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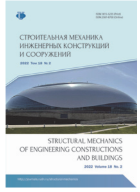

ТЕОРИЯ ТОНКИХ ОБОЛОЧЕК

THEORY OF THIN SHELLS

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REVIEW / НАУЧНЫЙ ОБЗОР

Tangential developable surfaces and their application in real structures

Sergey N. Krivoshapko✉, **Mathieu Gil-oulbé**

Peoples' Friendship University of Russia (RUDN University), Moscow, Russian Federation

✉ sn_krivoshapko@mail.ru

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Abstract. A class of developable surfaces contains cylindrical, conical, and tangential developable surfaces. Tangential developable surfaces are ruled surfaces of zero Gaussian curvature with cuspidal edges. They give great opportunities to architects and engineers for realization of their creative projects. Both the theoretical researches in the area of geometry of torses and strength analysis of shells and the influence of these researches on the application of torses in practice are shown. A presented research demonstrated that torses found the application in shipbuilding, aircraft construction, mechanical engineering, in architecture and building, engineering equipment and communications, in road building, in anti-erosive banks, topography, and cartography, clothing articles of light industry, in sculptural forms, and in modelling with developable surfaces. It was confirmed by the references on great number of published works on the subject, real examples from practice, and by handing in 14 illustrations of real objects.

Keywords: tangential developable surface, shipbuilding, mechanical engineering, architecture, building, engineering equipment, road building, anti-erosive banks, topography, clothing articles, light industry, sculptural forms

Торсовые поверхности и их применение в реальных конструкциях

С.Н. Кривошапко✉, **М. Жиль-улбе**

Российский университет дружбы народов, Москва, Российская Федерация

✉ sn_krivoshapko@mail.ru

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Аннотация. Класс развертывающихся поверхностей включает в себя цилиндрические, конические и торсовые поверхности. Торсовые поверхности – это линейчатые поверхности нулевой гауссовой кривизны с ребром возврата. Они дают большой простор архитекторам и инженерам для реализации твор-

Sergey N. Krivoshapko, DSc, Professor of the Department of Civil Engineering, Academy of Engineering, Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation; ORCID: 0000-0002-9385-3699, Scopus Author ID: 6507572305, eLIBRARY SPIN-code: 2021-6966; sn_krivoshapko@mail.ru

Mathieu Gil-oulbé, Candidate of Technical Sciences, Associate Professor of the Department of Civil Engineering, Academy of Engineering, Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation; ORCID: 0000-0003-0057-3485, Scopus Author ID: 57209566642, eLIBRARY ID: 27261171; gil-oulbem@hotmail.com

Кривошапко Сергей Николаевич, доктор технических наук, профессор департамента строительства, Инженерная академия, Российский университет дружбы народов, Российская Федерация, 117198, Москва, ул. Миклухо-Маклая, д. 6; ORCID: 0000-0002-9385-3699, Scopus Author ID: 6507572305, eLIBRARY SPIN-код: 2021-6966; sn_krivoshapko@mail.ru

Жиль-улбе Матье, кандидат технических наук, доцент департамента строительства, Инженерная академия, Российский университет дружбы народов, Российская Федерация, 117198, Москва, ул. Миклухо-Маклая, д. 6; ORCID: 0000-0003-0057-3485, Scopus Author ID: 57209566642, eLIBRARY ID: 27261171; e-mail: gil-oulbem@hotmail.com

ческих планов. Представлены как теоретические исследования в области геометрии и прочности торсов, так и образцы применения торсов в реальных структурах. Проведенный анализ источников показал, что торсы нашли применение в судо-, самолето- и машиностроении, в архитектуре и строительстве, в формах инженерного оборудования, в дорожном строительстве, при строительстве противоэрозионных валов, в топографии и картографии, в текстильной промышленности, в скульптурных формах и при аппроксимации сложных поверхностей. Это подтверждается ссылками на большое число опубликованных работ по теме и реальными примерами возведенных сооружений и объектов. Приведено 14 иллюстраций соответствующих структур.

Ключевые слова: торсовая поверхность, судостроение, машиностроение, архитектура пространственных систем, инженерное оборудование, дорожное строительство, антиэрозионный вал, топография, изделия легкой промышленности, скульптурная форма

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Introduction

At present time, hundreds of scientific papers, tens of monographies, devoted to investigations of torsal surfaces (torses, tangential developables) and shells, were published and many theses were defended on the considered subject. French scientist G. Monge began theoretical investigation of torsal surfaces in 1805. However, they were known since Archimedes when Archimedes screw has been obtained by experimental means. Interest for study of torsal surfaces does not calm down till present time. The chronology of published scientific-and-technical papers is evidence of it [1]. The themes of published papers include the following branches of human activities: shipbuilding, aircraft construction, agricultural implements, machine building, architecture and building, road building, anti-erosive banks and cartography, clothing articles of light industry, sculptural forms.

Later on, let us consider only non-degenerated developable surfaces, i.e. tangential developable surfaces, leaving the degenerated developable surfaces, i.e. cylindrical and conical surfaces, out of this review [2]. Tangential developable surfaces, cones, and cylinders are the ruled surfaces of zero Gaussian curvature and they permit uncoiling on plane without any folds and breakages. A tangent developable is a developable surface constructed by the union of the tangent lines of a space curve.

Potentialities and advantages of torsos are presented and described in many works [1; 3–6], but only small part of real torses were put into practice as factory-made goods, thin-walled structures, and shell erections in spite of large number of designs, recommendations of scientists on introduction of new types of surfaces, and the sketches of erection.

A brief overview of publications on theoretical studies of torse surfaces and shells

In this section, we will indicate only works in which there is a large bibliography on the topic of the article, and the article itself contains interesting results on torses. Otherwise, it becomes necessary to indicate hundreds of sources.

The main theorems for developable non-degenerate surfaces and methods of their construction are presented in a paper [7] with 59 references. These issues are covered in more detail in the monograph [1] with 386 references. All currently known ruled surfaces of negative and zero Gaussian curvature are indicated in an article [2].

A method of construction of developments of tangential developable surfaces is well studied. Geometricians presented seven analytical methods [8] and six graphical methods. A method of triangulation is the most popular.

There are works devoted to approximation of torses by hipped plate constructions [1]. Approximation simplifies considerably a process of manufacturing of torse article, but at present time, approximation is used only for substitution of cones by pyramids and cylinders by prisms.

The study of the process of rolling torses against each other is still of purely theoretical significance with recommendations for practical use [9; 10]. This problem is studied in works of V.S. Obukhova, A.L. Martirosov, S.F. Pilipaka, and G.S. Rachkovskaya. However, all their papers were published before 2008. New researches on this theme were not discovered.

As the presented studies show, all known torse surfaces are specified in a curvilinear non-orthogonal conjugate coordinate system, which complicates the calculation of thin shells with a middle torse surface for strength or stability. About a dozen works are known, for example [1; 11; 12], where the equilibrium equations, physical and geometric equations for the analytical calculation of torse shells are given. These equations have been used only for open developable helicoid [1; 13; 14]. The momentless theory for calculating torse shells given in curvilinear non-orthogonal coordinates is well-developed [1].

Only one type of tangent developable surfaces just equal slope surfaces can be easily given in lines of principal curvature. But now nobody solved the problem of strength analysis in analytical form. O.O. Aleshina analysed an equal slope shell with the ellipse at the base on action of own weight with the help of a FEM and with the help of a variational difference energy method [15].

Published works on the application of torses in real structures make up a separate section in the research of torses. Based on these works, one can judge the demands of society for the application of torse forms and its awareness of the state of affairs in the theoretical results in this area.

The application of torse forms in real items, structures, and buildings

Tangential developable surfaces have applications not only in mathematics but also in engineering. Consider the main branches of human activity where torses have found a confirmed application and have been embodied in real items, structures, and buildings.

Shipbuilding. This branch of the national economy uses torse surfaces as a means of approximating the base surface of the skin of surface ships. The method is used to construct tape torses based on two parallel curves, which in turn are parallel to the waterline (Figure 1). Yacht and boat builders who work at home with plywood and aluminium [16], use widely developable surfaces.

Agricultural implements. The smoothness of the plow surface is of great importance to prevent soil sticking to the plow blade. The possibilities of using torse surfaces as plowshares and a description of methods for obtaining torse plows were considered in many works, for example, in [17; 18]. The production association "Selkhozkhimiya" (Kherson) proved that tools of this type were suitable for the cultivation of salt licks [19].

In order not to repeat the materials of previously published works, let us point out the course of lectures [20], which describes the main agricultural machines currently in use, and in the works [1; 5; 7; 9; 17–19] numerous sources are given, where the information on prototypes of the working bodies of these agricultural machines in the shape of torsal surfaces is presented. In all cases, parabolic bending of a flat metal workpiece is used without changing its thickness [21].

Figure 1. The erection of steel ship hull
(Available from: ladverf.ru (accessed: 30.04.2021))

Mechanical engineering. Torse surfaces have been used in the design of gears [22]. There are works by V.S. Lyukshin, where the questions of the application of open developable helicoids for the formation of cutting tools are discussed in great detail and the treatment of an involute helical surface by the rolling method is described.

In mechanical engineering some surfaces have to be produced from sheet metal without deep-drawing, merely by bending the sheet. A developable car designed by Gregory Epps is a piecewise-smooth surface which can be decomposed into planar, cylindrical, conical and general tangent-surface-type developables [23]. One can assemble a model of real helicopter from fragments of developable surfaces [24]. In machine building, cylindrical and conical surfaces are used very widely in comparison with tangential developable surfaces.

Architecture and building. On the topic of this section, several structures were found (Figures 2 and 3), where the method of constructing a torse along two predetermined edge curves was used. In modern architecture, their properties were most successfully applied by Hans Hollein, Frank O. Gehry (Figure 2, *b*), Santiago Calatrava. Besides the building shown in Figure 2, *b*, F.O. Gehry used developable surfaces for forming of the overlap of the MARTa Herford Museum in Herford, Germany, and in the Guggenheim Museum in Bilbao, Spain. Developable walls of Disney concert hall in Los Angeles are well known.

Structures, shown in Figures 2, *a* and 3, *b*, were made by parabolic bending of initial plane thin sheet into the given position [21]. Additional examples of application of torsal surfaces in contemporary architecture are presented in a paper [4]. Georg Glaeser [4] writes that tangential developable surfaces are claimed by vanguard architecture where smooth surfaces are used and limitation on cost of building is especially taken into account.

In general, geometric knowledge in combination with new methods of structural computation opens up new approaches to manufacturing and fabrication of freeform surfaces with approximation of them by developable surfaces.

Several sketches of buildings that can be the basis of architectural designs are available additionally (Figure 4) [5].

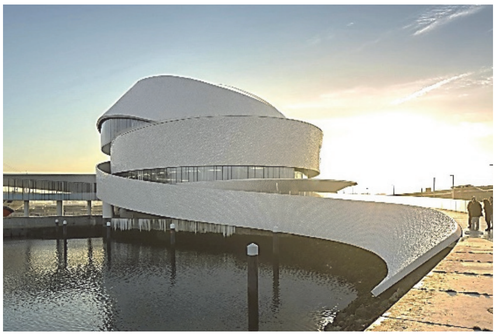
If we assume that at the base of the structure shown in Figure 4, an ellipse lies, and the vertex has the shape of a circle with a radius *R*, then the parametric equations of this torse surface can be taken in the monograph [1]. If we assume, that in Figure 4 the surface of an equal slope with an ellipse at the base is shown, then its equation is given in [25].

As an example, we can also cite a structure where the covering is made in the form of a torse with an edge of regression on a circular cone (Figure 5). This solution was repeated in several buildings.

a

b

Figure 2. Pavilion Luxembourg, Expo2020 (*a*) & Hotel Marques de Riscal, Spain, arch. Frank Gerhy (*b*)
(Available from: https://www.luxembourgexpo2020dubai.lu/en/le-design-2/le-design/ (*a*); http://arx.novosibdom.ru/node/1853 (*b*) (accessed: 30.01.2021))



a

b

Figure 3. A cruise terminal in the port of Leixoes, Portugal, arch. Luis Pedro Silva, 2015 (*a*) & a building "The planet KVN", Moscow (*b*)
(photo *a* available from: https://www.pinterest.ru/pin/272608583674986563/ (accessed: 10.08.2019); photo *b* by E.A. Grinko)

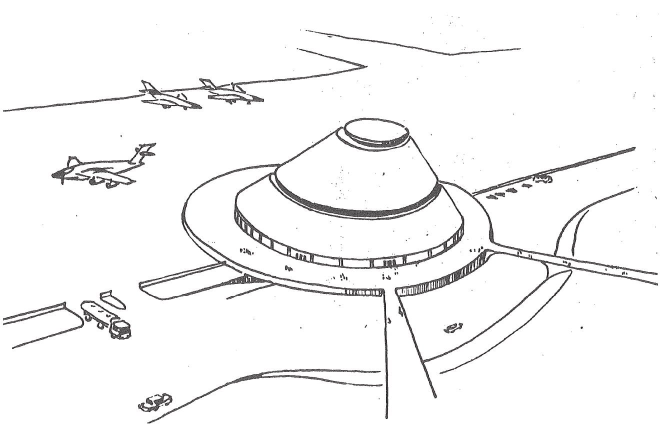
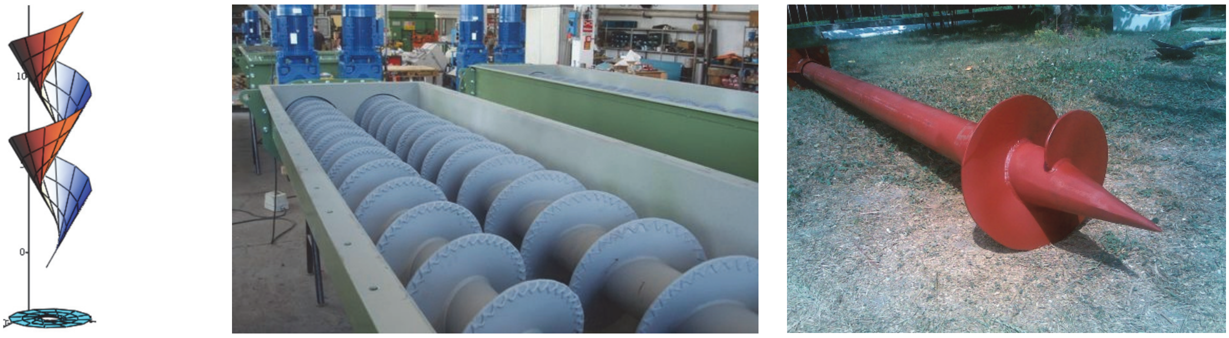


Figure 4. A sketch of building in airport
(a sketch was made by S.N. Krivoshapko)

Figure 5. Torse covering in the form of a conical helicoid
(Available from: https://www.kcur.org/show/central-standard/2015-02-12/the-bloody-history-of-mormonism-in-jackson-county (accessed: 10.05.2021))

Engineering equipment and communications. When constructing industrial buildings, much attention is paid to engineering equipment, in particular, pipelines. Pipelines can cross at different angles, large diameter pipelines can be split into multiple pipelines of different diameters, etc. In this case, torsal surfaces become indispensable in the design of mating units. This is clearly shown in the book [26]. Designers try to use cylindrical and conical surfaces, but sometimes torsal surfaces greatly simplify the design of the assembly. These connections are made by parabolic bending of a flat metal sheet with a preliminary construction of the developments of its elements [21].

An open developable helicoid (torse-helicoid or evolvent helicoid) is often used (Figure 6, *a*). It can be used as auger, screw, vertical or horizontal conveyor for transporting bulk, straw and semi-liquid loads (Figure 6, *b*), support anchor (Figure 6, *c*) [27]. Open helicoid is one continuous length as opposed to sectional flight, which consists of individual turns or segments that need to be welded together. Many scientific articles are devoted to the study of the stress-strain state of this construction.



a *b* *c*

Figure 6. A model of screw conveyer (*a*), a horizontal multi screw conveyer [IndiaMART] (*b*), a boring pile (*c*)
(Available from: https://www.exportersindia.com/product-detail/screw-conveyor-4292236.htm (accessed: 12.08.2021))

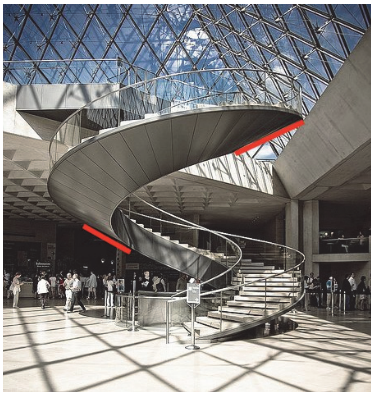


Figure 7. The Louvre, the Pyramid, Paris (Available from: https://www.pinterest.ru/pin/le-louvre-under-the-pyramid--469641067371696783/ (accessed: 12.08.2021))

Figure 8. MRL with light source [28]

Figure 9. Strengthening of slope of the embankment when grade and rounding of the road (Available from: https://www.geo-allianz.ru/objects/ukreplenie_otkosov_na_uchastke_dorogi_m-11/ (accessed: 30.04.2021))

The joint use of the developable and right helicoids in one spiral reinforced concrete staircase was first carried out in the museum-panorama “Stalingrad Battle” (Volgograd). A spiral staircase, the lower surface of which is a torse-helicoid, is used for ascent of visitors from the first floor to the observation deck. A developable helicoid can be seen also in entrance staircase of the Louvre by the Pyramid (Figure 7).

In [28], two example of designed lighting equipment with tangent surface were introduced at first (Figure 8). ‘Mountain Range Light’ (MRL), was designed to resemble the mountains surrounding the old city of Kyoto, Japan. Because the mainframe of MRL is the directing line of shade surfaces, the surfaces become developable ones.

Road building. The known geometric properties of the surface of the equal slope made possible to use them for approximation of topographic surface [29]. The same properties are used when designing embankment slopes when grade and rounding of roads, where the edge of the roadway is taken as the guiding curve (Figure 9). Examples of using surfaces of the equal slope when designing slopes in railway construction are available.

Anti-erosive banks, topography, and cartography. For the first time, torse surfaces were used for topographic purposes by G. Monge in 1764 working at the l'École Royale du Génie de Mézières, when he was given the task of determining the necessary height of an outer wall in a design of fortification [3].

Anti-erosion shafts on slopes have a curved shape, and they try to secure coincidence of banks with the equidistant curved lines of the topographic surface. Within two parallel curved lines, it is easy to construct a torse surface of a general type or a surface of the equal slope [20], which is used for approximation of a topographic surface.

Modern computer programs "DEVELOPABLE-MESH" and "UNFOLD program" give possibility to build developable surfaces according to separate scattered numerical marks, approximating geological structures with a given degree of accuracy. The method proposed in an article [30] was applied in real conditions (Red Mountain area in the Ventura basin).

In cartography, the problem of projecting the surface of the Earth's fragments onto a developable surface, which is then flattened onto a plane, has been completely solved [31].

Clothing articles of light industry. The application of the results of geometric modelling of light industry products and methods of cutting fabrics is described in papers [6; 32] with the involvement of a large number of used literature.

Figure 10. A sculpture from sheet materials, Austria (photo by N.S. Krivoshapko)

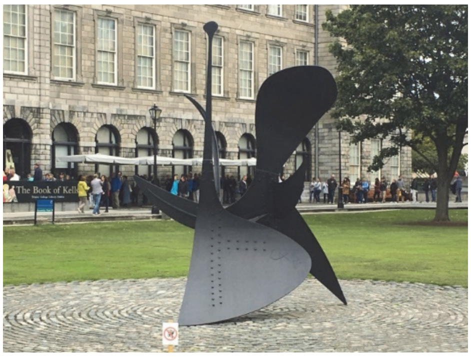


Figure 11. Abstract composition, Ireland (photo by N.S. Krivoshapko)

Figure 12. A ruled Möbius strip, Ekaterinburg, Russia (photo by N.E. Misyura)

Figure 13. Unfolding structure in the park (photo was taken in Internet)

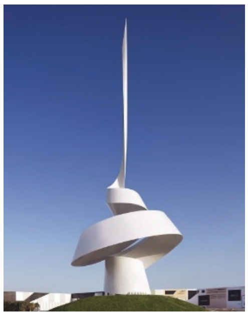


Figure 14. The Scroll, 2019, UAE (Available from: https://www.e-architect.com/dubai/the-scroll-sculpture-in-sharjah-uae (accessed: 30.04.2021))

Sculptural forms. Ilhan Koman was one of the innovative sculptors of the 20th century [33]. He frequently used mathematical concepts in creating his sculptures and discovered a wide variety of sculptural forms. He focused on developable sculptural forms and worked during a period that covers the late 1970's and early 1980's. Bruno Postle in conjunction with Gerry Judah [34] created and described sculpture in the form of a trefoil knot from sheet materials for Goodwood Festival of Speed in 2012. The full sculpture consists of eleven segments. The only structural components are the three surfaces themselves. These surfaces were developed in software into 2D cutting patterns and later cut from 6 mm steel plate. The plates were then welded together into eleven transportable sections and site-welded into the finished piece. The analogous sculpture was placed in Austria (Figure 10).

An interesting installation can be obtained by parabolic bending of a thin sheet of metal (Figure 11). Designers often use the Mobius strip (Figure 12) and other tape developable structures (Figure 13) to decorate parks and squares.

Gerry Judah has created a large sculpture titled "The Scroll" (Figure 14), 2019, before House of Wilson Library (Foster + Partners) in Sharjah, United Arab Emirates. This sculpture was designed in the form of developable strip.

There are many really erected sculptural forms in the form of torse elements. All of them were fulfilled in the style of contemporary art. This demonstrates the great interest of designers in the geometry of the torsal surfaces.

Modelling with developable surfaces. The reason why one wants to approximate complex surface by developable surfaces is manufacturing. The reinforced concrete shell requires formworks. This under construction is much easier to make if straight elements can be used. Approximation of non-developable surfaces by developable is studied in many works. Triangular fragments easily substitute developable surface due to its ability to develop on the plane. This problem is investigated in a work [35]. Johannes Wallner [36] presents several methods of approximation of complex surfaces by developable surfaces and gives the examples of substitution of developable surfaces by discrete conjugate nets consisting of finitely many discrete developables. Such a discrete surface is called a semi discrete conjugate net. A paper [37] studies geometric design of developable surfaces that consist of consecutive Bézier patches.

Tens of scientific papers are devoted to approximation of complex surfaces by tangential developable surfaces and tangential developable surfaces by plane triangular and quadrangular figures. Some of these works are indicated in the monograph [1] and in articles [9; 31; 36; 37] with a large amount of used literature.

Results

In all branches of the industries considered, construction and architecture, only thin-walled torse products and metal structures obtained by bending a thin workpiece have found real application. Torses have also found a real embodiment in mechanical engineering for the design of gears, in topography and in road construction. All other torse products, structures and erections are presented in sketches, projects, models and wishes of the authors of scientific and technical articles.

The purpose of this work is not to review all available scientific and technical literature on torses and torse shells. This was done in a monography [1] with 386 references, in a paper [38] with 44 sources, etc. The purpose of the article is to show the influence of scientific and technical information about torses on their use in real structures and buildings.

Additional information on recommendations for the application of torses, on products and structures in the form of torses, on the application of torses in aircraft construction, on the application of torse surfaces to study the geometry of complex surfaces and on the generalization of the concept of a torse to a multidimensional case is given in [1; 4; 5; 7; 9; 26]. The article does not repeat the content of previously published works, indicated in the used literature, but supplements them in terms of the application of torses in various branches of the national economy.

Conclusion

The provided information has shown that now, the study of torses and torse shells is carried out in the following directions: 1) construction of developments of torses on a plane with using a computer; 2) rolling of torses over each other; 3) application of the theory of torses to the geometry of surfaces of nonzero Gaussian curvature; 4) the use of torses in real structures; 5) analysis of thin torse shells for strength, stability, dynamics; 6) generalization of the concept of a torse surface to a multidimensional case.

The results obtained in all the indicated directions influence the recommendations for the application of torse in real products, structures and erections. Despite the successes in all these areas, the application of tangential developable surfaces in real structures, with the exception of shipbuilding, has not yet reached a satisfactory state.

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