load-bearing capacity of the structure [23].

Using the criterion of critical levels of internal deformation energy, it becomes to develop an algorithm and solve the problem of the progressive limit state of the building's load-bearing structures. The formulation of the problem proposed in the article allows us, step by step, to find the most loaded element of the structure (“weak link”), where the limit state will occur first of all, and to exclude it from operation in the calculation scheme. This process can be continued until a geometrically changeable system of elements is obtained.

This paper studies simple rod systems using the criterion of critical levels of internal deformation energy based on the equations of the displacement method. The results of determining the residual energy of deformation of the structure are given. A particular example of the progressive failure process of a structure is considered.

2. Methods

The criterion of critical energy levels is based on the separation of the energy of external influences and the energy of internal deformations [22]. This criterion can be formulated in the form of equations describing the requirement of a minimum variation of the deformation energy of a structure, including the condition of orthonormality of the design parameters of the structure, and boundary conditions for the range of permissible design parameters:

$$\delta^2 U(\chi) = 0; \qquad \Gamma(\chi) = 0, \tag{1}$$

where $U(\chi)$ – the potential energy of deformation of the structure; χ – extremal internal design parameters (generalized displacements and forces).

The structure may have several levels of critical energy. During the transition from one level to another, the state of self-stress of the structure changes.

The potential energy of deformation at each loading level of the structure can be decomposed into the sum of the potential energy balancing the work of external forces U_{ex} and the remaining part of the potential energy of deformation U_{cr} (which is in a self-balanced state):

$$U(\chi) = U_{\text{cr}}(\chi) + U_{\text{ex}}(\chi). \tag{2}$$

The limit state of the structure is considered to be maintained as long as the work of external forces does not exceed the potential energy of deformation $U(\chi)$, and it is balanced by a part of the potential energy of deformation U_{ex}. The remaining part of the self-balanced energy U_{cr} can be used for further increasing the load.

Further we will investigate the critical deformation energy of the structure U_{cr}. For this purpose, we will create a small perturbation of the internal field of forces or deformations. At the same time, we consider that external forces (or displacements) do not perform any actual work, since they are compensated by the internal forces.

From the variational principle (1), we obtain the condition of the crucial state of the structure in the form of the displacement method [23–25]:

$$[K]\{\delta Z\} = \left[\lambda^R\right]\{\delta Z\}, \tag{3}$$

where $[K]$ – is the stiffness matrix of the structure; $\{\delta Z\}$ – the vector of generalized displacements in the nodes of the structure for the state of self-stress, which is represented by a set of orthonormal functions; $[\lambda^R]$ – the matrix of eigenvalues, which has the meaning of single nodal reacting forces.

The vector of maximum nodal reacting forces in the nodes of the structure $[C]$ is calculated as:

$$\{\Phi_{\max}\} = [\lambda^R_{\min}]\{\delta Z_{\min}\}. \tag{4}$$

The algorithm for determination of the potential energy of deformation is formulated through the well-known matrix procedures of structural mechanics.

For example, the stiffness matrix can be obtained from the internal stiffness matrix of the structure and the static matrix of the task:

$$[K] = [A]^T [C][A]. \tag{5}$$

Having constructed the matrix, we solve the eigenvalue problem (3) and find the vector of nodal reacting forces (4). This vector is used to determine the forces *N* in each of the rods.

The potential energy of deformation of the structural elements is found from the vector of forces $\{N\}$ in the rods as:

$$U = \{N\}^T [L]\{N\} / 2. \tag{6}$$

The work of external forces (for which the design of the structure was carried out) is calculated using classical methods of structural mechanics.

3. Results and discussion

The proposed methodology is applied to the calculation of a statically indeterminate truss. The rod design scheme helps to describe the possible limit states in the simplest way and to demonstrate the self-stressing states of the structure.

The inequalities describing the limit state of the structure are as follows:

$$U(\Phi,\xi) \le U_{\text{ult}}; \quad \{\Phi_{\max}\} \le \{\Phi_{\text{ult}}\}; \quad \{\xi_{\max}\} \le \{\xi_{\text{ult}}\}, \tag{7}$$

where $\{\Phi\}$ is a vector of generalized forces; $\{\xi\}$ is a vector of generalized displacements.

The indexes correspond to the maximum and ultimate values.

The extremal values of energy of deformation $U(\Phi, \xi)$, generalized forces $\{\Phi\}$ and displacements $\{\xi\}$, including their values at the boundary of the range of admissible parameters, are determined from the task on the eigenvalues (3). They depend only on the geometric and mechanical characteristics of the structure, as well as on the conditions of supporting.

We consider that structural elements can no longer resist external influences at the occurrence of the ultimate state (violation of one of the conditions (7)) in one or more rods. This may be, for example, due to the occurrence of yielding state of the rod's material. In such cases, we will speak hereinafter about "rod deletion."

We will consider the simplest case, when the dimensions of the rods are selected in such a way that they do not lose their stability under compression. We assume that the constraints on displacements at all points of the structure are not violated. Then we deal only with limitations on strength. Moreover, these conditions are formulated uniformly for tensile and compressed rods.

Consider the truss shown in Figure 1. We assume that all rods have the same stiffness *EA* = 1. The length and width of all panels of the truss are the same. Figure 1 also shows the numbering of elements and nodes of the farm, in straight and oblique font respectively.

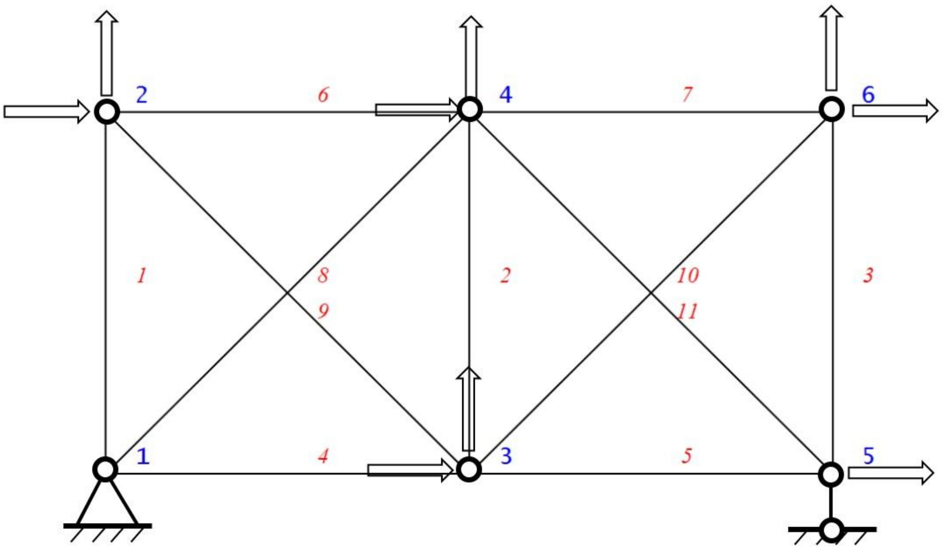

Figure 1. Design scheme of a statically indeterminate truss

Using the proposed methodology, we will find the first self-stress state of the structure. The calculations will be carried out in the software complex for the analysis of structures by the method of critical levels of energy "CLE," which implements this methodology [26].

The forces in the truss rods will be obtained from the unit vectors of displacements applied in the direction of the degrees of freedom indicated in Figure 1 by arrows. The results of determining the forces in the rods are shown in Figure 2. Figure 3 contains the extreme values of forces calculated from the maximum nodal displacements.

Comparing the forces in the rods of the truss shown in Figures 2 and 3, we note that the forces arising from the maximum values of node displacements are much greater than the forces from the action of single nodal displacements. The unit nodal displacements in this case are one of the possible cases of external actions on the truss. The main values of forces in the rods are always larger than from possible external impacts, as demonstrated by the obtained results.

The value of the maximum possible potential energy of deformation for this truss is equal to: $U_{\text{cr}}^{\max,I} = 30.07EA/l$. This value is calculated from the main (maximum) forces in the truss rods. The ratio of work of external unit forces to the maximum possible energy of deformation for the truss is: $W_{\text{ex}} / U_{\text{cr}}^{\max,I} = 0.055$. The relative residual value of the deformation energy of the truss is equal to: $U_{\text{res}}^{I} / U_{\text{cr}}^{\max,I} = (U_{\text{cr}}^{\max,I} - W_{\text{ex}}) / U_{\text{cr}}^{\max,I} = 0.94$. It means that the residual resource of the truss load-bearing capacity is a significant part of its maximum value.

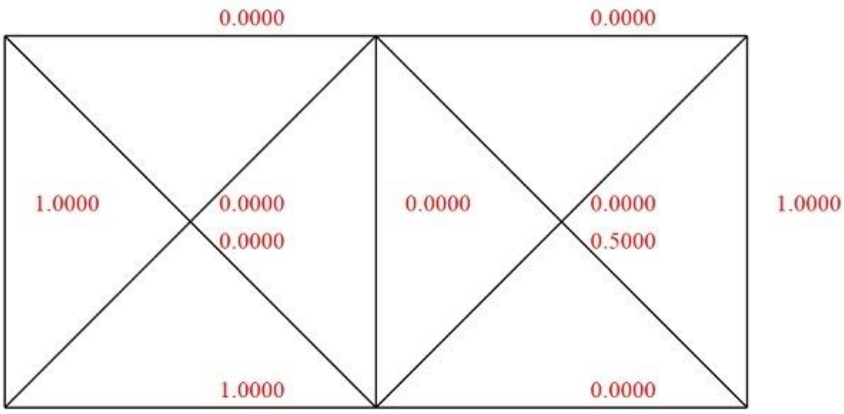

Figure 2. Self-stressing forces in the rods of a statically indeterminate truss from unit variations of displacements in the nodes

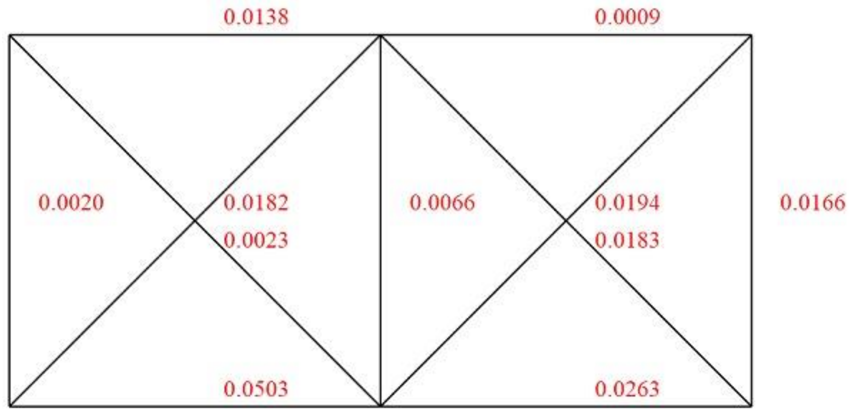

Figure 3. Main values of self-stress forces in the rods of a statically indeterminate truss from maximal nodal displacements

The maximum values of the main forces in the rods indicate which rod (or rods) is the "weak link" and will be removed due to violation of one of the ultimate limit state conditions. At this stage, this is rod 4 (Figure 3).

To find out which of the rods will be out of action at the next stage of loading of the truss, we remove this rod from the calculation scheme, and perform the calculation again. The results of the calculation are shown in Figures 4 and 5. Comparing the values of forces in Figures 4 and 5, we can see that the main (maximum) nodal forces also have larger values than from the single forces in the nodes that model the possible external load.

In a similar way, we continue the calculations until the structure becomes geometrically changeable. As a result, we investigate the process of progressive destruction of the structure step by step.

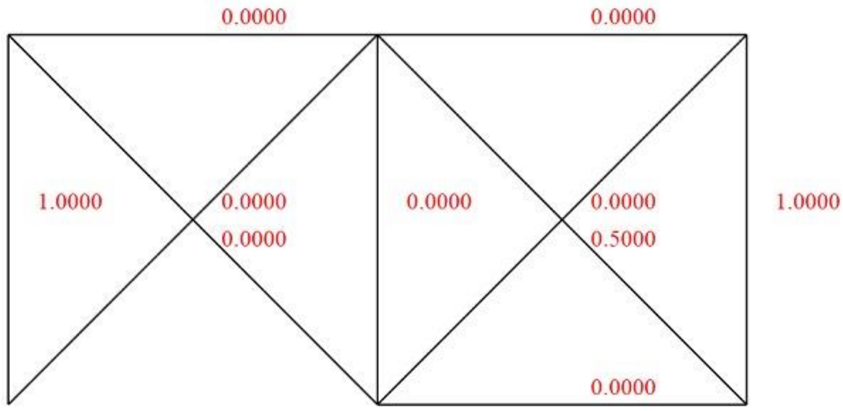

Figure 4. Self-stress forces in the rods of a statically indeterminate truss from unit variations of displacements in the nodes at the second stage of self-stressing

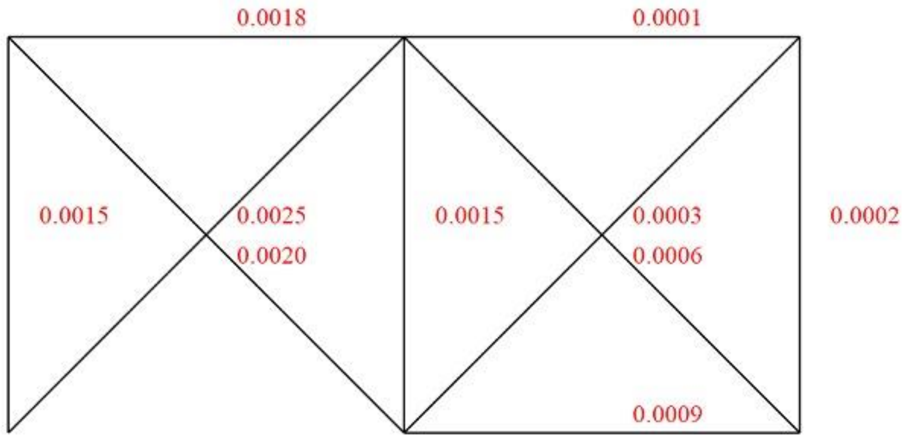

Figure 5. The main values of self-stressing forces in the rods of a statically indeterminate truss from the maximum nodal displacements at the second stage of self-stressing

At the second stage of self-stressing, the potential energy of deformation of the truss is equal to: $U_{\text{cr}}^{\max,II} = 26.94EA/l$. The ratio of the work of nodal displacements at the second stage of self-stressing to the maximum potential energy of deformation: $W_{\text{ex}}/U_{\text{cr}}^{\max,II} = 0.043$. The relative residual potential energy of deformation, which characterizes the residual resource at this stage, is equal to: $U_{\text{res}}^{II}/U_{\text{cr}}^{\max,II} = (U_{\text{cr}}^{\max,II} - W_{\text{ex}})/U_{\text{cr}}^{\max,II} = 0.96$.

In the second stage, the maximum values of the main forces appear in the rod 8. This rod is excluded from the load operation, and then the design scheme of the truss becomes geometrically changeable. The calculation is completed at this stage.

4. Conclusion

The authors propose a methodology based on the variational principle of critical energy levels of a deformable structure, which allows to solve a number of problems that cannot be solved on the basis of the minimum total energy principle for a structure.

A single criterion describing the limit state of a structure, based on the principle of change in the self-stress of the structure when passing through the critical level of the deformation energy of the structure, is used to solve the tasks of calculation for structures.

The example of the solution of the truss calculation problem is used to demonstrate the possibility of calculating the value of the structure's maximum possible potential deformation energy and, on its basis, the residual potential deformation energy of the structure after the application of external loads. A significant reserve of residual bearing capacity of the structure is revealed.

The method of detection the “weak link” of the structure in the form of displacement method is given, that allows to investigate the process of progressive ultimate limit state of the structure.

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