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Surfaces with a main framework of three given curves which include one circle

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Abstract. Superellipses are becoming more and more in demand in various branches of science and national economy due to their versatility. They found the most application in shipbuilding. Suggestions for the use of superellips in architecture and construction have appeared recently. The author proposes explicit and parametric equations of surfaces with a main framework of three predetermined superellips lying in three coordinate planes. These equations describe a large set of analytical shapes suitable for the formation middle surfaces of thin building shells. One of the superellipses is taken in a form of a circle. The shells can be designed on circular and rhombic plans, and also on plans in the shape of superellips of general type with convex and concave sides. All recommended surfaces are illustrated in 24 examples using computer graphics. A network of curvilinear non-orthogonal coordinates is generated on the surfaces using dimensionless independent parameters. The considered surfaces can become a part of the reserve of surfaces for further application in real structures and facilities.

Keywords: superellipse, rhombus, cylindroid, cone, translation surface

Поверхности с главным каркасом из трех заданных кривых, одна из которых – окружность

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Аннотация. Благодаря своей универсальности суперэллипсы становятся все более востребованными в различных отраслях науки. Наибольшее применение они нашли в судостроении. В последнее время появились предложения по использованию суперэллипсов в архитектуре и строительстве.

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Предлагаются явные и параметрические уравнения поверхностей с главным каркасом из трех заранее заданных суперэллипсов, лежащих в трех координатных плоскостях. Эти уравнения описывают большой набор аналитических форм, пригодных для формирования срединных поверхностей тонких строительных оболочек. Один из суперэллипсов взят в виде окружности. Оболочки можно проектировать на круглом и ромбическом планах, а также на планах в форме суперэллипсов общего вида с выпуклыми и вогнутыми сторонами. Все рекомендуемые поверхности проиллюстрированы на 24 примерах средствами компьютерной графики. С помощью безразмерных независимых параметров на поверхностях сформирована сеть криволинейных неортогональных координат. Рассматриваемые поверхности могут войти в резерв поверхностей для дальнейшего использования в реальных конструкциях и сооружениях.

Ключевые слова: суперэллипс, ромб, цилиндроид, круговой конус, поверхность переноса

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1. Introduction

Until recently, surfaces with a given three flat curves of the main framework in three coordinate planes were in demand in shipbuilding to form the hulls of surface and underwater vessels [1–3]. Then there were proposals to use superellips as flat curves of the main framework [4–6]. This made it possible to significantly expand the number of shapes for ship hulls.

In [7–8] it was first proposed to use thin shells with median surfaces containing three flat super ellipses of the main frame in construction and architecture. For the first time, a calculation was carried out for a distributed load of the self-weight type using the SCAD computer complex [7]. An attempt was made to find the most optimal shell from the three shells with the same main frame.

It is noted in [9] that surfaces containing superellipses are becoming more and more in demand due to their universality in various branches of science [10] and national economy [11]. However, some geometric problems for such surfaces are still unsolved.

It was assumed in [6; 7; 12] that the plane curves of the main frame of the considered surfaces are given in the form:

– the first curve of the main frame in the plane xOy (curve $z = 0$):

$$|y|^r = W^r\left(1 - \frac{|x|^t}{L^t}\right); \tag{1}$$

– the second curve of the main frame in the plane yOz (curve $x = 0$):

$$|z|^n = T^n\left(1 - \frac{|y|^m}{W^m}\right); \tag{2}$$

– the third curve of the main frame in the plane xOz (curve $y = 0$):

$$|z|^s = T^s\left(1 - \frac{|x|^k}{L^k}\right), \tag{3}$$

where for convex curves $r, t, n, m, s, k > 1$; for concave curves $r, t, n, m, s, k < 1$.

The curves (1)–(3) are superellipses [13]. In formulas (1)–(3) it is necessary to take $r = t$, $n = m$, $s = k$.

Using the methodology described in [6; 12; 14], it is possible to obtain explicit equations for three algebraic surfaces with the same main skeleton (1)–(3) and with the generating family of the same-type sections x = const:

$$|z| = T\left(1 - |x|^k/L^k\right)^{1/s}\left[1 - |y/W|^m/(1 - |x/L|^t)^{m/r}\right]^{1/n}, \tag{4}$$

with the generating family of the same-type sections y = const:

$$|z| = T(1 - |y|^m/W^m)^{1/n}\left[1 - |x/L|^k/(1 - |y/W|^r)^{k/t}\right]^{1/s}, \tag{5}$$

and with the generating family of the same-type sections z = const:

$$|y| = W(1 - |z|^n/T^n)^{1/m}\left[1 - |x/L|^t/(1 - |z/T|^s)^{t/k}\right]^{1/r}, \tag{6}$$

where $-L \le x \le L$, $-W \le y \le W$, $0 \le z \le T$.

The explicit surface equations (4)–(6) can be transferred to a parametric form of setting:

$$x = x(u) = \pm uL,\ y = y(u, v) = vW[1 - u^t]^{1/r},\ z = z(u, v) = T[1 - u^k]^{1/s}[1 - |v|^m]^{1/n}; \tag{7}$$

$$x = x(u, v) = vL[1 - u^r]^{1/t},\ y = y(u) = \pm uW,\ z = z(u) = T[1 - u^m]^{1/n}[1 - |v|^k]^{1/s}; \tag{8}$$

$$x = x(u, v) = vL[1 - u^s]^{1/k},\ y = y(u, v) = \pm W[1 - u^n]^{1/m}[1 - |v|^t]^{1/r},\ z = z(u) = uT, \tag{9}$$

where $0 \le u \le 1$, $-1 \le v \le 1$; u, v – dimensionless parameters.

Equations (4)–(9) were used in [15] to create five groups of new linear surfaces. Some of these linear surfaces were taken as the middle surfaces of thin building shells, for which their stress-strain state under the action of the static load of the self-weight type was studied in [16; 17].

2. Materials and methods

2.1. Possible triples of surfaces with a main framework of three flat curves, one of which is a circle

As is known, the circle is one of the main curves used in the design of curvilinear surfaces of thin shells in construction. Let suppose that the superellips (2) is a semicircle, then we have:

$$n = m = 2 \text{ and } T = W,\ z \ge 0. \tag{10}$$

With this assumption, we can obtain four groups of surfaces containing the semicircle (10). Each group will contain three surfaces with the same main frame.

2.1.1. The first triple of surfaces. If the superellipses (1) and (3) decompose into rhombuses, then $r = t = s = k = 1$. The triple surface equations (4)–(6) with the same main frame will have the form:

– a surface formed by a family of straight lines in the planes x = const:

$$z = \left[\left(1 - \frac{|x|}{L}\right)^2 T^2 - y^2\right]^{1/2}; \tag{11}$$

– a surface formed by a family of straight lines in the planes y = const:

$$z = T\left(1 - \frac{y^2}{W^2}\right)^{\frac{1}{2}}\left[1 - \frac{|x|}{L\left(1 - \frac{|y|}{W}\right)}\right]; \tag{12}$$

– a surface formed by a family of straight lines in the planes z = const:

$$|y| = W\left(1 - \frac{z^2}{T^2}\right)\left(1 + \frac{z}{T}\right)\left(1 - \frac{z}{T} - \frac{|x|}{L}\right). \tag{13}$$

The explicit surface equations (11)–(13) can be written in the parametric form (7)–(9).

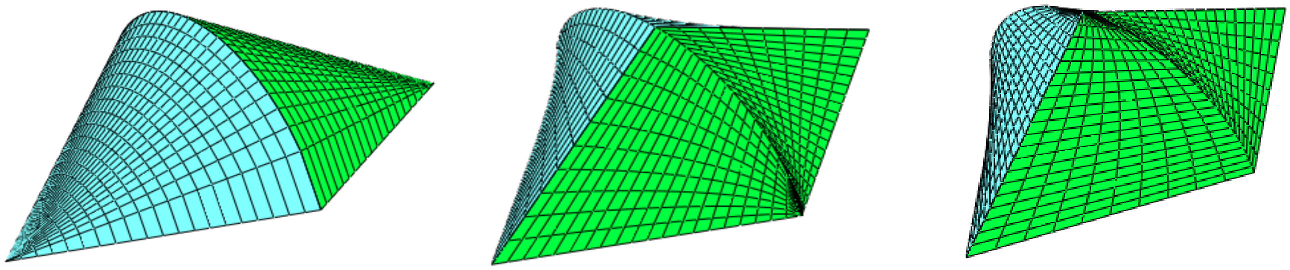
$$x = x(u) = \pm uL,\ y = y(u, v) = vW[1 - u],\ z = z(u, v) = T[1 - u][1 - v^2]^{1/2}; \tag{11a}$$

$$x = x(u, v) = vL[1 - u],\ y = y(u) = \pm uW,\ z = z(u) = T[1 - u^2]^{1/2}[1 - |v|]; \tag{12a}$$

$$x = x(u, v) = vL[1 - u],\ y = y(u, v) = \pm W[1 - u^2]^{1/2}[1 - |v|],\ z = z(u) = uT. \tag{13a}$$

Figure 1 shows a triple of surfaces with the same main frame with dimensions $T = W = 5$ m, $L = 7$ m.

The presented triple of surfaces consists of a composite conical surface (Figure 1, *a*) [18] and two cylindroids (Figure 1, *b*, *c*) [19].



a *b* *c*

Figure 1. Analytical surfaces on a rhombic plan (the first triple):
a – the surface is created using the formulas (11a); *b* – the surface is created using the formulas (12a); *c* – the surface is created using the formulas (13a)

2.1.2. The second triple of surfaces. Let the superellips (1) decompose into a rhombus, then $r = t = 1$, $n = m = 2$. The equations of the triples of surfaces (4)–(6) with the same main frame will take the form: surface formed by a family of circles in the planes x = const:

$$|z| = T\left(1 - |x|^k/L^k\right)^{1/s}[1 - (y/W)^2/(1 - |x|/L)^2]^{1/2}, \tag{14}$$

with a generating family of sections of the same type y = const:

$$|z| = T(1 - |y|^2/W^2)^{1/2}[1 - |x/L|^k/(1 - |y|/W)^k]^{1/s}, \tag{15}$$

and with a generating family of sections of the same type z = const:

$$|y| = W(1 - |z|^2/T^2)^{1/2}\left[1 - |x/L|/(1 - |z/T|^s)^{1/k}\right], \tag{16}$$

where $-L \le x \le L$, $-W \le y \le W$, $0 \le z \le T$.

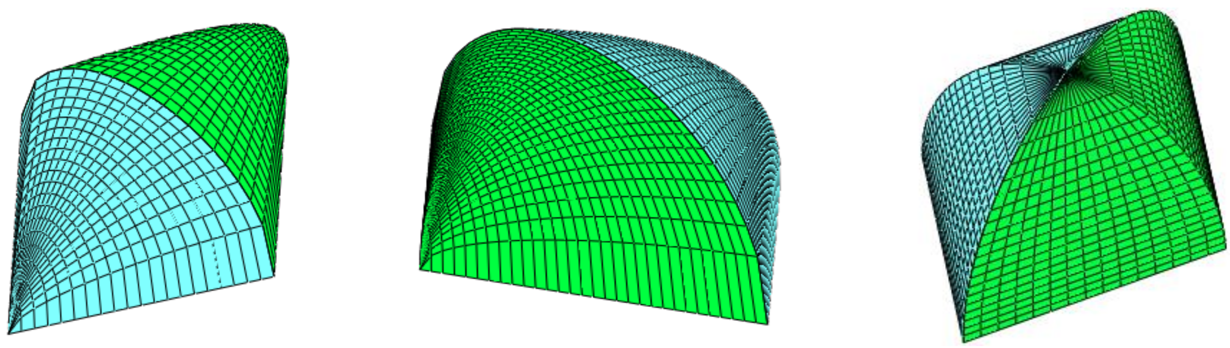
The explicit surface equations (14)–(16) can be transfered into a parametric form of setting:

$$x = x(u) = \pm uL, y = y(u, v) = vW[1 - u], z = z(u, v) = T[1 - u^k]^{1/s}[1 - v^2]^{1/2}; \tag{14a}$$

$$x = x(u, v) = vL[1 - u], y = y(u) = \pm uW, z = z(u) = T[1 - u^2]^{1/2}[1 - |v|^k]^{1/s}; \tag{15a}$$

$$x = x(u, v) = vL[1 - u^s]^{1/k}, y = y(u, v) = \pm W[1 - u^2]^{1/2}[1 - |v|], z = z(u) = uT, \tag{16a}$$

where $0 \le u \le 1$, $-1 \le v \le 1$; u, v – dimensionless parameters.



a *b* *c*

Figure 2. Analytical surfaces on a rhombic plan (the second triple):
a – the surface is created using the formulas (11a); *b* – the surface is created using the formulas (12a); *c* – the surface is created using the formulas (13a)

Figure 2 shows a triple of surfaces with the same main frame with dimensions $T = W = 5$ m, $L = 7$ m and with $s = k = 2$. The superellips (3) is taken in the form of a simple ellipse.

2.1.3 The third triple of surfaces. Let the superellipse (3) decompose into a rhombus, then $s = k = 1$, $n = m = 2$. The equations for the triplet of surfaces (4)–(6) with the same main framework will take the form: surface formed by a family of circles in the planes x = const:

$$|z| = T(1 - |x|/L)\left[1 - |y/W|^2/(1 - |x/L|^t)^{2/r}\right]^{1/2}, \tag{17}$$

with a generating family of sections of the same type y = const:

$$|z| = T(1 - |y|^2/W^2)^{1/2}\left[1 - |x|/L/(1 - |y/W|^r)^{1/t}\right], \tag{18}$$

and with a generating family of sections of the same type z = const:

$$|y| = W(1 - z^2/T^2)^{1/2}[1 - |x/L|^t/(1 - |z/T|^s)^t]^{1/r}, \tag{19}$$

where $-L \le x \le L$, $-W \le y \le W$, $0 \le z \le T$.

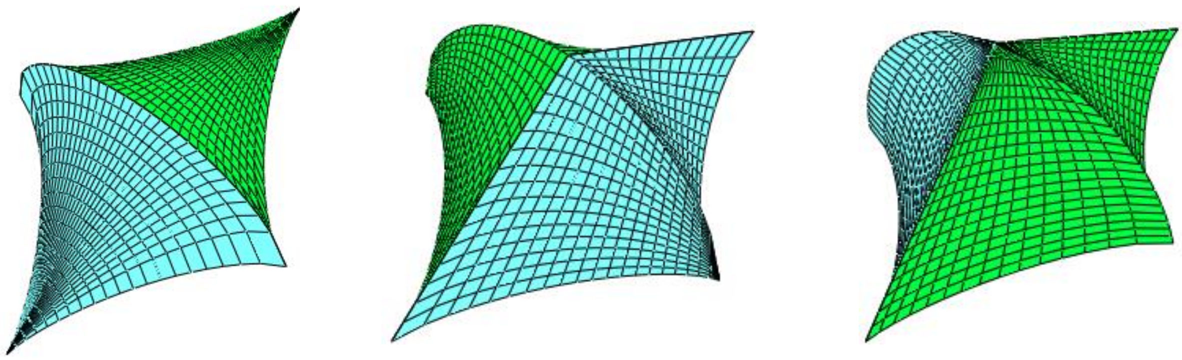
The explicit surface equations (17)–(19) can be transferred into a parametric form of setting:

$$x = x(u) = \pm uL, y = y(u, v) = vW[1 - u^t]^{1/r}, z = z(u, v) = T[1 - u][1 - v^2]^{1/2}; \tag{17a}$$

$$x = x(u, v) = vL[1 - u^r]^{1/t}, y = y(u) = \pm uW, z = z(u) = T[1 - u^2]^{1/2}[1 - |v|]; \tag{18a}$$

$$x = x(u, v) = vL[1 - u], y = y(u, v) = \pm W[1 - u^2]^{1/2}[1 - |v|^t]^{1/r}, z = z(u) = uT, \tag{19a}$$

where $0 \le u \le 1$, $-1 \le v \le 1$; u, v – dimensionless parameters.



a *b* *c*

Figure 3. Analytical surfaces on the plan in the superellipsoidal form (the third triple):
a – the surface is created using the formulas (17a); *b* – the surface is created using the formulas (18a); *c* – the surface is created using the formulas (19a)

Figure 3 shows a triple of surfaces with the same main framework with dimensions $T = W = 5$ m, $L = 7$ m, and with $r = t = 3/4$.

2.1.4. The fourth triple of surfaces. To generate the fourth triple of surfaces, meet only conditions (10). In this case, equations (4)–(6) take the form:

– with the forming family of circles lying in the sections x = const:

$$|z| = T\left(1 - |x|^k/L^k\right)^{1/s}\left[1 - |y/W|^2/(1 - |x/L|^t)^{2/r}\right]^{1/2}; \tag{20}$$

– with a generating family of sections of the same type y = const:

$$|z| = T(1 - y^2/W^2)^{1/2}\left[1 - |x/L|^k/(1 - |y/W|^r)^{k/t}\right]^{1/s}; \tag{21}$$

– with a generating family of sections of the same type z = const:

$$|y| = W(1 - z^2/T^2)^{1/2}\left[1 - |x/L|^t/(1 - |z/T|^s)^{t/k}\right]^{1/r}, \tag{22}$$

where $-L \le x \le L$, $-W \le y \le W$, $0 \le z \le T$.

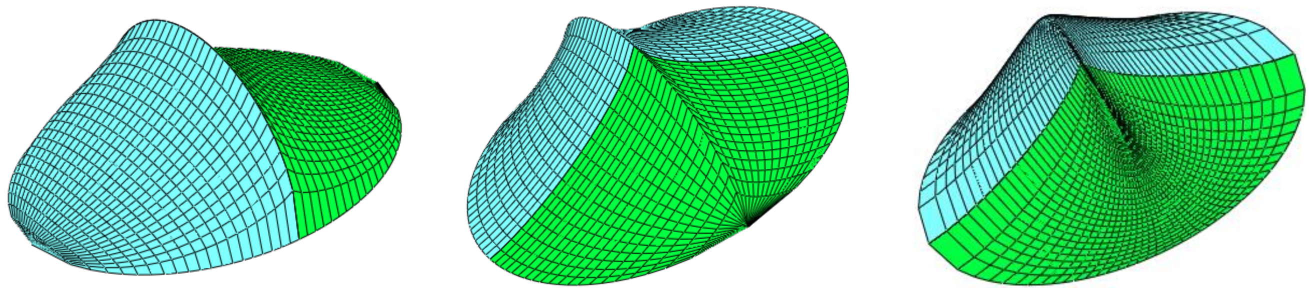
The explicit surface equations (20)–(21) can be transferred into a parametric form of setting:

$$x = x(u) = \pm uL, y = y(u, v) = vW[1 - u^t]^{1/r}, z = z(u, v) = T[1 - u^k]^{1/s}[1 - v^2]^{1/2}; \tag{20a}$$

$$x = x(u, v) = vL[1 - u^r]^{1/t}, y = y(u) = \pm uW, z = z(u) = T[1 - u^2]^{1/2}[1 - |v|^k]^{1/s}; \tag{21a}$$

$$x = x(u, v) = vL[1 - u^s]^{1/k}, y = y(u, v) = \pm W[1 - u^2]^{1/2}[1 - |v|^t]^{1/r}, z = z(u) = uT, \tag{22a}$$

where $0 \le u \le 1$, $-1 \le v \le 1$; u, v – dimensionless parameters.



a *b* *c*

Figure 4. Analytical surfaces on an elliptical plan (the fourth triple):
a – the surface is created using the formulas (20a); *b* – surface is created using the formulas (21a); *c* – surface is created using the formulas (22a)

Figure 4 shows a triple of surfaces with the same main framework with dimensions $T = W = 5$ m, $L = 7$ m and with $r = t = 2$, $s = k = 3/4$.

2.2. Triples of surfaces with the same main framework on a round plan

These surfaces are generated using the same formulas given above. It is only necessary to change the area of changes in the variable parameters u, v in the parametric equations and the area of changes in the Cartesian coordinates in the explicit equations. It is assumed that the Ox axis is directed upward, and that there is a circle in the yOz plane. When generating the new surfaces, the dimensions are left unchanged $T = W = 5$ m, $L = 7$ m, and $0 \le x \le L$, $-W \le y \le W$, $-T \le z \le T$, $n = m = 2$.

For the considered case, equations (7)–(9) will take the form:

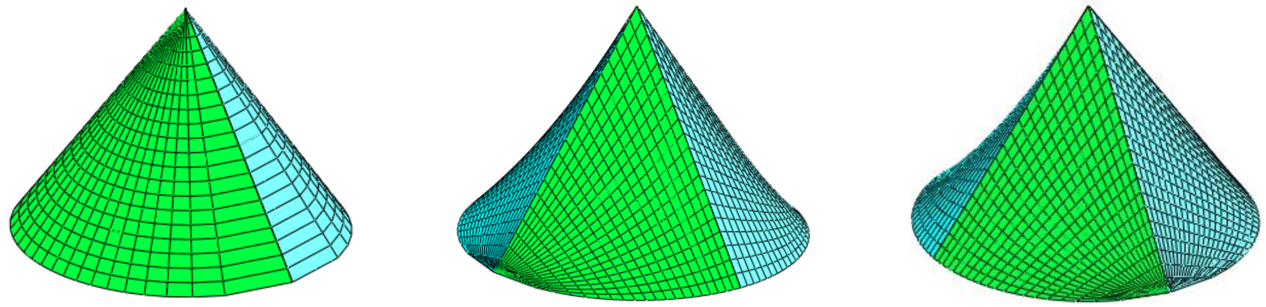
$$x = x(u) = uL, y = y(u, v) = vW[1 - u^t]^{1/r}, z = z(u, v) = \pm T[1 - u^k]^{1/s}[1 - |v|^m]^{1/n}; \tag{23}$$

$$x = x(u, v) = |v|L[1 - u^r]^{1/t}, y = y(u) = \pm uW, z = z(u) = \pm T[1 - u^m]^{1/n}[1 - |v|^k]^{1/s}; \tag{24}$$

$$x = x(u, v) = |v|L[1 - u^s]^{1/k}, y = y(u, v) = \pm W[1 - u^n]^{1/m}[1 - |v|^t]^{1/r}, z = z(u) = \pm uT, \tag{25}$$

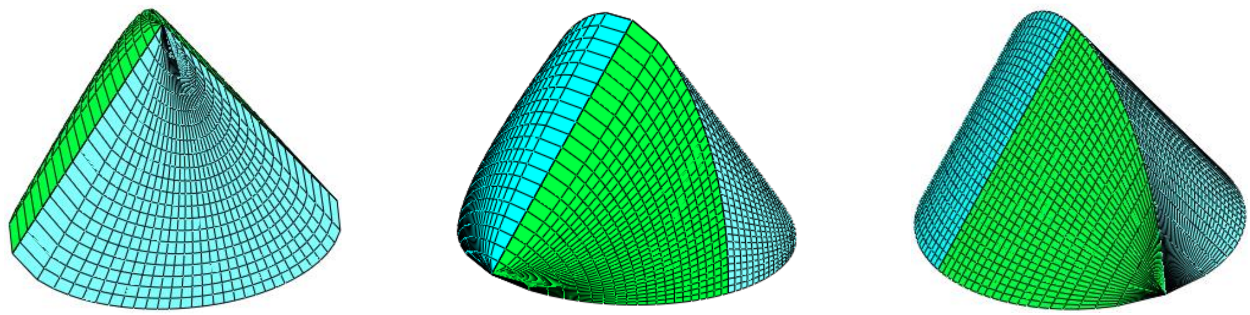
where $0 \le u \le 1$, $-1 \le v \le 1$; u, v are dimensionless parameters.

Using parametric equations (23)–(25), the first triple of surfaces at $r = t = s = k = 1$ (Figure 5), the second triple of surfaces at $r = t = 1$, $s = k = 2$ (Figure 6), the third triple of surfaces at $s = k = 1$, $r = t = 3/4$ (Figure 7) and the fourth triple of surfaces at $r = t = 2$, $s = k = 3/4$ (Figure 8) are generated.



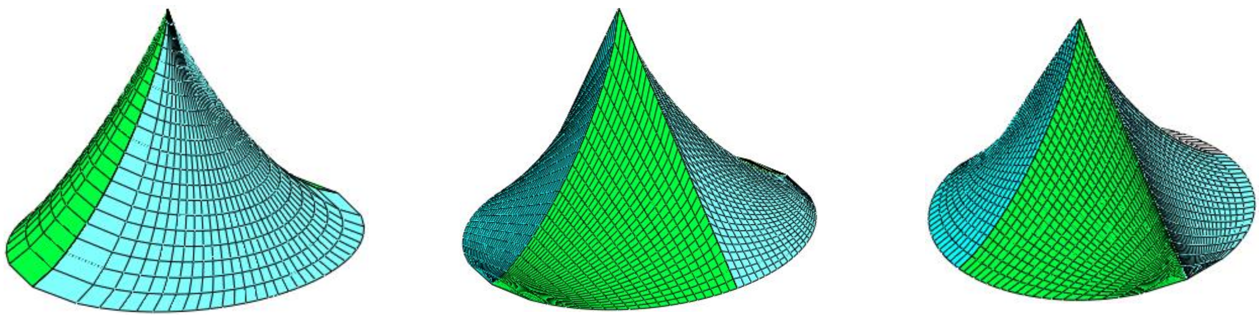
a *b* *c*

Figure 5. Analytical surfaces on a round plan (the first triple):
a – the surface is created using the formulas (23); *b* – the surface is created using the formulas (24); *c* – the surface is created using the formulas (25)



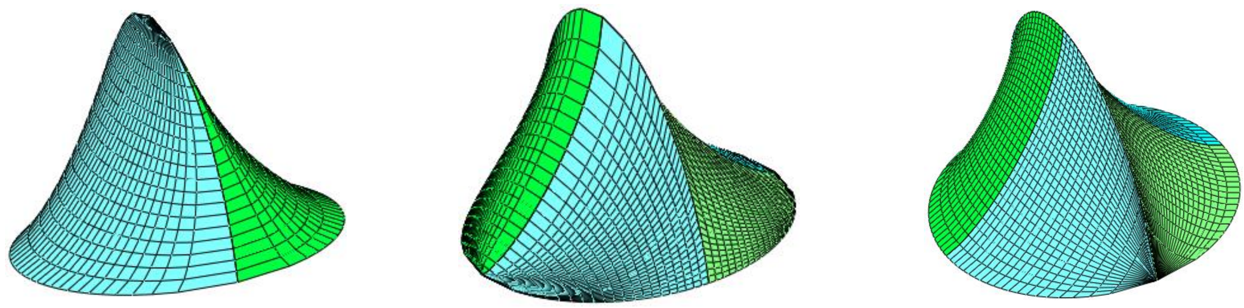
a *b* *c*

Figure 6. Analytical surfaces on a round plan (the second triple):
a – the surface is created using the formulas (23); *b* – the surface is created using the formulas (24); *c* – the surface is created using the formulas (25)



a *b* *c*

Figure 7. Analytical surfaces on a round plan (the third triple):
a – the surface is created using the formulas (23); *b* – the surface is created using the formulas (24); *c* – the surface is created using the formulas (25)



a *b* *c*

Figure 8. Analytical surfaces on a round plan (the fourth triple):
a – the surface is created using the formulas (23); *b* – the surface is created using the formulas (24); *c* – the surface is created using the formulas (25)

All the surfaces presented in this section are located on a circular basis on the plane yOz ($x = 0$). Several examples of surfaces on a circular plane with two superellips in intersecting planes are considered in [20].

3. Results and discussion

The paper shows the formation of four triples of surfaces, based on previously obtained analytical and parametric equations of surfaces with the main framework of three superellipses. All the 12 surfaces contain a circle as one of the flat curves of the main framework. The presented surfaces are visualized graphically for better recognition of them by architects and builders. In the author's opinion, these surfaces can be taken as the basis for the shapes of building and mechanical engineering objects, as well as the linear surfaces proposed earlier in [15]. At the very least, these surfaces may be in the reserve of surfaces waiting for their application [21] within one of the architectural styles [22].

4. Conclusion

The considered surfaces need further study, both from the point of view of differential geometry and structural mechanics of shells (surface areas, volume of internal space, stress-strain state of thin shells with middle surfaces in the form of proposed surfaces, finding optimal shells according to selected optimality criteria, etc.). The number of surfaces with circles can be significantly expanded by changing the exponents in formulas (1), (3).

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