2023. 19(1). 3–16

СТРОИТЕЛЬНАЯ МЕХАНИКА ИНЖЕНЕРНЫХ КОНСТРУКЦИЙ И СООРУЖЕНИЙ

STRUCTURAL MECHANICS OF ENGINEERING CONSTRUCTIONS AND BUILDINGS

ISSN 1815-5235 (Print), 2587-8700 (Online)

HTTP://JOURNALS.RUDN.RU/STRUCTURAL-MECHANICS

РАСЧЕТ И ПРОЕКТИРОВАНИЕ СТРОИТЕЛЬНЫХ КОНСТРУКЦИЙ

ANALYSIS AND DESIGN OF BUILDING STRUCTURES

DOI 10.22363/1815-5235-2023-19-1-3-16
EDN: FFMQQR
UDC 624.012

RESEARCH ARTICLE / НАУЧНАЯ СТАТЬЯ

Calculation of the formation of normal cracks in a reinforced concrete element based on the deformation theory of plasticity of concrete by G.A. Geniev

Ngoc Tuyen Vu[1], **Natalia V. Fedorova**[2]

[1]*Moscow State University of Civil Engineering (National Research University), Moscow, Russian Federation*
[2]*Research Institute of Building Physics of the Russian Academy of Architecture and Building Sciences, Moscow, Russian Federation*
ngoctuyennd91@gmail.com

Article history
Received: October 7, 2022
Revised: December 21, 2022
Accepted: December 25, 2022

Abstract. The authors present a refined method of determining the moment of cracking in reinforced concrete bar constructions using the diagram of deformation of concrete built on the basis of the deformation theory of plasticity by G.A. Geniev in which the stress and strain invariants of concrete are linked by nonlinear dependences. In the resulting defining equations, the hypothesis of flat sections, as well as the premise of reaching the limit values of concrete deformations on the stretched fibers of the cross-section are used. Stresses in concrete are determined through deformation values in accordance with the nonlinear deformation diagram of concrete. On the basis of the assumptions accepted, analytical dependences for determining the moment of cracking in the sections of bending elements with single and double reinforcement have been acquired. The formulas obtained were used in the analysis of various factors influence on crack resistance of bendable reinforced concrete elements. It was found out that the moment of crack formation practically does not change when percentage of reinforcement of longitudinal tensile or compressed reinforcement changes. The most effective method of crack resistance improvement is the increase of concrete strength. The proposed methodology is verified by comparison with experimental results on reinforced concrete prototypes. It is concluded that the use of the diagram of nonlinear deformation of concrete on the basis of the theory of plasticity by G.A. Geniev allows to estimate more strictly the crack resistance of reinforced concrete rod elements.

Keywords: deformation diagrams, plasticity, relative deformations, reinforced concrete structures, cracking moment

For citation
Vu N.T., Fedorova N.V. Calculation of the formation of normal cracks in a reinforced concrete element based on the deformation theory of plasticity of concrete by G.A. Geniev. *Structural Mechanics of Engineering Constructions and Buildings.* 2023;19(1):3–16. http://doi.org/10.22363/1815-5235-2023-19-1-3-16

Ngoc Tuyen Vu, Candidate of Technical Sciences, senior lecturer, Department of Fundamental Education, Moscow State University of Civil Engineering (National Research University), 26 Yaroslavskoye Shosse, Moscow, 129337, Russian Federation; ORCID: 0000-0001-5755-8345, Scopus Author ID: 57215934802, eLIBRARY SPIN-code: 5948-4496; ngoctuyennd91@gmail.com

Natalia V. Fedorova, Doctor of Technical Sciences, Professor, leading researcher, Department No. 40 "Perspective Priority Directions in Construction Equipment", Research Institute of Building Physics of the Russian Academy of Architecture and Building Sciences; 21 Lokomotivny Proezd, Moscow, 127238, Russian Federation; ORCID: 0000-0002-5392-9150, Scopus Author ID: 57196437054, ResearcherID: O-8119-2015, eLIBRARY SPIN-code: 3365-8320; fedorovanv@mgsu.ru

Расчет по образованию нормальных трещин в железобетонном элементе на основе деформационной теории пластичности бетона Г.А. Гениева

Н.Т. Ву[1], **Н.В. Федорова**[2]

[1]*Национальный исследовательский Московский государственный строительный университет, Москва, Российская Федерация*
[2]*Научно-исследовательский институт строительной физики Российской академии архитектуры и строительных наук, Москва, Российская Федерация*
✉ ngoctuyennd91@gmail.com

История статьи
Поступила в редакцию: 7 октября 2022 г.
Доработана: 21 декабря 2022 г.
Принята к публикации: 25 декабря 2022 г.

Для цитирования
Vu N.T., Fedorova N.V. Calculation of the formation of normal cracks in a reinforced concrete element based on the deformation theory of plasticity of concrete by G.A. Geniev // Строительная механика инженерных конструкций и сооружений. 2023. Т. 19. № 1. С. 3–16. http://doi.org/10.22363/1815-5235-2023-19-1-3-16

Аннотация. Представлена уточненная методика определения момента трещинообразования в железобетонных стержневых конструкциях с использованием диаграммы деформирования бетона, построенной на основе деформационной теории пластичности Г.А. Гениева, в которой инварианты напряженного и деформированного состояния бетона связаны между собой нелинейными зависимостями. В полученных определяющих уравнениях использованы гипотеза плоских сечений, а также предпосылка о достижении на растянутых волокнах поперечного сечения предельных значений деформаций бетона. Напряжения в бетоне определяются через значения деформаций в соответствии с нелинейной диаграммой деформирования бетона. На основе принятых предпосылок получены аналитические зависимости для определения момента трещинообразования в сечениях изгибаемых элементов с вариантами одиночного и двойного армирования. Полученные формулы применялись при анализе влияния различных факторов на трещиностойкость изгибаемых железобетонных элементов. Установлено, что при изменении процента армирования продольной растянутой или сжатой арматуры момент образования трещин практически не меняется. Наиболее эффективным методом повышения трещиностойкости является увеличение прочности бетона. Предложенная методика верифицирована сравнением с экспериментальными результатами на железобетонных опытных образцах. Сделан вывод о том, что использование диаграммы нелинейного деформирования бетона на основе теории пластичности Г.А. Гениева позволяет более строго оценить трещиностойкость железобетонных стержневых элементов.

Ключевые слова: диаграммы деформирования, пластичность, относительные деформации, железобетонные конструкции, момент трещинообразования

Introduction

It is known that in reinforced concrete structures under the action of loads and impacts, the formation of cracks is allowed due to partial or complete removal of the stretched concrete zone from the work [1–4]. The work of many scientists, such as V.I. Murashev [5], A.A. Gvozdev [6], A.S. Zalesov [7], V.N. Baikov [8], N.I. Karpenko [9], E.N. Kodysh [10], N.N. Trekin [11], V.A. Eryshev [12; 13], Vl.I. Kolchunov [14], A.G. Tamrazyan et al. [15] was devoted to the solution of this problem in different periods of time. A number of methods has been developed for determining the moment of crack formation, and successfully implemented in regulatory documents of different generations. Nevertheless, accurate prediction of the crack formation process has not lost its relevance and is important for improving the design of safe reinforced concrete structures, and in some cases it is critical and excludes the possibility of their further operation (for example, waterproof structures, structures operated in ground water, reservoirs, silos, coating shells [16–18], etc.). The accurate determination of the moment of crack formation for structures made of high-strength reinforced concrete and fiber reinforced concrete is of particular importance [19–22]. As shown in [23] the range of deformation to failure is relatively short and does not exceed 25–30% in such structures after the formation of cracks. Such studies

Ву Нгок Туен, кандидат технических наук, старший преподаватель, кафедра фундаментального образования, Национальный исследовательский Московский государственный строительный университет, Российская Федерация, 129337, Москва, Ярославское шоссе, д. 26; ORCID: 0000-0001-5755-8345, Scopus Author ID: 57215934802, eLIBRARY SPIN-код: 5948-4496; ngoctuyennd91@gmail.com

Федорова Наталия Витальевна, доктор технических наук, профессор, ведущий научный сотрудник, отдел № 40 «Перспективные приоритетные направления в строительной технике», Научно-исследовательский институт строительной физики Российской академии архитектуры и строительных наук, Российская Федерация, 127238, Москва, Локомотивный пр-д, д. 21; ORCID: 0000-0002-5392-9150, Scopus Author ID: 57196437054, ResearcherID: O-8119-2015, eLIBRARY SPIN-код: 3365-8320; fedorovanv@mgsu.ru

are also relevant for conventional mass-designed structures, since the formation of cracks entails a significant change in the parameters of the limit states of the second group – stiffness, crack opening, which in many cases become decisive when assigning reinforcement and the level of prestressing. In this regard, studies aimed at clarifying the level of load at which cracks form in the structure continue to be relevant

In the previous Russian Construction Standards and Regulations (SNiP) 2.03.01-84*[1] the structural model for determining the force perceived by the cross-sections of the bending elements normal to the longitudinal axis during the formation of cracks was built on the basis of the following provisions. The hypothesis of flat sections (Bernoulli's hypothesis) is adopted when the calculated section is deformed. It was assumed that at the moment of cracking, the relative deformation of the extreme stretched fiber of concrete reaches a maximum value equal to $2R_{\text{bt,ser}} / E_b$, and the stress in tensile concrete, regardless of its deformations, was assumed to be constant and equal to the tensile strength of concrete $R_{\text{bt,ser}}$. The stresses in the concrete at the compressed zone are determined taking into account the elastic deformations of the concrete, i.e. the stress diagram in the compressed zone has a triangular shape (Figure 1, *a*). According to the accepted provisions, the moment perceived by the section normal to the longitudinal axis of the element during the formation of a crack is determined by the formula:

$$M_{\text{crc}} = R_{\text{bt,ser}} W_{\text{pl}}. \tag{1}$$

The concept of elastic-plastic moment of resistance is introduced in order to take into account plastic deformations of tensile concrete W_{pl}:

$$W_{\text{pl}} = \frac{2\left(I_{b0} + \alpha I_{so} + \alpha I'_{so}\right)}{h - x} + S_{bo}. \tag{2}$$

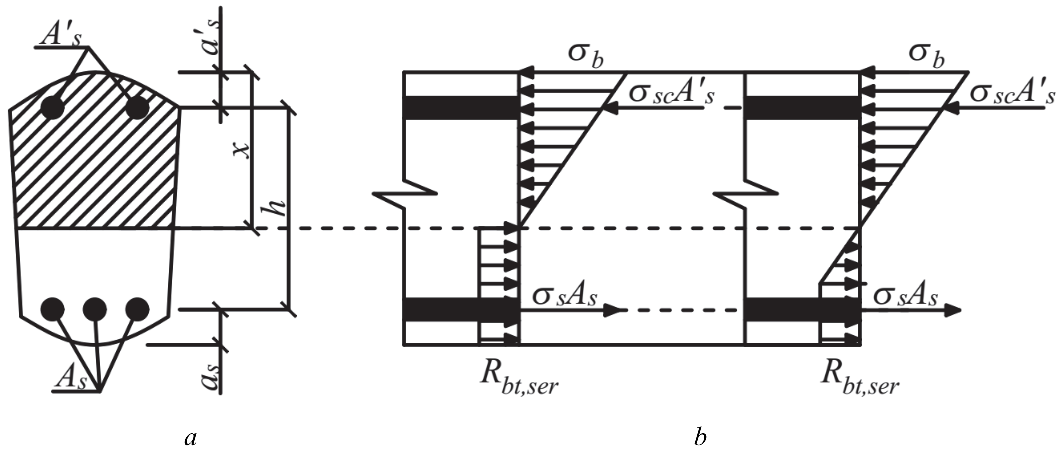


Figure 1. Schemes of the stress diagram in the cross section of the element when calculating it according to the formation of cracks normal to the longitudinal axis of the element: *a* – according to SNiP 2.03.01.84*; *b* – according to SP 63.13330.2018[2]

In the new code of rules currently in force SP 63.13330.2018, in contrast to SNiP 2.03.01.84*, some excellent starting points for constructing a calculation model are adopted: the stress diagram in the tensile zone of concrete takes a trapezoidal shape with stresses not exceeding the design values of concrete resistance stretching $R_{\text{bt,ser}}$; the relative deformation of the extreme stretched fiber of concrete is taken equal to its limit value $\varepsilon_{\text{bt,ult}} = 0{,}00015$ (Figure 1, *b*).

From the point of view of the theory of elasticity, the above approaches for determining the moment of cracking in the cross section of reinforced concrete elements violate the generalized Hooke's law for concrete [24]. It has also been proved by experimental data that the actual diagram of concrete stresses in the tensile and compressed zone differs from the accepted one and has a curvilinear shape even with a low level of loading [25–30]. Therefore, such approaches, described in domestic standards in order to simplify the calculation

[1] SNiP 2.03.01.84*. *Concrete and reinforced concrete structures*. Moscow: USSR Gosstroi Publ.; 1989. 84 p. (In Russ.)

[2] SP 63.13330.2018. *Concrete and reinforced concrete structures. Main provisions*. Moscow: Ministry of Construction and Housing and Communal Services of the Russian Federation; 2018. 124 p. (In Russ.)

apparatus, are conditional. In this regard, it is of interest to quantify the noted assumptions and develop a variant of a more rigorous calculation model, taking into account the plastic deformations of concrete to determine the limiting force perceived by the sections of bending elements normal to the longitudinal axis during the formation of cracks. The solution of the problem under consideration in this paper is based on the deformation theory of plasticity of concrete by G.A. Geniev.

Method

The stress-strain state of a bent reinforced concrete element before the formation of cracks will be determined based on the following assumptions:

1. Sections remain flat during bending, i.e. deformations along the height of the element change according to a linear law [31].
2. The stresses in the concrete of the compressed and tensile zones are determined taking into account the inelastic deformations described by the deformation theory of plasticity of concrete by G.A. Geniev [32; 33].
3. The condition for the formation of a crack in the section is the equality of the limiting deformations on the stretched side of concrete to the value of the limiting tensile strength of concrete $\varepsilon_{bt,ult}$.
4. Stresses in tensile reinforcement are taken depending on the relative deformations as for an elastic body.

In accordance with the adopted second hypothesis, the invariants of the stressed and deformed state of concrete are interconnected by nonlinear dependencies:

$$T = G_0\left(1-\frac{\Gamma}{2\Gamma_s}\right)\Gamma, \tag{3}$$

where the initial shear modulus G_0 is determined by the formula

$$G_0 = \frac{E_b}{2(1+\nu)}; \tag{4}$$

Γ, Γ_s – intensity of shear deformations and its limiting value respectively.

To determine the moment of formation of normal cracks, consider the case of pure bending of a reinforced concrete rod, for which only normal stresses occur in the cross section. In this case, we have the following boundary conditions: $\sigma_1 \neq 0,\ \sigma_2 = \sigma_3$.

Let us express the intensity of concrete shear deformations and its limiting value in terms of the main deformations:

$$\Gamma = \sqrt{\frac{2}{3}}\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2} = \sqrt{\frac{2}{3}}\sqrt{(\varepsilon_1-\nu\varepsilon_1)^2+(\nu\varepsilon_1-\nu\varepsilon_1)^2+(\nu\varepsilon_1-\varepsilon_1)^2} = \frac{2}{\sqrt{3}}(1-\nu)\varepsilon_1; \tag{5}$$

$$\Gamma_s = \frac{2}{\sqrt{3}}(1-\nu)\varepsilon_{1,\text{ult}}. \tag{6}$$

Let us express in (1) the intensity of shear stresses, the square of which is numerically equal to the second invariant of the stress deviator:

$$T = \frac{1}{\sqrt{6}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} = \frac{1}{\sqrt{6}}\sqrt{(\sigma_1-0)^2+(0-0)^2+(0-\sigma_1)^2} = \frac{1}{\sqrt{3}}\sigma_1. \tag{7}$$

Putting formulas (4)–(7) in (3) we get a nonlinear relationship between the main stress and the main deformation of concrete ε_1 in cross section with pure bending:

$$\sigma_1 = E_b\left(1-\frac{\varepsilon_1}{2\varepsilon_{1,\mathrm{ult}}}\right)\varepsilon_1. \tag{8}$$

Formula (8) for compressed and tensile concrete can be written as

$$\sigma_b(\varepsilon_b) = E_b\left(1-\frac{\varepsilon_b}{2\varepsilon_{b,\mathrm{ult}}}\right)\varepsilon_b; \tag{9}$$

$$\sigma_{\mathrm{bt}}(\varepsilon_{\mathrm{bt}}) = E_b\left(1-\frac{\varepsilon_{\mathrm{bt}}}{2\varepsilon_{\mathrm{bt,ult}}}\right)\varepsilon_{\mathrm{bt}}. \tag{10}$$

Here, the ultimate strains of concrete in tension and compression are determined by the formulas

$$\varepsilon_{b,\mathrm{ult}} = 2\frac{R_{b,\mathrm{ser}}}{E_b}; \tag{11}$$

$$\varepsilon_{\mathrm{bt,ult}} = 2\frac{R_{\mathrm{bt,ser}}}{E_b}. \tag{12}$$

In accordance with the accepted third and fourth prerequisites for calculating the stress and strain diagrams in the cross section of a reinforced concrete element, they have the form shown in Figure 2.

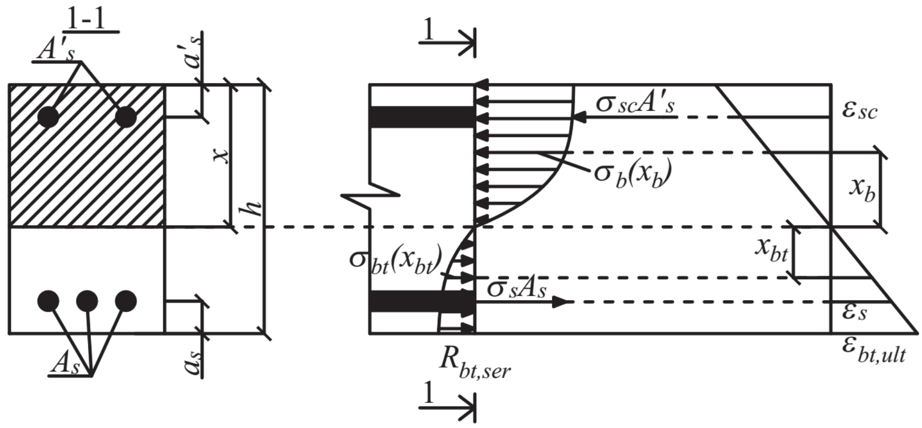


Figure 2. The adopted scheme for the distribution of strains and stresses in the section of the element when determining the moment of cracking

Using the accepted first premise, we determine the deformation of compressed concrete at a distance *xb* from the neutral axis:

$$\varepsilon_b = \frac{x_b}{h-x}\varepsilon_{\mathrm{bt,ult}}. \tag{13}$$

Similarly, we determine the deformation of tensile concrete at a distance x_{bt} from the neutral axis:

$$\varepsilon_{\mathrm{bt}} = \frac{x_{\mathrm{bt}}}{h-x}\varepsilon_{\mathrm{bt,ult}}. \tag{14}$$

Using formulas (11) and (13) into formula (9), we obtain the dependence of the stress of compressed concrete on x_b:

$$\sigma_b(x_b)=\frac{R_{\mathrm{bt,ser}}x_b\left[2R_{b,\mathrm{ser}}\left(h-x\right)-R_{\mathrm{bt,ser}}x_b\right]}{R_{b,\mathrm{ser}}\left(h-x\right)^2}. \tag{15}$$

Similarly, putting (12) and (14) into formula (10), we obtain the stress of tensile concrete, depending on x_{bt}:

$$\sigma_{\mathrm{bt}}(x_{\mathrm{bt}})=\frac{R_{\mathrm{bt,ser}}x_{\mathrm{bt}}\left[2\left(h-x\right)-x_{\mathrm{bt}}\right]}{\left(h-x\right)^2}. \tag{16}$$

We also determine the deformations of tensioned and compressed reinforcement from the first premise about flat sections:

$$\varepsilon_s=\frac{h-x-a_s}{h-x}\varepsilon_{\mathrm{bt,ult}}=\frac{2R_{\mathrm{bt,ser}}}{E_b}\frac{h-x-a_s}{h-x}; \tag{17}$$

$$\varepsilon_{\mathrm{sc}}=\frac{x-a_s^{/}}{h-x}\varepsilon_{\mathrm{bt,ult}}=\frac{2R_{\mathrm{bt,ser}}}{E_b}\frac{x-a_s^{/}}{h-x}. \tag{18}$$

From the condition that the sum of the projections of all longitudinal forces is equal to zero, we obtain the following expression:

$$\sum X=0,\ E_s\varepsilon_s A_s+\int_0^{h-x}\sigma_{\mathrm{bt}}\left(x_{\mathrm{bt}}\right)bd\left(x_{\mathrm{bt}}\right)=\int_0^{x}\sigma_b\left(x_b\right)bd\left(x_b\right)+E_s\varepsilon_{\mathrm{sc}}A_s^{/}. \tag{19}$$

Using formulas (15)–(18) into formula (19) we get

$$\left(1+\beta\right)bx^3+3\left[2\alpha\left(A_s+A_s^{/}\right)+bh\right]x^2+6\left[\alpha\left(A_sa_s-A_s^{/}a_s^{/}\right)-\alpha h\left(2A_s+A_s^{/}\right)-bh^2\right]x+$$
$$+2h\left\{bh^2+3\alpha\left[A_s\left(h-a_s\right)+A_s^{/}a_s^{/}\right]\right\}=0, \tag{20}$$

here,

$$\alpha=\frac{E_s}{E_b};\ \beta=\frac{R_{\mathrm{bt,ser}}}{R_{b,\mathrm{ser}}}. \tag{21}$$

From the solution of the cubic equation (20), the height of the compressed zone of the section is determined. After that, we can determine the moment of internal forces relative to the zero line. It will be equal to the external moment:

$$M_{\mathrm{crc}}=E_s\varepsilon_sA_s\left(h-x-a_s\right)+\int_0^{h-x}\sigma_{\mathrm{bt}}\left(x_{\mathrm{bt}}\right)bx_{\mathrm{bt}}d\left(x_{\mathrm{bt}}\right)+\int_0^{x}\sigma_b\left(x_b\right)bx_bd\left(x_b\right)+E_s\varepsilon_{\mathrm{sc}}A_s^{/}\left(x-a_s^{/}\right). \tag{22}$$

Being used in formula (22) the found formulas for stresses in concrete and deformations in reinforcement, we obtain:

$$M_{\text{crc}} = \frac{2\alpha J_s R_{\text{bt,ser}}}{h-x} + \frac{5S_{\text{bt}} R_{\text{bt,ser}}}{6} + \frac{\beta J_b \left[8R_{b,\text{ser}}\left(h-x\right) - 3R_{\text{bt,ser}} x\right]}{4\left(h-x\right)^2} + \frac{2\alpha J_s^{/} R_{\text{bt,ser}}}{h-x}, \tag{23}$$

где

$$J_s = A_s \left(h - x - a_s\right)^2 \tag{24}$$

moment of inertia of tensile reinforcement to the neutral axis;

$$S_{\text{bt}} = \frac{b\left(h-x\right)^2}{2} \tag{25}$$

static moment of tensioned concrete to the neutral axis;

$$J_b = \frac{bx^3}{3} \tag{26}$$

moment of inertia of compressed concrete to the neutral axis;

$$J_s^{/} = A_s^{/}\left(x - a_s^{/}\right) \tag{27}$$

moment of inertia of compressed reinforcement to the neutral axis.

Let us consider a special case of a single reinforcement. In this case, when $A_s^{/} = a_s^{/} = 0$, the equation for determining the height of the compressed zone takes the form

$$\left(1+\beta\right)bx^3 + 3\left(2\alpha A_s + bh\right)x^2 + 6\left[\alpha A_s\left(a_s - 2h\right) - bh^2\right]x + 2h\left[bh^2 + 3\alpha A_s\left(h - a_s\right)\right] = 0. \tag{28}$$

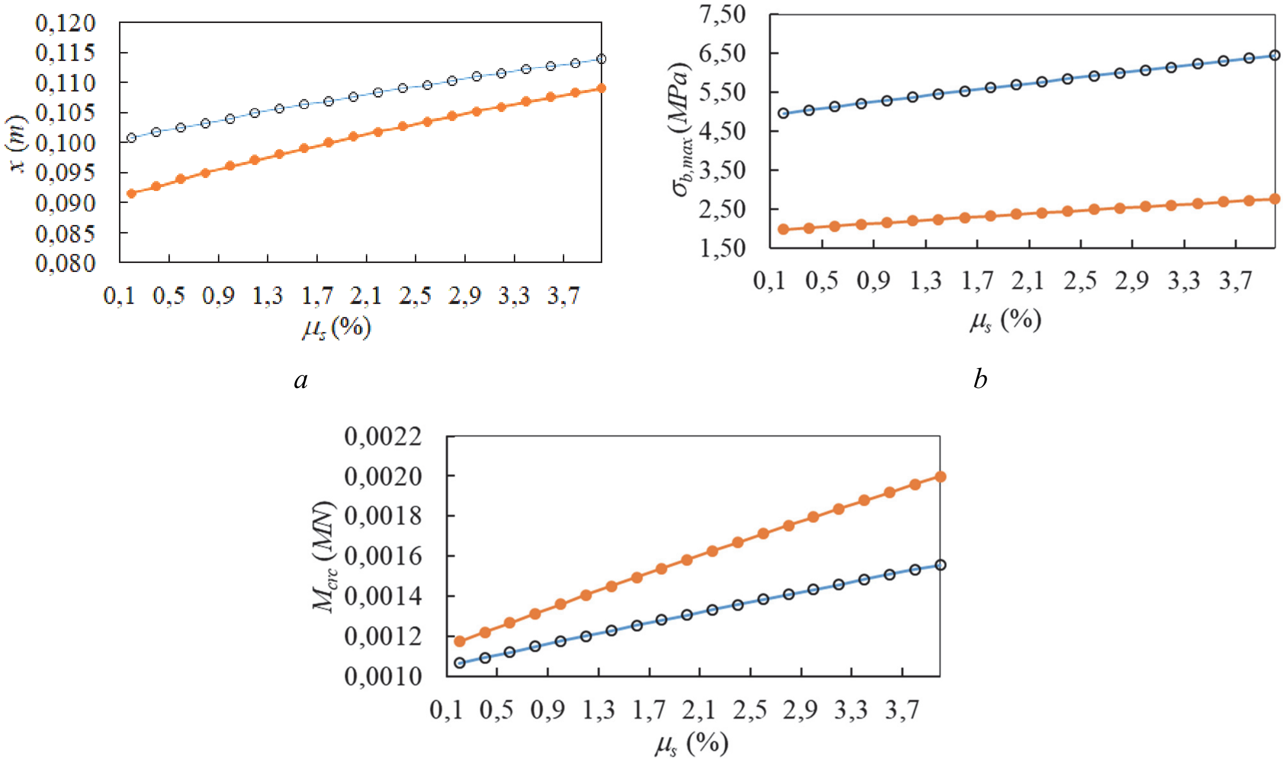
The moment of cracking is determined by the expression:

$$M_{\text{crc}} = \frac{2\alpha J_s R_{\text{bt,ser}}}{h-x} + \frac{5S_{\text{bt}} R_{\text{bt,ser}}}{6} + \frac{\beta J_b \left[8R_{b,\text{ser}}\left(h-x\right) - 3R_{\text{bt,ser}} x\right]}{4\left(h-x\right)^2}. \tag{29}$$

Results and discussion

According to the obtained calculated dependences, the calculation of the moment of crack formation in a reinforced concrete bending element of a rectangular section with section dimensions: height h = 200 mm, width b = 100 mm was performed. A short load action was considered. A500 class reinforcement with modulus of elasticity is adopted $E_s = 2\cdot 10^5$ MPa. At the same time, the percentage of reinforcement of the longitudinal tensile reinforcement varied $\mu_s = A_s / bh$ from 0,2 to 4%, percentage of reinforcement of longitudinal compression reinforcement $\mu_s^{/} = A_s^{/} / bh$ от 0,2 до 4%, and the class of concrete varied from B10 to B100.

By analyzing the results of the calculation, it was established (Figure 3) that with an increase in the percentage of reinforcement μ_s by 20 times, the value of the height of the compressed zone of concrete x slightly increases by 1.18 times when calculated according to the calculated dependencies SP 63.13330.2018 and by 1.13 times when calculated using formula (20). In this case, the relationship between them is linear (Figure 3, *a*).



a *b* *c*

Figure 3. Influence of the percentage of reinforcement of longitudinal tension reinforcement:
a – on the height of the compressed zone of concrete; *b* – maximum stress in compressed concrete $\sigma_{b,\max}$; *c* – moment of cracking;
according to the proposed method; according to SP 63.13330.2018

A similar linear relationship is observed between the percentage of reinforcement with longitudinal tensile reinforcement and the stress in the upper compressed fiber of concrete (Figure 3, *b*). Quantitative maximum compressive stress $\sigma_{b,\max}$ slightly depends on μ_S .

Insignificant changes are observed in the quantitative values of the moment of cracking in the considered reinforced concrete element M_{crc} at different values of reinforcement percentage μ_S . With an increase in the percentage of reinforcement μ_S by 20 times moment of cracking, perceived by the section, calculated according to SP 63.13330.2018, increases by 1.45 times, and calculated by the method under consideration – by 1.66 times. An analysis of the influence of the percentage of reinforcement with longitudinal compressed reinforcement was also carried out μ_S' on the stressed state of the reinforced concrete section at the time of crack formation (Figure 4). It has been established that the height of the compressed zone of concrete *x* decreases with an increase in the percentage of reinforcement μ_S' (see Figure 4, *a*), but this decrease is insignificant and when calculating according to SP 63.1330.2018 and when calculated by formula (18) equals to 1.17 times. The maximum stress in the concrete of the compressed zone is practically independent of the percentage of reinforcement μ_S' (see Figure 4, *b*). In this case, the moment of cracking, determined according to SP 63.1330.2018, increases by 3.6 times. The value of this moment, calculated by the above method, does not change (Figure 4, *c*). The independence of the value of the moment of cracking from the percentage of reinforcement by longitudinal compressive reinforcement can be explained as follows. As the percentage of reinforcement increases μ_S' from the equilibrium equation in the direction of the normal to the cross section (see Figure 2), the height of the compression zone will decrease. In this case, an additional compressive force occurs in the concrete compression zone, caused by the compressed reinforcement. Therefore, the value of the moment of cracking, calculated by integrating the compressive stress in concrete and reinforcement over the area of the compressed zone, will practically not change.

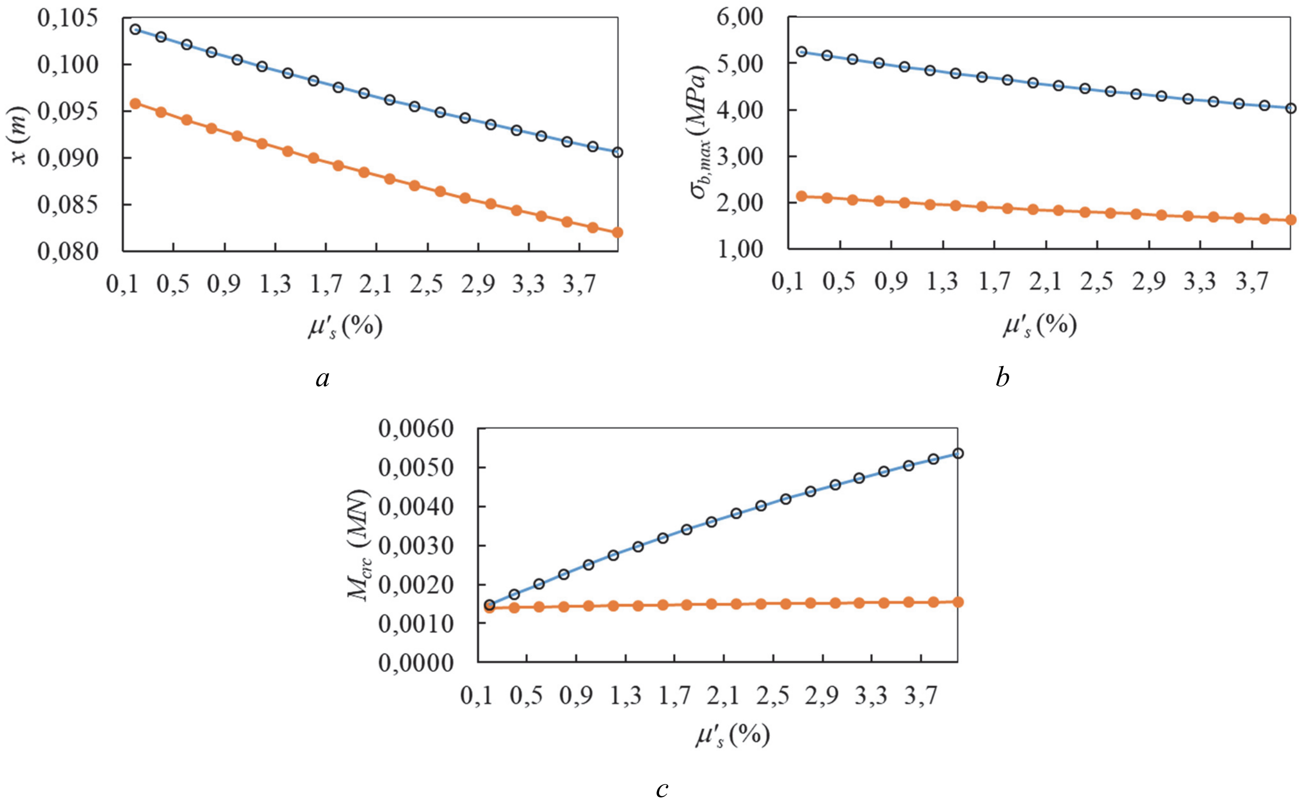


Figure 4. Influence of the percentage of reinforcement of longitudinal compression reinforcement:
a – on the height of the compressed zone of concrete x; *b* – maximum stress in compressed concrete $\sigma_{b,\max}$; *c* – moment of cracking M_{crc};
according to the proposed method; according to SP 63.13330.2018

Analysis of the change in the height of the compressed zone of concrete, the maximum compressive stress in concrete, as well as the moment of cracking, depending on the class of concrete used, showed the following (Figure 5). The height of the compressed zone of the section of the element, calculated according to SP 63.13330.2018 and according to the proposed method, does not depend on the value of the concrete class (Figure 5, *a*)

The value of the maximum compressive stress in concrete $\sigma_{b,\max}$ when changing the class of concrete, it increases significantly according to the proposed method, and when calculating according to SP 63.13330.2018 this value is practically unchanged (Figure 5, *b*).

The values of the moments of cracking in the calculation for SP 63.13330.2018 and according to the proposed method, depending on the class of concrete, they change significantly (Figure 5, *c*). t can be seen from the graphs that with a 10-fold increase in the class of concrete, the moment of cracking increases when calculating according to SP 63.13330.2018 by 3,93 times and when calculating according to the proposed method – 3.95 times.

Investigating the dependence of the cracking moment on the above factors, we can conclude that reinforcement does not play a significant role in improving the crack resistance of reinforced concrete structures. The most effective way to increase the crack resistance of reinforced concrete elements is to increase the class of concrete used. The difference between the moments of cracking, calculated according to SP 63.13330.2018 and according to the proposed method, depending on various factors, varies from 9.3 to 16.7%.

To assess the degree of reliability of the obtained dependences, a numerical comparison was made of the results of calculating the fracture toughness moment calculated by the proposed method with the experimental data of Vl.I. Kolchunov, Al-Hashimi Omar [34], obtained using modern tools, which allowed a more detailed and accurate study of the process of crack formation in reinforced concrete structures. The results of a comparison of experimental data on the moment of crack formation, calculation data obtained by formula (23), and according to the method of SP 63.13330.2018 for beams with a section of 100×250 mm, with a total length of 1200 mm, supported on two supports and loaded with concentrated loads, from ordinary and lightweight concrete are presented in Table.

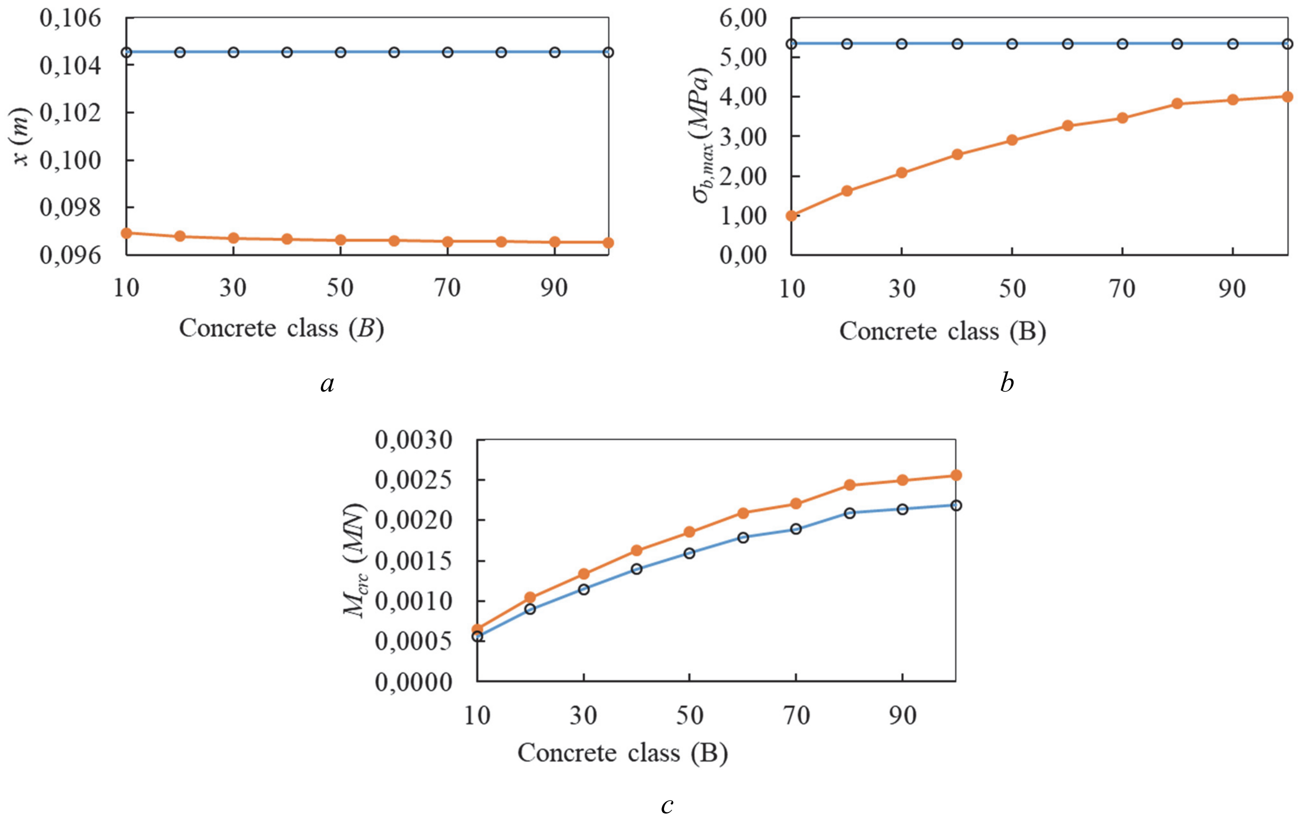


Figure 5. Influence of concrete class:
a – on the height of the compressed zone of concrete; *b* – maximum stress in compressed concrete; *c* – moment of cracking;
according to the proposed method; according to SP 63.13330.2018

As follows from the table, the calculated value of the moment of crack formation according to SP 63.13330.2018 turned out to be significantly underestimated compared to the experimental values by an average of 30% for beam structures made of lightweight autoclaved cellular concrete and by 22.5% for beams made of heavy concrete. When calculating according to the proposed method, the agreement between the calculated and experimental average values of the moment of crack formation is closer.

Calculation results of the cracking moment in bending elements

Construction code	Concrete	Reinforcement	Load of first crack formation, kg	M^{exp}_{crc}	M^{SP}_{crc}		M^{t}_{crc}	
				kH·m	kH·m	Δ_{SP}, %	kH·m	Δ_t, %
Б2-1-2	Heavy concrete B25: R_b = 23 MPa, E_b =25 330 MPa	4Ø10 A240 R_s = 380 MPa	2400	4.8	3.6	25.1	3.9	16.8
Б5-1-2	Heavy concrete B25: R_b = 23 MPa, E_b = 25 330 MPa	4Ø10 A240 R_s = 380 MPa	2300	4.6	3.6	20.3	3.9	15.6
Ба-4	Autoclaved cellular concrete R_b = 3.5 MPa	2Ø8 A240 R_s = 300 MPa	1150	2.3	1.6	30	1.8	20.1

Note: M^{exp}_{crc}, M^{SP}_{crc}, M^{t}_{crc} – the moment of fracturing, respectively experimental, by SP 63.13330.2018 and the proposed method; Δ_{SP} – the discrepancy between the experimental data and the results of calculations by SP 63.13330.2018; Δ_t – the same and the calculation results in accordance with the received formula (23).

From this we can conclude that the use in the calculation of the diagram and analytical dependences of the nonlinear deformation of concrete, obtained on the basis of the theory of plasticity by G.A. Geniev, allows a more rigorous assessment of the crack resistance of bent reinforced concrete elements.

Conclusion

1. An alternative variation of the calculation model was proposed and analytical dependencies were obtained to determine the moment of crack formation in a reinforced concrete bending element reinforced with double reinforcement based on the deformation theory of plasticity of concrete by G.A. Geniev.

2. The numerical analysis of the influence of the percentage of reinforcement of longitudinal tensioned and compressed reinforcement and the class of concrete on the moment of cracking in reinforced concrete bending elements showed the different influence of these factors when using the SP 63.13330.2018 method for calculation and the dependences of the proposed version of the calculation model. The quantitative values of the moment of cracking when calculated according to the proposed formulas with Integrals are higher from 9.3 to 16.7% compared to the calculation according to SP 63.13330.2018. At the same time, the calculated values obtained by the proposed method are closer to the values of the cracking moment obtained experimentally for beams made of heavy and light concretes.

References

1. Al-Fakih A., Hisbany Mohd Hashim M., Alyousef R., Mutafi A., Hussein Abo Sabah S., Tafsirojjaman T. Cracking behavior of sea sand RC beam bonded externally with CFRP plate. *Structures*. 2021;33:1578–1589. http://doi.org/10.1016/J.ISTRUC.2021.05.042

2. Nasser H., Vandewalle L., Verstrynge E. Effect of pre-existing longitudinal and transverse corrosion cracks on the flexural behaviour of corroded RC beams. *Construction and Building Materials*. 2022;319:126141. http://doi.org/10.1016/J.CONBUILDMAT.2021.126141

3. Jebasingh Daniel J. Experimental and numerical study on the cracking behavior and flexural strength of RC shallow beams with rectangular opening and varying length. *Structures*. 2022;40:460–468. http://doi.org/10.1016/J.ISTRUC.2022.04.040

4. Nayak C.B., Narule G.N., Surwase H.R. Structural and cracking behaviour of RC T-beams strengthened with BFRP sheets by experimental and analytical investigation. *Journal of King Saud University – Engineering Sciences.* 2022;34(6):398–405. http://doi.org/10.1016/J.JKSUES.2021.01.001

5. Murashev V.I. *Crack resistance, stiffness and strength of reinforced concrete*. Moscow: Mashstroyizdat Publ.; 1950. (In Russ.)

6. Gvozdev A.A., Borishansky M.S. To the question of the calculation of bending elements according to the stage of destruction. *Project and Standard*. 1934;(8):7–12. (In Russ.)

7. Zalesov A.S., Mukhamediev T.A., Chistyakov E.A. Calculation of crack resistance of reinforced concrete structures according to new regulatory documents. *Beton i Zhelezobeton* [*Concrete and Reinforced Concrete*]. 2002;(5):15–19. (In Russ.)

8. Baikov V.N., Sigalov E.E. *Reinforced concrete structures. General course.* Moscow: Stroyizdat Publ.; 1991. (In Russ.)

9. Karpenko N.I., Radaikin O.V. On the improvement of concrete deformation diagrams for determining the moment of cracking and breaking moment in bending reinforced concrete elements. *Construction and Reconstruction*. 2012;41(3):10–16. (In Russ.)

10. Trekin N.N., Kodysh E.N., Trekin D.N. Improvement of the method for assessing the crack resistance of bent reinforced concrete elements. *Beton i Zhelezobeton* [*Concrete and Reinforced Concrete*]. 2020;601(1):61–64. (In Russ.)

11. Trekin N.N., Kodysh E.N., Trekin D.N. Calculation of the formation of normal cracks based on the deformation model. *Industrial and Civil Construction.* 2016;(7):74–78. (In Russ.)

12. Yeryshev V.A., Kazakov M.Yu. To the method of determining the moment of cracking of bent reinforced concrete elements by a nonlinear deformation model. *Bulletin NGIEI.* 2017;79(12):32–42. (In Russ.)

13. Mukhamediev T.A. Taking into account the inelastic properties of concrete when calculating reinforced concrete structures for the formation of cracks. *Structural Mechanics and Analysis of Constructions.* 2018;280(5):24–29. (In Russ.)

14. Kolchunov Vl.I. Numerical-analytical method in reinforced concrete mechanics. *Structural Mechanics of Engineering Constructions and Buildings*. 2022;18(6):525–533. (In Russ.) http://doi.org/10.22363/1815-5235-2022-18-6-525-533

15. Tamrazyan A.G., Chernik V.I., Matseevich T.A., Manaenkov I.K. Analytical model of deformation of reinforced concrete columns based on fracture mechanics. *Structural Mechanics of Engineering Constructions and Buildings.* 2022;18(6):573–583. http://doi.org/10.22363/1815-5235-2022-18-6-573-583

16. Li Z., Zhu H., Du C., Gao D., Yuan J., Wen C. Experimental study on cracking behavior of steel fiber-reinforced concrete beams with BFRP bars under repeated loading. *Composite Structures*. 2021;267:113878. http://doi.org/10.1016/J.COMPSTRUCT.2021.113878

17. Wei C., Zhang Q., Yang Z., Li M., Cheng Z., Bao Y. Flexural cracking behavior of reinforced UHPC overlay in composite bridge deck with orthotropic steel deck under static and fatigue loads. *Engineering Structures*. 2022;265:114537. http://doi.org/10.1016/J.ENGSTRUCT.2022.114537

18. Cui S., Xu X., Chen Z., Zeng G., Ouyang Q., Li G. Effect of different sizing agent-treated basalt fibers on bending and cracking performance of reinforced BFRC beams. *Construction and Building Materials*. 2023;365:130037. http://doi.org/10.1016/J.CONBUILDMAT.2022.130037

19. Zhao M., Li C., Su J., Shang P., Zhao S. Experimental study and theoretical prediction of flexural behaviors of reinforced SFRELC beams. *Construction and Building Materials*. 2019;208:454–463. http://doi.org/10.1016/J.CONBUILDMAT.2019.03.037

20. Luo J., Shao X., Fan W., Cao J., Deng S. Flexural cracking behavior and crack width predictions of composite (steel + UHPC) lightweight deck system. *Engineering Structures.* 2019;194:120–137. http://doi.org/10.1016/J.ENGSTRUCT.2019.05.018

21. Głodkowska W., Ziarkiewicz M. Cracking behavior of steel fiber reinforced waste sand concrete beams in flexure – experimental investigation and theoretical analysis. *Engineering Structures*. 2018;176:1–10. http://doi.org/10.1016/J.ENGSTRUCT.2018.08.097

22. Ismail M.K., Hassan A.A.A., AbdelAleem B.H., El-Dakhakhni W. Flexural behavior and cracking of lightweight RC beams containing coarse and fine slag aggregates. *Structures*. 2023;47:1005–1019. http://doi.org/10.1016/J.ISTRUC.2022.11.065

23. Travush V.I., Karpenko N.I., Kolchunov Vl.I., Kaprielov S.S., Demyanov A.I., Bulkin S.A., Moskovtseva V.S. Results of experimental studies of complex-stressed beams of circular cross-section made of high-strength fibro-reinforced concrete. *Structural Mechanics of Engineering Constructions and Buildings*. 2020;16(4):290–297. http://doi.org/10.22363/1815-5235-2020-16-4-290-297

24. Korsun V.I., Karpenko S.N., Makarenko S.Yu., Nedorezov A.V. Modern strength criteria for concrete under volumetric stress conditions. *Construction and Reconstruction*. 2021;97(5):16–30. (In Russ.)

25. Fedorova N.V., Vu N.T., Medyankin M.D., Phan D.Q. Determination of the Viscosity modulus of concrete under static-dynamic loading regimes. *International Scientific Siberian Transport Forum*. 2022;403(1):1294–1302.

26. Yuan P., Xiao L., Wang X., Xu G. Failure mechanism of corroded RC beams strengthened at shear and bending positions. *Engineering Structures*. 2021;240:112382. http://doi.org/10.1016/J.ENGSTRUCT.2021.112382

27. Yu X.Y., Jiang C., Zhang W.P. Failure mode-based calculation method for bending bearing capacities of corroded RC beams strengthened with CFRP sheets. *Engineering Structures*. 2022;271:114946. http://doi.org/10.1016/J.ENGSTRUCT.2022.114946

28. Zinoviev V.N. Dilatation effect and state diagram of concrete under uniaxial and triaxial compression. Part 2. *Beton i Zhelezobeton* [*Concrete and Reinforced Concrete*].2015;(2):27–31. (In Russ.)

29. Zinoviev V.N., Smolyagov O.O., Grigoriev A.A. Methods for studying microcrack formation in concrete under uniaxial compression. *Beton i Zhelezobeton* [*Concrete and Reinforced Concrete*]. 2014;(1):27–31. (In Russ.)

30. Zinoviev V.N. Combined state diagram and parametric levels of microcracking in concrete. Part 3. *Beton i Zhelezobeton* [*Concrete and reinforced Concrete*]. 2015;(3):28–31. (In Russ.)

31. Kolchunov V.I., Yakovenko I.A. On the use of the hypothesis of flat sections in reinforced concrete. *Construction and Reconstruction*. 2011;38(6):16–23. (In Russ.)

32. Geniev G.A. Variant of the deformation theory of plasticity of concrete. *Beton i Zhelezobeton [Concrete and Reinforced Concrete].* 1969;(2):18–19. (In Russ.)

33. Geniev G.A., Kissyuk V.N., Tyupin G.A. *Theory of plasticity of concrete and reinforced concrete*. Moscow: Stroyizdat Publ.; 1974. (In Russ.)

34. Kolchunov V.I., Al-Hashimi O.I., Protchenko M.V. Rigidity of reinforced concrete structures in bending with transverse and longitudinal forces. *Construction and Reconstruction*. 2021;(6):5–19. (In Russ.)

Список литературы

1. *Al-Fakih A., Hisbany Mohd Hashim M., Alyousef R., Mutafi A., Hussein Abo Sabah S., Tafsirojjaman T.* Cracking behavior of sea sand RC beam bonded externally with CFRP plate // Structures. 2021. Vol. 33. Pp. 1578–1589. http://doi.org/10.1016/J.ISTRUC.2021.05.042

2. *Nasser H., Vandewalle L., Verstrynge E.* Effect of pre-existing longitudinal and transverse corrosion cracks on the flexural behaviour of corroded RC beams // Construction and Building Materials. 2022. Vol. 319. Article 126141. http://doi.org/10.1016/J.CONBUILDMAT.2021.126141

3. *Jebasingh D.J.* Experimental and numerical study on the cracking behavior and flexural strength of RC shallow beams with rectangular opening and varying length // Structures. 2022. Vol 40. Pp. 460–468. http://doi.org/10.1016/J.ISTRUC.2022.04.040

4. *Nayak C.B., Narule G.N., Surwase H.R.* Structural and cracking behaviour of RC T-beams strengthened with BFRP sheets by experimental and analytical investigation // Journal of King Saud University – Engineering Sciences. 2022. Vol. 34. No. 6. Pp. 398–405. http://doi.org/10.1016/J.JKSUES.2021.01.001

5. *Мурашев В.И.* Трещиноустойчивость, жесткость и прочность железобетона. М.: Машстройиздат, 1950. 268 с.

6. *Гвоздев А.А., Боришанский М.С.* К вопросу о расчете изгибаемых элементов по стадии разрушения // Проект и стандарт. 1934. № 8. С. 7–12.

7. *Залесов А.С., Мухамедиев Т.А., Чистяков Е.А.* Расчет трещиностойкости железобетонных конструкций по новым нормативным докуменам // Бетон и железобетон. 2002. № 5. С. 15–19.

8. *Байков В.Н., Сигалов Э.Е.* Железобетонные конструкции. Общий курс. М.: Стройиздат, 1991. 767 с.

9. *Карпенко Н.И., Радайкин О.В.* К совершенствованию диаграмм деформирования бетона для определения момента трещинообразования и разрушающего момента в изгибаемых железобетонных элементах // Строительство и реконструкция. 2012. Т. 41. № 3. С. 10–16.

10. *Трекин Н.Н., Кодыш Э.Н., Трекин Д.Н.* Совершенствование метода оценки трещиностойкости изгибаемых железобетонных элементов // Бетон и железобетон. 2020. Т. 601. № 1. С. 61–64.

11. *Трекин Н.Н., Кодыш Э.Н., Трекин Д.Н.* Расчет по образованию нормальных трещин на основе деформационной модели // Промышленное и гражданское строительство. 2016. № 7. С. 74–78.

12. *Ерышев В.А., Косков М.Ю.* К методике определения момента трещинообразования изгибаемых железобетонных элементов по нелинейной деформационной модели // Вестник НГИЭИ. 2017. Т. 79. № 12. С. 32–42.

13. *Мухамедиев Т.А.* Учет неупругих свойств бетона при расчете железобетонных конструкций по образованию трещин // Строительная механика и расчет сооружений. 2018. Т. 280. № 5. С. 24–29.

14. *Колчунов Вл.И.* Численно-аналитический метод в механике железобетона // Строительная механика инженерных конструкций и сооружений. 2022. Т. 18. № 6. С. 525–533. http://doi.org/10.22363/1815-5235-2022-18-6-525-533

15. *Тамразян А.Г., Черник В.И., Мацеевич Т.А., Манаенков И.К.* Аналитическая модель деформирования железобетонных колонн на основе механики разрушения // Строительная механика инженерных конструкций и сооружений. 2022. Т. 18. № 6. С. 573–583. http://doi.org/10.22363/1815-5235-2022-18-6-573-583

16. *Li Z., Zhu H., Du C., Gao D., Yuan J., Wen C.* Experimental study on cracking behavior of steel fiber-reinforced concrete beams with BFRP bars under repeated loading // Composite Structures. 2021. Vol. 267. Article 113878. http://doi.org/10.1016/J.COMPSTRUCT.2021.113878

17. *Wei C., Zhang Q., Yang Z., Li M., Cheng Z., Bao Y.* Flexural cracking behavior of reinforced UHPC overlay in composite bridge deck with orthotropic steel deck under static and fatigue loads // Engineering Structures. 2022. Vol. 265. Article 114537. http://doi.org/10.1016/J.ENGSTRUCT.2022.114537

18. *Cui S., Xu X., Chen Z., Zeng G., Ouyang Q., Li G.* Effect of different sizing agent-treated basalt fibers on bending and cracking performance of reinforced BFRC beams // Construction and Building Materials. 2023. Vol. 365. Article 130037. http://doi.org/10.1016/J.CONBUILDMAT.2022.130037

19. *Zhao M., Li C., Su J., Shang P., Zhao S.* Experimental study and theoretical prediction of flexural behaviors of reinforced SFRELC beams // Construction and Building Materials. 2019. Vol. 208. Pp. 454–463. http://doi.org/10.1016/J.CONBUILDMAT.2019.03.037

20. *Luo J., Shao X., Fan W., Cao J., Deng S.* Flexural cracking behavior and crack width predictions of composite (steel + UHPC) lightweight deck system // Engineering Structures. 2019. Vol. 194. Pp. 120–137. http://doi.org/10.1016/J.ENGSTRUCT.2019.05.018

21. *Głodkowska W., Ziarkiewicz M.* Cracking behavior of steel fiber reinforced waste sand concrete beams in flexure – experimental investigation and theoretical analysis // Engineering Structures. 2018. Vol. 176. Pp. 1–10. http://doi.org/10.1016/J.ENGSTRUCT.2018.08.097

22. *Ismail M.K., Hassan A.A.A., AbdelAleem B.H., El-Dakhakhni W.* Flexural behavior and cracking of lightweight RC beams containing coarse and fine slag aggregates // Structures. 2023. Vol. 47. Pp. 1005–1019. http://doi.org/10.1016/J.ISTRUC.2022.11.065

23. *Travush V.I., Karpenko N.I., Kolchunov Vl.I., Kaprielov S.S., Demyanov A.I., Bulkin S.A., Moskovtseva V.S.* Results of experimental studies of complex-stressed beams of circular cross-section made of high-strength fibro-reinforced concrete // Строительная механика инженерных конструкций и сооружений. 2020. Т. 16. № 4. С. 290–297. http://doi.org/10.22363/1815-5235-2020-16-4-290-297

24. *Корсун В.И., Карпенко С.Н., Макаренко С.Ю., Недорезов А.В.* Современные критерии прочности для бетонов при объемных напряженных состояниях // Строительство и реконструкция. 2021. Т. 97. № 5. С. 16–30.

25. *Fedorova N.V., Vu N.T., Medyankin M.D., Phan D.Q.* Determination of the viscosity modulus of concrete under static-dynamic loading regimes // International Scientific Siberian Transport Forum. 2022. Vol. 403. No. 1. Pp. 1294–1302.

26. *Yuan P., Xiao L., Wang X., Xu G.* Failure mechanism of corroded RC beams strengthened at shear and bending positions // Engineering Structures. 2021. Vol. 240. http://doi.org/10.1016/J.ENGSTRUCT.2021.112382

27. *Yu X.Y., Jiang C., Zhang W.P.* Failure mode-based calculation method for bending bearing capacities of corroded RC beams strengthened with CFRP sheets // Engineering Structures. 2022. Vol. 271. Article 114946. http://doi.org/10.1016/J.ENGSTRUCT.2022.114946

28. *Зиновьев В.Н.* Эффект дилатации и диаграмма состояний бетона при одноосном и трехосном сжатии. Ч. 2 // Бетон и железобетон. 2015. № 2. С. 27–31.

29. *Зиновьев В.Н., Смолягов О.О., Григорьев А.А.* Методы исследования микротрещинообразования бетона при одноосном сжатии // Бетон и железобетон. 2014. № 1. С. 27–31.

30. *Зиновьев В.Н.* Объединенная диаграмма состояний и параметрические уровни микротрещинообразования бетона. Ч. 3 // Бетон и железобетон. 2015. № 3. С. 28–31.

31. *Колчунов В.И., Яковенко И.А.* Об использовании гипотезы плоских сечений в железобетоне // Строительство и реконструкция. 2011. Т. 38. № 6. С. 16–23.

32. *Гениев Г.А.* Вариант деформационной теории пластичности бетона // Бетон и железобетон. 1969. № 2. С. 18–19.

33. *Гениев Г.А., Киссюк В.Н., Тюпин Г.А.* Теория пластичности бетона и железобетона. М.: Стройиздат, 1974. 316 с.

34. *Колчунов В.И., Аль-Хашими О.И., Протченко М.В.* Жесткость железобетонных конструкций при изгибе с поперечной и продольной силами // Строительство и реконструкция. 2021. № 6. С. 5–19.