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REVIEW / НАУЧНЫЙ ОБЗОР

Relaxation of stress in elements of reinforced concrete structures

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Abstract. The calculation and prediction of the long-term safety of building structures is associated with the dynamics of the stress state of their composite elements and leads to relaxation problems for assessing the redistribution of stresses between the components that make up the structural element. In this study, reinforced concrete elements and the redistribution of stress from concrete to reinforcement are considered. To solve the corresponding relaxation problem an approach based on the concept of the strength structure of materials is proposed, which considers them as a union of their fractions (layers, fibers) with statistically distributed strengths. The loss of the ability of force resistance caused by loading by part of the fractions of the element entails a redistribution of stresses to its entire fractions. As a result of this, a nonlinear dependence of deformations on the design stresses arises, calculated under the assumption of equal strength of all fractions. For a material isotropic in strength, the relaxation problem is reduced to solving a linear integral equation conjugated with its linear rheological equation. The linear integral equation relatively structural stresses is reduced. After solving it, the desired stress is determined as the root of the algebraic equation connecting the structural and design stresses. The proposed approach significantly simplifies the obtaining of necessary for the long-term safety prediction of structures stress estimates in the components of structural elements.

Keywords: stress relaxation, creep of constructions, deformation of constructions elements, long-term safety of constructions

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Релаксация напряжений в элементах железобетонных конструкций

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Аннотация. Расчет и прогноз длительной безопасности строительных конструкций сопряжен с динамикой напряженного состояния их композитных элементов и приводит к задачам релаксации для оценки перераспределения напряжений между составляющими конструктивный элемент компонентами. В исследовании рассматриваются железобетонные элементы и перераспределение напряжения с бетона на арматуру. Для решения соответствующей релаксационной задачи предлагается подход, основанный на концепции прочностной структуры материалов, рассматривающей их как объединение своих фракций (слоев, волокон) со статистически распределенными прочностями. Порождаемая нагружением потеря способности силового сопротивления частью фракций элемента влечет перераспределение напряжений на его целые фракции. В результате возникает нелинейная зависимость деформаций от расчетных напряжений, рассчитанных в предположении равнопрочности всех фракций. Для изотропного по прочности материала релаксационная задача сводится к решению линейного интегрального уравнения, сопряженного с его линейным реологическим уравнением. Выводится линейное интегральное уравнение относительно так называемого структурного напряжения способной к силовому сопротивлению частью элемента. После его решения искомое напряжение определяется как корень алгебраического уравнения, связывающего структурные и расчетные напряжения. Предлагаемый подход существенно упрощает получение необходимых в прогнозе длительной безопасности сооружений оценок напряжений в компонентах конструкционных элементов.

Ключевые слова: релаксация напряжений, ползучесть бетона, деформация элементов конструкций, длительная безопасность конструкций

Introduction

In a global sense, the relaxation phenomenon represents the process of thermodynamic equilibrium establishing in a system consisting of a large number of particles. In structural mechanics, relaxation is understood as a reduction in stresses when the initial deformation is fixed by the bonds. From the point of view of physical chemistry, stress reduction occurs due to intermolecular displacements and reorientation of the intramolecular structure, and therefore the similarity of relaxation and creep phenomena is manifested. In this paper on the basis of the accepted concept of the strength structure of materials the modification of known in the linear creep theory L. Boltzmann's superposition principle [1] is obtained allowing its applicability under nonlinear dependence of deformations on design stresses.

Relaxation problems are associated with the phenomenon of creep – an increase in deformation $\varepsilon(\tau)$-generated by stress $\sigma(t_0)$ when $\tau > t_0$. Stresses $\sigma(\tau)$ decrease over time with constant deformation $\varepsilon(t_0)$ and this phenomenon is called stress relaxation. The phenomenon of relaxation is a consequence of

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the development of creep deformation in the material $\varepsilon_п(\tau, t_0)$, because with a constant total deformation $\varepsilon(t_0) = \varepsilon_м(t) + \varepsilon_п(t, t_0)$ due to an increase in deformation $\varepsilon_п(t, t_0)$, the portion $\varepsilon_м(t)$ of instantaneous deformation decreases and the stress $\sigma(t) = E(t)\varepsilon_м(t)$ decreases. This means that creep and relaxation phenomena take place simultaneously.

Continuous redistribution of stresses between concrete and reinforcement is essential for their current stress-strain state. Stress relaxation in the concrete component of the structural element entails an increase in the stress in the reinforcement and its significant excess over the calculated one can lead to breaking of the reinforcement and to serious consequences in critical structures (in reinforced concrete reactor shells, water ducts).

The standard method for solving the considered problems is the preliminary determination of the relaxation kernel by a given creep kernel. However, the solution of the corresponding integral equation is expressed in a very slowly converging series [2]. The application of the Laplace transform and the Poincare small parameter method is busy and time-consuming [3].

The solution of relaxation problems is greatly simplified when the corresponding integral equations of state are reduced to their differential forms. In this paper, the linear integral equation of state of uniaxially loaded concrete is reduced to a simple linear differential equation of the first order. This takes into account the presence of a single aging function that determines the evolution of the elastic modulus and the creep measure.

Remark 1. The key point of this method is the linearity of the integral equation of state. In the nonlinear statement, the transformation to a linear differential equation becomes possible after the derivation of the linear relative to structural stress (based on a modification of L. Boltzmann's principle) statement equation of concrete.

Rheological equations of mechanical state

Creep deformation under simple loading $\sigma(\tau)$,

$$\varepsilon_п(t, \tau) = C^*(t, \tau)\sigma(\tau). \qquad (1)$$

Take the measure of creep of concrete in the form

$$C^*(t, \tau) = C(\infty, 28)\theta(\tau)f(t - \tau), \qquad (2)$$

where $\theta(\tau)$ – the aging function; $f(t - \tau)$ – the creep strain accumulation function; $\lim_{t\to\infty} C^*(t, \tau) = C^*(\infty, \tau)$.

Based on experimental data [4], the following structure of the aging function was established in [5]:

$$\theta(\tau) = \frac{R(28)}{R(\tau)} = \frac{E(28)}{E(\tau)} = \frac{C^*(\infty,\tau)}{C(\infty,28)}. \qquad (3)$$

Denoting $C^*(\infty, 28)f(t - \tau)$=$C^*{}_0 f(t - \tau)$ и $\theta(\tau)\sigma(\tau)$= $\hat{\sigma}(\tau)$, according to (2) and (3) we have

$$C^*(t, \tau) = \theta(\tau)C_0^*(t, \tau); \;\; \hat{\sigma}(\tau) = R(28)\eta(\tau), \qquad (4)$$

where $\eta(\tau) = \frac{\sigma(\tau)}{R(\tau)}$ – the stress level; $R(28)$ – the strength of concrete at the age of 28 days.

Corresponding to the increment of the stress level $\Delta\eta(\tau_i) = \frac{\Delta\sigma(\tau_i)}{R(28)}$ at the moment τ_i a partial increment of creep deformations at the moment *t* is

$$\Delta\varepsilon_п(t, \tau_i) = C^*(t, \tau_i)R(28)\Delta\eta(\tau_i) = C_0^*(t, \tau_i)\Delta\hat{\sigma}(\tau_i). \qquad (5)$$

Since each increment $\Delta\varepsilon_п(t, \tau_i)$ depends only on the magnitude of the level and $t - \tau_i$, then the increment

$$\Delta\hat{\sigma}(t) = \sum_{i=1}^{n} \Delta\hat{\sigma}(\tau_i) \qquad (6)$$

responds to creep deformation increment

$$\Delta\varepsilon_п(t, t_0) = \sum_{i=1}^{n} C_0^*(t, \tau_i)\Delta\hat{\sigma}(\tau_i). \qquad (7)$$

By passing in (7) to the limit and integrating in parts we have

$$\Delta\varepsilon_{п}(t,t_0) = C_0^*(t,t)\hat{\sigma}(t) - C_0^*(t,t_0)\hat{\sigma}(t_0) - \int_{t_0}^{t}\hat{\sigma}(\tau)\frac{\partial C_0^*(t,\tau)}{\partial\tau}d\tau. \quad (8)$$

The increment of the total deformation $\varepsilon(t,t_0) = \varepsilon_{м}(t) - \varepsilon_{м}(t_0) + \Delta\varepsilon_{п}(t,t_0)$ and, adding to $\Delta\varepsilon_{п}(t,t_0)$ the deformation, caused by stress $\sigma(t_0)$, we obtain a linear rheological equation of concrete under uniaxial loading

$$\varepsilon(t,t_0) = \frac{\sigma(t)}{E(t)} + C_0^*(t,t)\sigma(t) - \int_{t_0}^{t}\theta(\tau)\sigma(\tau)\frac{\partial C_0^*(t,\tau)}{\partial\tau}d\tau. \quad (9)$$

When $\theta(\tau) = 1$ the parameters $E(\tau)$ and $C^*(t,\tau)$ are time-invariant for ageless concrete

$$\varepsilon(t,t_0) = \frac{\sigma(t)}{E(t)} + C_0^*(t,t)\sigma(t) - \int_{t_0}^{t}\sigma(\tau)\frac{\partial C_0^*(t,\tau)}{\partial\tau}d\tau. \quad (10)$$

Remark 1. Boltzmann's superposition principle is formulated for ageless concrete (ideal) material. When taking into account aging, the superposition of partial creep deformations is realized by partial increments of the reduced stress $\hat{\sigma}(\tau) = \theta(\tau)\sigma(\tau)$, corresponding to increments of the level $\eta(\tau) = \frac{\sigma(\tau)}{R(\tau)}$ of stresses $\sigma(\tau)$.

The creep deformation accumulation function is selected in the form

$$f(t-\tau) = \left[1 - ke^{-\gamma(t-\tau)}\right]; \quad 0 < k \le 1$$

and thus [6]:

$$C^*(t,\tau) = C(\infty,28)\theta(\tau)\left[1 - ke^{-\gamma(t-\tau)}\right]. \quad (11)$$

Remark 2. When in (11) we have $k < 1$, a summand $C^*(t,t)\sigma(t) \ne 0$ and it is called short-term creep. Taking measure (11) with $k < 1$, some authors [7] assume the inertia-free nature of the creep phenomenon, while other authors [8] (since the creep measure determines precisely delayed deformations), assuming the inertia in time of this phenomenon, take the creep measure (11) with $k = 1$

$$C^*(t,\tau) = C(\infty,28)\theta(\tau)\left[1 - e^{-\gamma(t-\tau)}\right]. \quad (12)$$

According to (9) and (12) we have $C^*(t,t) = 0$

$$\varepsilon(t,t_0) = \frac{\sigma(t)}{E(t)} - \int_{t_0}^{t}\theta(\tau)\sigma(\tau)\frac{\partial C_0^*(t,\tau)}{\partial\tau}d\tau. \quad (13)$$

The measure (12) is taken in this paper.

Since with measure (12) we have $-\theta(\tau)\frac{\partial C_0^*(t,\tau)}{\partial\tau} = \frac{\partial C^*(t,\tau)}{\partial t}$, then according to (13)

$$\varepsilon(t,t_0) = \frac{\sigma(t)}{E(t)} + \int_{t_0}^{t}\sigma(\tau)\frac{\partial C_0^*(t,\tau)}{\partial t}d\tau. \quad (14)$$

The value of

$$\delta(t,\tau) = \frac{1}{E(\tau)} + C(t,\tau) \quad (15)$$

in building codes it is taken as the general compliant of instantaneous and delayed deformations and under simple loading $\sigma(\tau)$

$$\varepsilon(t,\tau) = \frac{\sigma(\tau)}{E(\tau)} + C(t,\tau)\sigma(\tau). \quad (16)$$

The imposition of deformations is carried out with general compliance

$$\delta^*(t,\tau) = \frac{1}{E(t)} + C^*(t,\tau) \quad (17)$$

and when loading $\sigma(\tau)$, we assume

$$\varepsilon(t,\tau) = \frac{\sigma(\tau)}{E(t)} + C^*(t,\tau)\sigma(\tau). \quad (18)$$

According to (16) and (18)

$$\frac{1}{E(\tau)} + C(t,\tau) = \frac{1}{E(t)} + C^*(t,\tau),$$

$$C^*(t,\tau) = C(t,\tau) + \frac{1}{E(\tau)} - \frac{1}{E(t)}. \quad (19)$$

Equality (19) means, that with compliance $\delta(t,\tau)$ the measure $C^*(t,\tau)$of creep deformations decreases by an amount

$$[\frac{1}{E(\tau)} - \frac{1}{E(t)}].$$

In the works [7–9], assuming $E(\tau) = E(t);\ t_0 \le \tau \le t$ the rheological equation is derived by applying partial increments of creep deformation

$$\varepsilon(t,t_0) = \frac{\sigma(t)}{E(t)} - \int_{t_0}^{t} \sigma(\tau)\frac{\partial C^*(t,\tau)}{\partial \tau} d\tau, \quad (20)$$

equivalent according to (19) to equation

$$\varepsilon(t,t_0) = \frac{\sigma(t)}{E(t)} - \int_{t_0}^{t} \sigma(\tau)\frac{\partial}{\partial \tau}\frac{1}{E(\tau)} d\tau - \int_{t_0}^{t} \sigma(\tau)\frac{\partial C(t,\tau)}{\partial \tau} d\tau. \quad (21)$$

Remark 3. Equations (20) and (21) describe the same mechanical state of concrete, which was not noticed by the authors of [10]. This led to the statement [10] that the second component in (21) is unnecessary and to an incorrect representation of the superposition principle "as a fundamental error in the theory of creep" [11].

The force increasing by a normal cross-section $N(\tau)$ entails the destruction of a part of the fractions, reducing the cross-sectional area A to $A(\tau)$, formed by whole fractions at the moment τ. The value associated with structural damage

$$\sigma_c(\tau) = \frac{N(\tau)}{A(\tau)} \quad (22)$$

is called the structural, and the value of

$$\sigma(\tau) = \frac{N(\tau)}{A} \quad (23)$$

is called the calculated normal voltage in the structural element. According to (22) and (23)

$$\sigma_c(\tau) = \frac{A}{A(\tau)}\sigma(\tau) = S^0(\tau)\sigma(\tau), \quad (24)$$

where the function $S^0(\tau) = \frac{A}{A(\tau)}$ describes the process of destruction of fractions, accompanied by a redistribution of loading $N(\tau)$ on the area $A(\tau)$, because only fractions that are entire at this moment exert force resistance. In contrast to the linear formulation, which means the equal strength of all fractions of the element, the stepwise increment of stress $\Delta\sigma = \sum_{i=1}^{n} \Delta\sigma(\tau_i)$ does not correspond to mutually independent increments of creep deformations – the action $\Delta\sigma(\tau_i)$ at the moment $\tau = \tau_i$ is enhanced by the action of $\Delta\sigma(\tau_j)$, where $j > i$ [12]. Note also that the area $A(\tau_i)$is determined by all increments $\Delta\sigma(\tau_k)$; $k < i$.

This circumstance leads to the need to modify the principle of L. Boltzmann's superposition. Since the cause of the dependence between $\Delta\varepsilon_{п}(t, \tau_i)$ is the equal strength of the fractions, we will (mentally) select in the concrete component $\boldsymbol{V}$ of the structural element its part $\boldsymbol{V_t}$, consisting of entire fractions in the segment $[t_0, t]$. Just for this part the rheological equation describes the stress-strain state (SSS) taking into account the rheology at $[t_0, t]$.

Since the (SSS) of the part $\boldsymbol{V_t}$ at the current moment$\tau \in [t_0, t]$ coincides with the (SSS) in the part $\boldsymbol{V_\tau}$, consisting of entire fractions at the moment τ, the stress in $\boldsymbol{V_t}$ is a structural stress $\sigma_c(\tau)$. Strength balance of the fractions $\boldsymbol{V_t}$ to $\tau = t$ entails mutual independence of increments

$$\Delta\varepsilon_{п}(t, \tau_i) = C^*(t, \tau_i)\Delta\sigma_c(\tau_i)$$

and stress $\Delta\sigma_c(t) = \sum_{i=1}^{n} \Delta\sigma_c(\tau_i)$ generates deformation

$$\Delta\varepsilon_{п}(t, t_0) = \sum_{i=1}^{n} C^*(t, \tau_i)\Delta\sigma_c(\tau_i) = \sum_{i=1}^{n} C_0^*(t, \tau_i)\theta(\tau_i)\Delta\sigma_c(\tau_i). \quad (25)$$

The relation (25) allows (by repeating the above constructions) the derivation of the rheological equation of the mechanical state in a nonlinear formulation [13; 14]

$$\varepsilon(t, t_0) = \frac{\sigma_c(t)}{E(t)} - \int_{t_0}^{t} \theta(\tau)\sigma_c(\tau)\frac{\partial C_0^*(t,\tau)}{\partial\tau}d\tau, \quad (26)$$

$$\varepsilon(t, t_0) = \frac{S^0(\tau)\sigma(t)}{E(t)} - \int_{t_0}^{t} \theta(\tau)S^0(\tau)\sigma_c(\tau)\frac{\partial C_0^*(t,\tau)}{\partial\tau}d\tau. \quad (27)$$

Remark 4. Relation (25) represents the principle of superposition of creep deformations for an aging material (in the concept of its strength structure) in a nonlinear formulation.

In [9], assuming the interdependence of partial increments, the equation is derived

$$\varepsilon(t, t_0) = \frac{S_{м}\left[\frac{\sigma(t)}{R(t)}\right]}{E(t)} - \int_{t_0}^{t} S_{п}\left[\frac{\sigma(\tau)}{R(\tau)}\right]\frac{\partial C^*(t,\tau)}{\partial\tau}d\tau. \quad (28)$$

In (28) $S_{м}\left[\frac{\sigma(t)}{R(t)}\right]$ and $S_{п}\left[\frac{\sigma(\tau)}{R(\tau)}\right]$ – nonlinear functions of instantaneous deformations and creep deformations.

Remark 5. In the physical aspect, both types of deformations are generated by a single force factor – structural stress $\sigma_c(\tau)$ – and therefore $S_{м}[\eta(t)] = \sigma_c(t)$ and $S_{п}[\eta(\tau)] = \sigma_c(\tau)$. According to $\sigma_c(\tau) = S^0(\tau)\sigma(\tau)$ the unified stress function is represented as $S[\sigma(\tau)] = S^0(\tau)\sigma(\tau)$.

The nonlinear function $S^0(\tau)$ in applications is given by the equation [15]

$$S^0(\tau) = 1 + V\left[\frac{\sigma(\tau)}{R(\tau)}\right]^m, \quad (29)$$

where V and m are empirical parameters and for concrete $m = 4$ is usually assumed.

Equation (9) does not take into account the nonlinearity of the diagram $\sigma - \varepsilon$ observed in experiments and, as A.A. Gvozdev first noted, is not suitable for the theory of reinforced concrete. Assuming the dependence of the instantaneous deformation on the linear one $\sigma(\tau)$, and the creep deformation as the nonlinear one, he considered the surge in the initial section $\sigma - \varepsilon$ as a consequence of the rapidly flowing creep. According to A.A. Gvozdev [16], creep is two-component and deformation $\varepsilon_{п}(t, t_0)$ consists of the so-called partially reversible deformation of the 2nd kind and irreversible deformation of the 1st kind generated by force damage. Equation of two-component creep theory under uniaxial loading [3]

$$\varepsilon(t, t_0) = \frac{\sigma(t)}{E(t)} - \int_{t_0}^{t} \sigma(\tau)\frac{\partial}{\partial\tau}\left[\frac{1}{E(\tau)}\right]d\tau - \int_{t_0}^{t} \sigma(\tau)\frac{\partial C(t,\tau)}{\partial\tau}d\tau + \int_{0}^{\max\sigma} f(\sigma)F[T(\sigma, t)]\frac{\partial C_0^*(t,\tau)}{\partial\tau}d\sigma, \quad (30)$$

where $f(\sigma)$ is a nonlinear stress function; $F[T(\sigma, t)]$ – a function of the total duration $T(\sigma, t)$ of the stress to the moment *t*.

Denoting $A_d(\tau)$ as a part of the area A, corresponding to the fractions destroyed at the moment τ

$$A = A(\tau) + A_d(\tau);\;\; S^0(\tau) = 1 + \frac{A_d(\tau)}{A(\tau)};\;\; \alpha(\tau) = \frac{A_d(\tau)}{A(\tau)};\;\; S^0(\tau) = 1 + \alpha(\tau);$$

$$\sigma_c(\tau) = \sigma(\tau) + \alpha(\tau)\,\sigma(\tau). \tag{31}$$

The representation (31) of the structural stress corresponds to the equations

$$\varepsilon_{\text{м}}(t) = \varepsilon_{\text{мл}}(t) + \varepsilon_{\text{мн}}(t);\;\; \varepsilon_{\text{п}}(t, t_0) = \varepsilon_{\text{пл}}(t, t_0) + \varepsilon_{\text{пн}}(t, t_0), \tag{32}$$

meaning that instantaneous and delayed deformations are composed of their linear and nonlinear parts. According to (30) and (32), the last summand on the right side of the equality is the sum $\varepsilon_{\text{мн}}(t) + \varepsilon_{\text{пн}}(t, t_0)$ and is represented in [3] as a creep deformation of the 1st kind.

Remark 6. The incorrectness of considering the nonlinear part of instantaneous deformation as creep deformation is also noted in [10].

In [17], the creep equation of concrete is given on the basis of nonlinear Eurocode diagrams

$$\varepsilon(t, t_0) = f_2[\sigma(t)] - \int_{t_0}^{t} f_1[\varepsilon_{\text{м}}(\tau)]\frac{\partial C(t,\tau)}{\partial \tau}d\tau, \tag{33}$$

where $\sigma(t) = f_1[\varepsilon_{\text{м}}(\tau)]$ and $\varepsilon_{\text{м}}(\tau) = f_2[\sigma(\tau)]$ represent the direct and inverse function of a nonlinear diagram $\sigma - \varepsilon_{\text{м}}$.

According to (33), in the equation of state, along with the nonlinear dependence of instantaneous deformations on $\sigma(\tau)$, creep is represented as linearly dependent on $\sigma(\tau)$.

Relaxation problems

Stress relaxation in concrete under uniaxial loading

To determine the design and structural stresses for a given deformation according to equations (9) and (26), we have the following integral equations

$$\sigma(t) = E(t)\varepsilon(t, t_0) + E(t)\int_{t_0}^{t}\theta(\tau)\sigma(\tau)\frac{\partial C_0^*(t,\tau)}{\partial \tau}d\tau, \tag{34}$$

$$\sigma_c(t) = E(t)\varepsilon(t, t_0) + E(t)\int_{t_0}^{t}\theta(\tau)\sigma_c(\tau)\frac{\partial C_0^*(t,\tau)}{\partial \tau}d\tau. \tag{35}$$

Both equations (9) and (26) and equations (34) and (35) have the same structure with the same parameters $E(\tau)$ and $C_0^*(t,\tau)$ in linear and nonlinear formulations. Within the framework of the concept we have adopted the nonlinearity is determined by the structure of the material and force loading, and parameters that do not depend on these factors – $E(\tau)$ and $C_0^*(t,\tau)$ are determined by physico-chemical processes.

The scheme of definition $\sigma(\tau)$ and $\sigma_c(\tau)$ is the same. Consider equation (34). Because $\theta(\tau) = \frac{E(28)}{E(\tau)}$ and $\sigma(\tau) = E(t)\varepsilon_y(\tau)$; $\varepsilon_y(\tau)$ – elastic deformation, then according to (9) with measure (12), we obtain the equality

$$\varepsilon(t) = \varepsilon_y(t) + \gamma\varphi e^{-\gamma t}\int_{t_0}^{t}\varepsilon_y(\tau)e^{\gamma\tau}d\tau;\;\; \varphi = E(28)C(\infty, 28), \tag{36}$$

that represent a linear integral equation with respect to elastic deformation $\varepsilon_y(\tau)$.

Equation (36) is reduced to the differential form [18]: multiply both parts (36) by $e^{\gamma t}$

$$e^{\gamma t}\varepsilon(t) = \varepsilon^{\gamma t}\varepsilon_y(t) + \gamma\varphi\int_{t_0}^{t}\varepsilon_y(\tau)e^{\gamma\tau}d\tau. \tag{37}$$

We differentiate all summands (37) by t, taking into account the known equality

$$\frac{d}{d(t)}\int_{t_0}^{t} f(\tau)d\tau = f(t),$$

$$\varepsilon^{\gamma t}\,[\dot{\varepsilon}(t)+\gamma\varepsilon(t)] = e^{\gamma\tau}[\dot{\varepsilon}_y(t)+\gamma\varepsilon_y(t)]+\gamma\varphi e^{\gamma\tau}\varepsilon_y(t). \tag{38}$$

Now, multiplying all the summands by $e^{-\gamma\tau}$, we obtain a linear differential equation of the first order with respect to $\varepsilon_y(t)$

$$\dot{\varepsilon}_y(t)+b\varepsilon_y(t) = \dot{\varepsilon}(t)+\gamma\varepsilon(t);\quad b=\gamma(1+\varphi). \tag{39}$$

The function $\varepsilon_{y0}(t)=Ce^{-bt}$ represents the general solution of a homogeneous equation $\dot{\varepsilon}(t)+b\varepsilon_y(t)=0$. We look for the general solution of equation (39) in the form $\varepsilon_y(t)=C(t)e^{-bt}$ and, substituting it into (39), we get

$$\dot{C}\,(t)e^{-bt}-bC(t)e^{-bt}+bCe^{-bt}=\varphi_0(t);\quad \varphi_0(t)=\dot{\varepsilon}(t)+\gamma\varepsilon(t);$$

$$\dot{C}\,(t)e^{-bt}=\varphi_0(t)\ \text{ и }\ C(t)=\int e^{bt}\,\varphi_0(t)dt=\Phi_0(t)+C.$$

So, the general solution of equation (39) is

$$\varepsilon_y(t)=Ce^{-bt}+e^{-bt}\Phi_0(t)=Ce^{-bt}+\Phi(t);\quad \Phi(t)=e^{-bt}\Phi_0(t). \tag{40}$$

Since according to (37) we have $\varepsilon_y(t_0)=\varepsilon(t_0)$ and $\varepsilon_y(t_0)=\Phi(t_0)+Ce^{-bt}$, then

$$C=[\varepsilon(t_0)-\Phi(t_0)]e^{bt_0}.$$

Thus,

$$\varepsilon_y(t)=\big[\varepsilon\big(t_0-\Phi(t_0)\big)\big]e^{-b(t-t_0)}+\Phi(t), \tag{41}$$

$$\sigma(t)=E(t)\big[\varepsilon\big(t_0-\Phi(t_0)\big)\big]e^{-b(t-t_0)}+E(t)\Phi(t). \tag{42}$$

In the nonlinear formulation, equation (26) is reduced to the form (39) using the above transformations and the structural stress is determined as

$$\sigma_c^*(t)=E(t)\big[\varepsilon\big(t_0-\Phi(t_0)\big)\big]e^{-b(t-t_0)}+E(t)\Phi(t). \tag{43}$$

According to the stress found by this formula $\sigma_c^*(t)$ the required calculated stress $\sigma^*(t)$is determined by the solution of the algebraic equation

$$S^0[\sigma(t)]\sigma(t)=\sigma_c^*(t). \tag{44}$$

When the nonlinear function $S^0[\sigma(t)]$ is taken as (29) for $m = 4$, according to (44) we have the equation

$$V\left[\frac{\sigma(t)}{R(t)}\right]^5+\sigma(t)=\sigma_c^*(t) \tag{45}$$

and the largest of the real roots is taken as an estimate of the calculated stress.

In [18], model cases of forced deformations $\varepsilon(t)=\varepsilon_0$ and $\varepsilon(t)=v(t-t_0)$ are considered; v is the constant rate of deformation.

In work [19–20] linear integral equations (34) and (35) are solved by iteration method.

Remark 1. In (17), equation (33) is reduced to a first-order differential equation. In this case, the function $f_1[\varepsilon_м(\tau)] = \sigma(t)$ according to the nonlinear Eurocode diagrams is taken as

$$\sigma(t) = \frac{a(t)\varepsilon_м(t)+\delta(t)\varepsilon_м^2(t)}{1+g(t)\varepsilon_м(t)},$$

where functions $a(t), \delta(t), g(t)$ are selected empirically.

As a result, in contrast to equation (39), a rather complex nonlinear equation is obtained [17], and the question of its solution arises.

Stress relaxation in bent reinforced concrete elements

Let's consider stress relaxation in a single reinforced concrete beam bent by a moment $M(t)$. At a distance h_a from the neutral axis *Ox*

$$\sigma_a(t) = \frac{1}{\mu}\left[\frac{M(t)h_a}{J_b} - \sigma_b(t,h_a)\right]. \tag{46}$$

Here $\mu = \frac{A_a}{A_b}$, A_a and A_b are the areas of normal sections of the reinforcement and the concrete component of the beam; $n_0 = \frac{J}{J_b}$, where J and J_b are the moments of inertia of the concrete part of the beam and the reduced normal section relative to the *Ox* axis; $\sigma_b(t,h_a)$ is stress in the concrete layer in contact with the reinforcement.

At the level h_a according to the condition of compatibility of deformations, we obtain the equation

$$\sigma_b(t,h_a) = \widehat{\sigma_b}(t,h_a) + \lambda(t)\int_{t_0}^{t}\theta(\tau)\sigma_b(\tau,h_a)\frac{\partial C_0^*(t,\tau)}{\partial\tau}d\tau, \tag{47}$$

where $\widehat{\sigma_b}(t,h_a) = \frac{M(t)h_a}{J_b(\mu n_0 m(t)+1)}$ – instant elastic stress; $\lambda(t) = \frac{M(t)E_a(t)}{\mu n_0 m(t)+1}$; $m(t) = \frac{E_a(t)}{E_b(t)}$; $E_a(t)$ – modulus of elastic deformations of reinforcement; $E_b(t)$ – modulus of elastic deformations of concrete.

For a given elastic stress $\widehat{\sigma_b}(t,h_a)$ the integral equation (47) is solved by reducing to a differential equation of the form (39) relatively to $\varepsilon_y(\tau)$ or by simple iterations with zero approximation $\sigma_{b0}(t,h_a) = \widehat{\sigma_b}(t,h_a)$.

At a constant bending moment M we have the equation

$$\widehat{\sigma_b}(t,h_a) + (\lambda+\gamma)\sigma_b(t,h_a) = \gamma\widehat{\sigma_b}(t,h_a). \tag{48}$$

According to the solution of equation (48) with an initial condition $\sigma_{b0}(t,h_a) = \widehat{\sigma_b}(t,h_a)$ for sufficiently large t the estimate of $\sigma_b(t,h_a)$ is

$$\sigma_b(\infty,h_a) = \frac{\gamma}{\lambda+\gamma}\widehat{\sigma_b}(0,h_a). \tag{49}$$

As a result of a prolonged redistribution of stresses from concrete to reinforcement its initial stress $\sigma_a(0) = \frac{Mh_a n_0 m}{J(\mu n_0 m+1)}$ increases to value

$$\sigma_a(\infty) = \frac{1}{\mu n_0}\left[\frac{M(t)h_a}{J_b} - \frac{\gamma}{\lambda+\gamma}\widehat{\sigma_b}(0,h_a)\right]. \tag{50}$$

Structural damage leads to an intensification of this process and makes a significant contribution to the assessment $\sigma_b(\infty,h_a)$ and $\sigma_a(\infty)$.

Remark 1. In [3], a combination of the small parameter method by introducing a multiplier ς into the last summand of equation (30) and the Laplace transform is used to solve relaxation problems. The structure of equation (30), together with the methods used, makes finding stresses $\sigma_b(\tau)$ and $\sigma_a(t)$ quite a difficult task.

Conclusion

In the long-term forecast of the safety of reinforced concrete structures and buildings, estimates of the maximum stress values in the reinforcement are essential, because its rupture can lead to serious consequences. These estimates are obtained by solving the problem of stress relaxation in the concrete component of the structural element, entailing an increase of stress in the reinforcement.

Based on the concept of statistical strength distribution of fractions which union forms an element of a reinforced concrete structure, an approach is proposed for solving relaxation problems by reducing integral equations of state to a differential form. At the same time, according to the linearity of the integral equation of state with respect to the so-called structural stresses and the generality of the aging function for the modulus of elasticity and the creep measure of concrete, a simple linear differential equation of the first order is obtained.

The approach proposed in this paper, based on the strength structure of constructive materials (concrete, steel, wood, plastic), is significantly simpler than the known methods of stress assessment in the components of reinforced concrete structural elements.

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