2023. 19(2). 119–129

СТРОИТЕЛЬНАЯ МЕХАНИКА ИНЖЕНЕРНЫХ КОНСТРУКЦИЙ И СООРУЖЕНИЙ

STRUCTURAL MECHANICS OF ENGINEERING CONSTRUCTIONS AND BUILDINGS

ISSN 1815-5235 (Print), 2587-8700 (Online)

HTTP://JOURNALS.RUDN.RU/STRUCTURAL-MECHANICS

АНАЛИТИЧЕСКИЕ И ЧИСЛЕННЫЕ МЕТОДЫ РАСЧЕТА КОНСТРУКЦИЙ

ANALYTICAL AND NUMERICAL METHODS OF ANALYSIS OF STRUCTURES

DOI: 10.22363/1815-5235-2023-19-2-119-129
EDN: KQOOOY
UDC 004.94:624.014

RESEARCH ARTICLE / НАУЧНАЯ СТАТЬЯ

Use of interpolation methods for modeling the stress-strain state of operated oil storage tanks

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Article history
Received: January 17, 2023
Revised: March 24, 2023
Accepted: March 27, 2023

Abstract. The aim of the research is the comparison of two approaches for computer modeling of the stress-strain state of thin-walled shells of engineering structures, considering the imperfections of the geometric shapes arising due to their operation. The object of the study is the operated steel vertical cylindrical reservoir with imperfections of the geometric shape intended for storage of petroleum products. The first, so-called classical, approach provides geometric modeling of the surface of the tank's shell with the subsequent import of the geometric model into one of the systems of finite element analysis to calculate the stress-strain state of the structure and determine its technical condition, and the possibility of further operation. The geometric modeling of the shell surface with imperfections was performed using a two-dimensional interpolation method based on the 1st order smoothness outlines implemented in the point calculus. The calculation of the stress-strain state of the shell was carried out in the SCAD Office computer complex, taking into account geometric and structural non-linearity on the basis of the octahedral tangential stress theory. The second approach assumes modeling of an array of functions of vertical deflection of the tank wall by means of interpolation, solution of an array of differential equations of the elastic cylindrical shell under axisymmetric loading, improved by introduction of vertical deflection functions of the wall, followed by two-dimensional interpolation and analysis of the deformed state of the shell based on displacements arising in the tank wall from the hydrostatic load. As a result of the effective use of two-dimensional interpolation in the process of implementing the second approach, it was possible to achieve a significant increase in the speed of the numerical solution while maintaining sufficient accuracy for engineering calculations.

Keywords: computer modeling, stress-strain state, thin-walled cylindrical shell, operated tank, interpolation

For citation
Konopatskiy E.V., Krysko A.A., Shevchuk O.A. Use of interpolation methods for modeling the stress-strain state of operated oil storage tanks. *Structural Mechanics of Engineering Constructions and Buildings.* 2023;19(2):119–129. http://doi.org/10.22363/1815-5235-2023-19-2-119-129

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Использование интерполяционных методов для моделирования напряженно-деформированного состояния эксплуатируемых резервуаров для хранения нефтепродуктов

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История статьи
Поступила в редакцию: 17 января 2023 г.
Доработана: 24 марта 2023 г.
Принята к публикации: 27 марта 2023 г.

Для цитирования
Konopatskiy E.V., Krysko A.A., Shevchuk O.A. Use of interpolation methods for modeling the stress-strain state of operated oil storage tanks // Строительная механика инженерных конструкций и сооружений. 2023. Т. 19. № 2. С. 119–129. http://doi.org/10.22363/1815-5235-2023-19-2-119-129

Аннотация. Цель исследования – сравнение двух подходов к компьютерному моделированию напряженно-деформированного состояния тонкостенных оболочек инженерных сооружений с учетом несовершенств геометрической формы, возникающих в результате их эксплуатации. Объект исследования – эксплуатируемый стальной вертикальный цилиндрический резервуар для хранения нефтепродуктов с несовершенствами геометрической формы. Первый, так называемый классический, подход предусматривает геометрическое моделирование поверхности оболочки резервуара с последующим импортом геометрической модели в одну из систем конечно-элементного анализа для расчета напряженно-деформированного состояния конструкции и определения ее технического состояния, а также возможности дальнейшей эксплуатации. Геометрическое моделирование поверхности оболочки с несовершенствами выполнено методом двумерной интерполяции на основе обводов 1-го порядка гладкости, реализованной в точечном исчислении. Расчет напряженно-деформированного состояния оболочки произведен в вычислительном комплексе SCAD Office с учетом геометрической и конструктивной нелинейности на основе теории октаэдрических касательных напряжений. Второй подход предусматривает моделирование массива функций отклонения стенки резервуара от вертикали с помощью интерполяции, решение массива дифференциальных уравнений упругой цилиндрической оболочки при осесимметричном нагружении, усовершенствованных за счет введения функций отклонения стенки от вертикали, с последующей двумерной интерполяцией и анализом деформированного состояния оболочки на основе радиальных перемещений, возникающих в стенке резервуара от действия гидростатической нагрузки. В результате эффективного использования двумерной интерполяции в процессе реализации второго подхода удалось достичь значительного повышения быстродействия численного решения при сохранении достаточной для инженерных расчетов точности.

Ключевые слова: компьютерное моделирование, напряженно-деформированное состояние, тонкостенная цилиндрическая оболочка, эксплуатируемый резервуар, интерполяция

1. Introduction

Modern studies of the stress-strain state of various structures cannot be represented without the use of mathematical, computer and other models [1–4]. Such studies are conducted throughout the entire lifecycle of a structure, starting from its design and ending with its utilization. The main means of such modeling are multivariate interpolation and approximation [5–7]. A general approach to modeling, without regard to the method of implementation, includes developing a geometric model [8]; developing a computer model [8]; forming a finite-element network [9]; and solving a system of differential equations using the finite-element method [10; 11]. The same approach is valid not only for modeling the stress-strain state, but also for the study of thermal, snow,

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wind and other types of loads. Each step of its implementation uses the tools of interpolation and approximation. But, considering the large amount of calculations required to solve the system of differential equations with accuracy that is sufficient for practical use, various use of these tools can significantly affect the speed of calculations, which is a significant factor. This raises a new problem of efficient use of technologies of parallel computations, the most effective of them nowadays are graphical processors [12; 13].

It is important to realize that different assumptions and approximations are often used at each step of the modeling process, starting from the creation of the geometric model to the direct use of the finite element method. For that reason, the final model will always be an approximate one, but when the number of finite elements is large and their size approaches to infinitesimal value, it can have high accuracy, which is more than sufficient for engineering calculations. For example, already at the stage of geometric modeling of the structural shell, you can use various continuous or piecewise curves (outlines, splines, etc.) that will pass through the node points of interpolation, but have different curvature between them. They are all valid from the modeling point of view with respect to the original data and they have a right to exist, but the result of the finite element analysis will be different, because the curvature between the node points of the interpolation is different. It is not always possible to determine which method is more precise. Of course, it is possible to compare the obtained models with each other. But it is difficult to choose an etalon model for the comparison. Based on these reasons this article considers the possibility of using multidimensional interpolation and approximation techniques to achieve a significant increase in the speed of calculations without significant loss of accuracy of simulation results by the example of modeling the stress state of the operated tank for the storage of petroleum products.

2. Materials and methods

The research was conducted on a model of the tank No. 1 for storage of gasoline with 1000 m^3 capacity of integrated assembly point “Beshevsky”, which was built based on a typical project TP 704-1-54 “Steel vertical cylindrical tank for oil and petroleum products with capacity of 1000 m^3”.[1]

All structural initial data required for computer modeling and numerical study of the tank, we accept according to the typical project TP 704-1-54. The initial paremeters for modeling are the geometric dimensions of the tank (tank radius r = 6.165 m; thickness of the shell h = 0.005 m), the height of the liquid level in the tank (d = 8.44 m), the physical properties of the stored liquid (usually tanks are tested with water, so the density of water was used γ = 1000 kg/m^3).

As for the original geometric information, it is not enough to have a typical project, because due to objective and subjective reasons, the actual surface of the tank is always different from the project. These reasons are various loads (the structure's own weight, hydrostatic pressure, vacuum, wind and snow loads), manufacturing errors, violation of operating conditions. To compensate the missing information, we will use the survey and assessment of the technical condition of tank No. 1 for storage of gasoline with volume of 1000 m^3 in integrated assembly point “Beshevsky”, which was conducted by the “Donbass diagnostic center of building structures, buildings and structures” of the Donbas National Academy of Civil Engineering and Architecture.[2]

The first of the proposed approaches can be classified as a classical one [14–17]. It involves determining the geometric shape of the shell surface of the studied engineering structures. Since this surface takes an irregular shape after usage, there arises the specific problem of determining such an irregular surface. The works [18; 19] use two-dimensional outlines of the first degree of regularity to determine the irregular surface of the tank shell for the storage of oil and petroleum products. In order to implement them, a special geometric scheme was developed (Figure 1), which takes into account the deviation of the wall from the vertical along the tank’s circumference.

In accordance with this geometrical scheme, geometrical and computational algorithms for modelling the irregular surface were developed. A new problem arose at this stage, which was how to form a finite-element network of the desired size from the obtained compound surface. As a result, a special program was written in VBA, which modeled the surface of the tank shell as an array of 3DDFace objects based on 4 points with visualization of the model in the AutoCAD software package. This allowed to develop a calculation scheme of the tank in the computational complex SCAD Office (Figure 2) and conduct a finite-element analysis of the tank for the storage of petroleum products, with the account of imperfections of its geometric shape.

[1] TP 704-1-54 “Steel vertical cylindrical tank for oil and petroleum products with capacity of 1000 m^3”. (In Russ.) Available from: https://meganorm.ru/Data2/1/4293782/4293782405.pdf (accessed: 25.04.2022).

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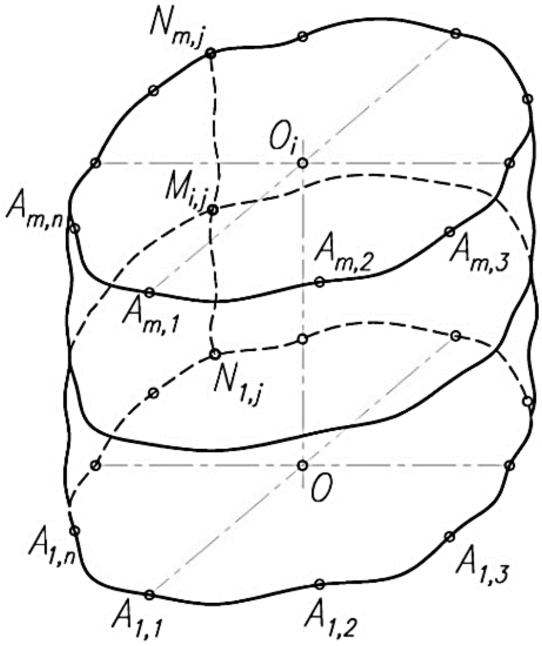


Figure 1. Geometric scheme of the tank surface with imperfections of the geometric shape

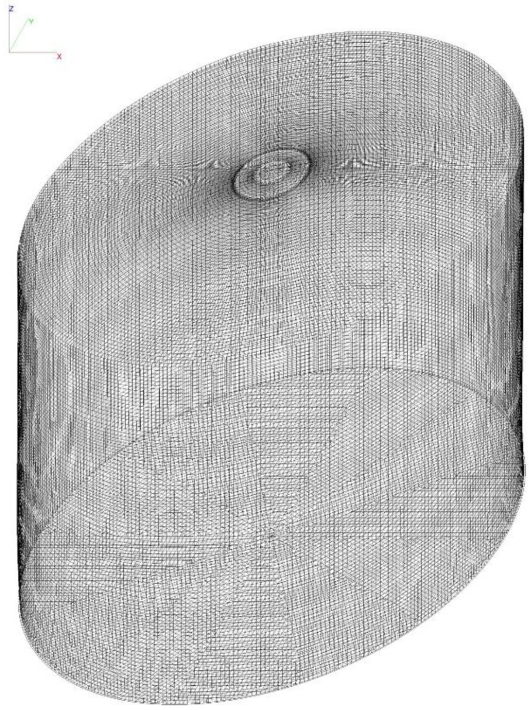


Figure 2. Calculation diagram for a tank of 1000 m^3 with geometric imperfections

Here another problem arises, which is the necessity to take into account not only geometrical, but also structural non-linearity. Even insignificant imperfections of geometric shape, represented by the deviation of tank wall from the vertical position, leads to the fact that the loading of the shell becomes non-axisymmetric. In addition, the presence of geometric nonlinearity in this case leads to the necessity to take also into account the structural nonlinearity associated with the change of the initial calculation scheme under the action of hydrostatic load occurring during the filling of the tank with liquid.

In order to compensate the impact of structural nonlinearity on the tank walls, a stage-by-stage tank loading scheme was implemented during modeling [19]. Calculations were carried out in the SCAD Office computational complex in accordance with the strength theory of octahedral tangential stresses (Huber – Hencki – Meiser energy theory). As a result, an analysis of the stress state of a petroleum product storage tank with imperfections was performed (Figure 3).

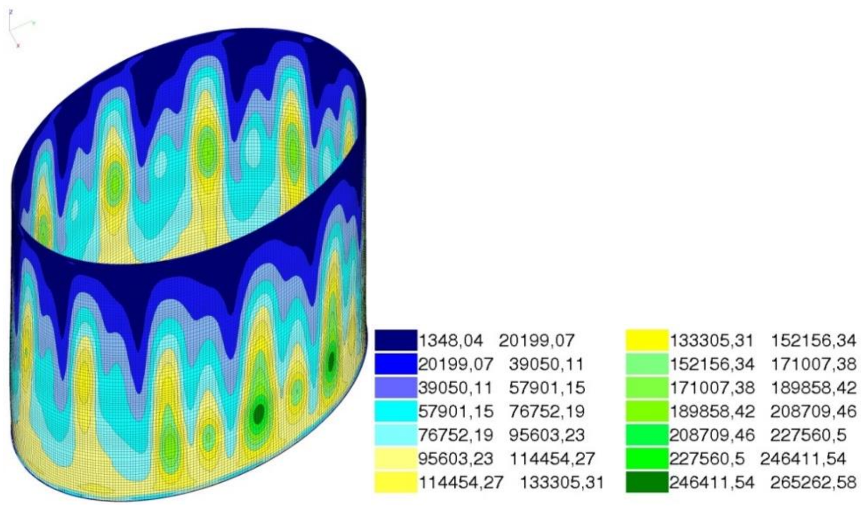


Figure 3. Values of the adjusted stresses in the tank wall under hydrostatic load with account of geometric and structural nonlinearity, kN/m^2

The disadvantages of this approach are large amount of calculations and, as a result, large time costs for their implementation. Calculation of the stress-strain state of the tank according to the calculation scheme (Figure 3), which contains a total of 65 854 finite elements in the form of rectangular plates, taking into account geometric and structural nonlinearity, took over 25 hours on a computer running Intel Core i5-2400, which for engineering surveys is long enough. Another disadvantage is that a model with many finite elements is quite difficult to operate in the AutoCAD software package.

It is possible to significantly reduce the calculation time by considering each of the 12 sections of the tank separately and then combining them into a common model using interpolation. It means to apply two-dimensional interpolation not to build a geometric model of the tank shell surface with geometric imperfections, but to build a response surface based on radial displacements arising under the action of hydrostatic load.

The difficulty is that the existing model for determining the stress state of an elastic cylindrical shell under axisymmetric loading [20; 21] is an idealized one. In our case, the presence of deviations of the tank wall from the vertical, leads to the fact that the loading of the shell is not axisymmetric. To consider this, the existing differential equation of equilibrium of the tank shell element was improved by introducing a function of initial deflections of the cylindrical tank from the vertical δ = δ(x), as shown in [22]:

$$D\frac{d^4w}{dx^4}+\frac{kEh\left(w+\delta\right)}{r^2\left(1-\frac{\alpha\mu}{2}\right)}=\gamma g\left(x-d\right), \tag{1}$$

where $w = w(x)$ – the calculated function of radial displacements from hydrostatic load; x – the coordinate of the wall in height, measured from the tank's butt weld, m; r – tank radius, m; h – tank wall thickness, m; δ = δ(x) – function of the initial tank deviations from the vertical; k – correcting factor, taking into account geometric and structural nonlinearity, as well as the stresses arising in the upper band of the shell due to its interaction with the tank roof (can be calculated by formula 2); $E = 2.1\cdot10^{11}$ Pa – Young's modulus for steel; μ = 0.3 – Poisson's ratio; α – the parameter, which at uniaxial stress state is taken equal to 0, and at internal gas pressure in a closed cylindrical vessel is taken equal to 0.5; $D=\frac{Eh^3}{12\left(1-\mu^2\right)}$ – the cylindrical stiffness, kg·m; γ – the density of the stored liquid, kg/m^3; $g = 9.81$ m/s^2 – acceleration of free fall; d – the height of the liquid level in the tank, m.

$$k=9.215\cdot10^{-10}\varphi^4-6.203\cdot10^{-7}\varphi^3+0.0001\varphi^2-0.0021\varphi+0.3888, \tag{2}$$

where φϵ(0; 360) – the angle around the circle of the tank.

In the same paper it is shown that the mathematically exact solution presented in [20; 21] for the differential equation with account of improvement gives significant inaccuracies, so its solution is performed by numerical method using geometrical interpolators [23], by analogy with the finite superelements method [24–26]. It allows, by analogy with the Isogeometric method [27–30], to eliminate the necessity of coordinating geometric information in the process of interaction between CAD and FEA systems by using geometric interpolators both for geometric modeling and for approximating the numerical solution of differential equations.

As a result, an array of functions characterizing the deviation of the tank wall from the vertical was formed, which was used to obtain an array of functions – numerical solutions of differential equations. The resulting solutions are curves located around the circle of the tank (Figure 4). These lines were used as guidelines to generate a surface of response characterizing the radial displacements arising in the tank wall from the action of hydrostatic load. A line of a closed outline of the first order of smoothness was used as the generating surface of the response. To visualize the results of modeling the deformed state of the tank shell, the response surface was represented by the means of the computer algebra system Maple in the form of a functional dependence of color on the sign and value of the radial displacements and was applied to the model of the surface of the storage tank for petroleum products (Figure 4).

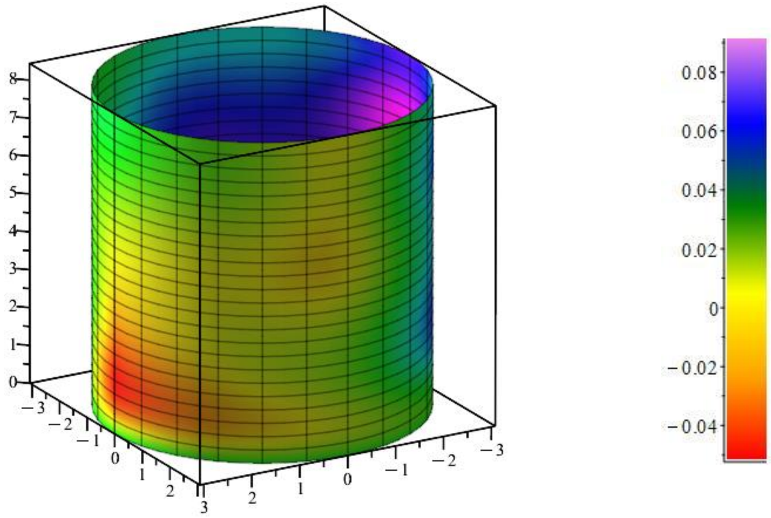


Figure 4. Values of radial displacements in the tank wall from the hydrostatic load, m

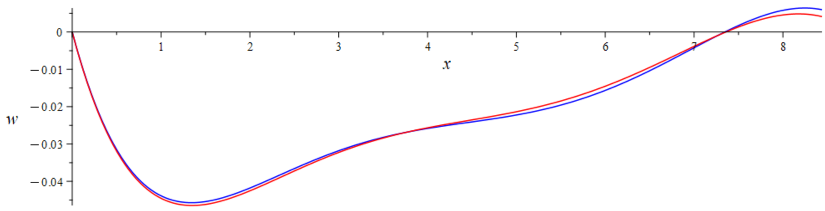
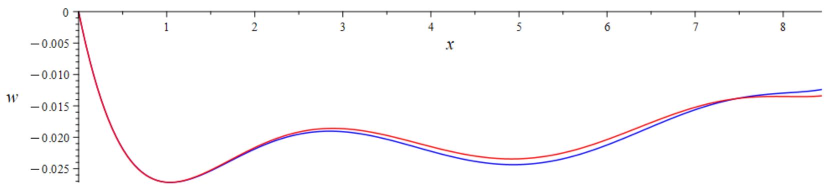
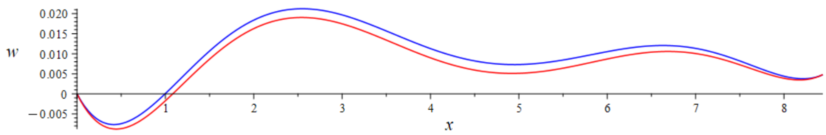
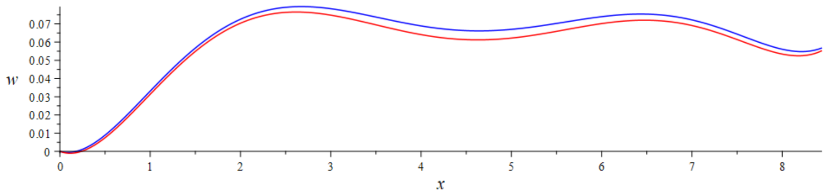
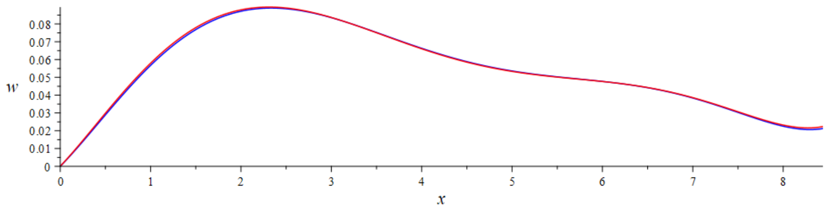
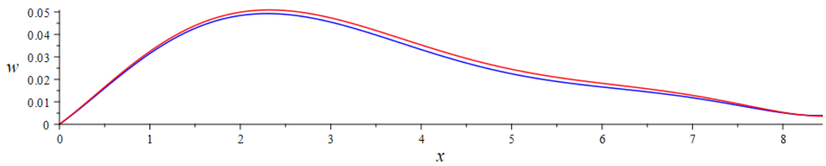
3. Results and discussion

The accuracy of the simulation results was established by comparing of both approaches with each other. In total, a comparison was made for 12 curved lines along the circumference of the tank, characterizing the total action of initial deflections and radial displacements from the action of hydrostatic load (see Table). Comparisons were made using two methods, the first of which is based on a visual comparison of the obtained curved lines. The second method is an innovative one [31] and involves the discretization of lines, as a result of which two sets of dots are formed. Comparison of the obtained sets of dots with each other was carried out using the coefficient of determination. As a result of the numerical comparison for all 12 lines, high values of the coefficient of determination were obtained, and this confirms the high accuracy of the calculations of the strained state of the tank shell. Some of the comparison results of the numerical solution of the differential equation (1) with the use of geometrical interpolants and computer modeling in the computing complex SCAD Office are shown in Table (the line obtained on the basis of the computer model of the tank with imperfections in SCAD Office is shown in blue; the numerical solution with the use of geometrical interpolants [22] is shown in red).

If we compare the results of modeling presented in Figures 3 and 4, they obviously have significant differences. And the matter is not just in the fact that the reduced stresses are applied on the cylindrical surface of the tank in one case (Figure 3), and the radial displacements are applied in the other case (Figure 4). It is possible to pass from displacements in the tank wall to stresses by multiplying by the quotient of the division of the Young's modulus by the radius of the tank, considering the function of the deviation of the wall from the vertical along the height of the tank $\delta = \delta(x)$. The main difference is that only 12 lines along the circumference of the tank were used as an experiment to construct the response surface (Figure 4), on the basis of these lines the validity of the obtained results was confirmed. In contrast, for the implementation of the finite element analysis in SCAD Office (Figure 3), 360 elements were involved in the circumference of the tank. In addition, Figure 4 shows only part of the tank wall up to the level of filling with liquid, implemented based on multiple numerical solution of differential equation (1) in the system of Maple computer algebra. And the same tank was designed as a whole with the roof in the finite element analysis system in SCAD Office (Figure 5).

At the same time, we managed to achieve accuracy of numerical modeling that is sufficient for engineering calculations and significantly increase their speed by using effective interpolation methods for solving the specified problem. Calculation of an array consisting of 12 numerical solutions of differential equations defining 12 guiding lines of the response surface, as well as construction of its form with the help of one-dimensional first-order smoothness contours, considering visualization, takes about 20 seconds even with no parallelization of computational streams. If necessary, the number of guiding lines of the response surface can be significantly increased to achieve greater accuracy of engineering calculations. But even in this case, the processing speed will be much higher than in the classical approach. For example, if we use not 12, but 1000 guiding lines, the calculation time using one core of the central processor will not exceed 1 hour. In this case, the distance between the guiding lines will be less than the size of the finite elements, on the basis of which it took 25 hours to calculate. As a result, not only the computational speed but also the accuracy of engineering calculations will be increased. The proposed method can also be easily implemented to separate computational operations, because it combines the potential of structural methods of geometric modeling, capable to provide separation of geometric constructions by tasks (message passing), and the mathematical apparatus "Point calculus", capable to implement separation by data (data parallel) by means of sub-coordinate calculations, which allows in perspective to use all available computing potential of modern multi-core processors and graphical systems [0].

Comparison of the accuracy of the modeling results

Angle on the circumference of the tank	Graphic visualization of comparison results	The value of the determination coefficient
0º		0.998
60º		0.985
120º		0.936
180º		0.973
240º		0.999
300º		0.99

Another advantage of the second approach is that the numerical solution of the improved differential equation is obtained already taking into account geometric and structural nonlinearity. As a result, there is no need to use the staged loading scheme [19], specially developed so that due to the discretization of the numerical solution in the system of finite element analysis SCAD to consider the initial geometric non-linearity of the deformed tank wall and the structural non-linearity arising in the tank wall in the process of staged filling of the tank with liquid.

According to the conducted research, the engineering methods of inspection of the technical condition of a tank for storage of petroleum products with imperfections in geometric shape, proposed in [18], have been refined. The essence of refinement is to compose and solve an array of differential equations of elastic cylindrical shell under axisymmetric loading, improved by introducing functions of wall deflection from the vertical, with subsequent two-dimensional interpolation of modeling results [23].

4. Conclusion

The general conclusions and results of the study are as follows:

1. Comparison of two approaches to computer modeling of the stress-strain state of thin-walled shells of engineering structures with account of imperfections in geometric shape has been carried out. As a result of effective use of two-dimensional interpolation at the stage of creating the response surface, characterizing the radial movements in the tank wall from the hydrostatic load, it was possible to achieve a sufficient accuracy for engineering calculations of numerical modeling, and significantly increase their speed even without the use of parallel calculations.

2. The specific feature of the proposed method is that the model using the interpolation methods can be easily described by a differential equation of the 4th order, which has a simple numerical solution in the form of a polynome of the 6th degree. In this case, the resulting model considers the stress state of the tank for storage of petroleum products, with account of both geometric and constructive nonlinearity.

3. The interpolation methods of computer modeling of the stress-strain state of thin-walled shells of engineering structures with the imperfections of geometric shape which are described in the paper can also be effectively used for modeling such types of loads as wind load, snow load, self-weight load of the structure, etc., which allows to avoid the need for expensive full-scale experiments, which in some cases are non-profitable or impossible.

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