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Surfaces of congruent sections of pendulum type on cylinders with generatrix superellipses

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Abstract. In 1972, I.I. Kotov proposed to separate the surfaces of congruent sections into a separate class and to include the surfaces of plane-and-parallel translation, surfaces of revolution, carved surfaces of Monge, cyclic surfaces with a generatrix circle of constant radius, rotative, spiroidal, and helical surfaces in it. The aim of the research is to obtain generalized parametric equations of surfaces of congruent sections of the pendulum type on right cylinders with plane-and-parallel translation of movable rigid superellipses. Analytical geometry methods are used. Computer systems MathCad and AutoCAD are applied to visualize surfaces. The results consist in the derivation of parametric equations of the studied surfaces in a general form convenient for the use of computer modeling methods. The technique is demonstrated on five examples with congruent mobile superellipses. The possibility of using obtained surface shapes in parametric architecture, free-form architecture, and in shaping of the surfaces of some technical products is noted.

Keywords: surfaces, congruent sections, superellipse, plane-parallel transfer, curves, astroid, surface assignment

Поверхности конгруэнтных сечений маятникового типа на цилиндрах с образующими суперэллипсами

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Аннотация. И.И. Котов в 1972 г. предложил выделить поверхности конгруэнтных сечений в отдельный класс и включить в него поверхности плоскопараллельного переноса, поверхности вращения, резные поверхности Монжа, циклические поверхности с образующей окружностью постоянно-

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го радиуса, ротативные, спироидальные и винтовые поверхности. Цель исследования – получение обобщенных параметрических уравнений поверхностей конгруэнтных сечений маятникового типа на прямых цилиндрах при плоскопараллельном переносе подвижных жестких суперэллипсов. Используются методы аналитической геометрии. Для визуализации поверхностей применяются компьютерные системы MathCad и AutoCad. Результаты заключаются в выводе параметрических уравнений изучаемых поверхностей в общем виде, удобном для использования методов компьютерного моделирования. Методика продемонстрирована на пяти примерах с конгруэнтными подвижными суперэллипсами. Отмечается возможность использования полученных форм поверхностей в параметрической архитектуре, архитектуре свободных форм и при формообразовании поверхностей некоторых технических изделий.

Ключевые слова: поверхность конгруэнтных сечений, суперэллипс, плоскопараллельный перенос кривых, астроида, задание поверхности

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Introduction

Recently, several papers have been published [1–4] devoted to the formation of surfaces of congruent sections of the pendulum type on arbitrary cylinders with forming plane curves in the form of circles [1; 2], parabolas [2; 3] and ellipses [4]. A surface of congruent sections is a surface bearing a continuous one-parameter family of plane lines. Such a surface is obtained as a result of moving some flat line (generatrix). The simplest types of surfaces of congruent sections are plane-and-parallel translation surfaces relative to the projection plane [5]. A plane-and-parallel transfer of a figure relative to the plane of projection is its movement in space, in which each of its points moves in its plane of level. Varieties of plane-and-parallel translation surfaces are right translation surfaces [6] (Figure 1).

Figure 1. The circular translation surface
(Cheremushkinsky Market, Moscow, photo by I.A. Mamieva)

The number of surfaces under consideration can be significantly expanded if we accept congruent plane curves, given in the form

$$|z|^n = T^n\left(1 - \frac{|y|^m}{W^m}\right),$$

where *n* and *m* are constant non-negative numbers.

By giving different values to the parameters n and m, it is possible to obtain various closed and open plane curves. For $n = m$, closed curves called superellipses are obtained [7]. Superellipses with $T = W$ are called Lame curves[1], for $n = m = 2$ and $T = W$, a circle is obtained, and for $n = m = 2$ and $T \neq W$, an ellipse. The more the value of the parameter $n = m$, the more precisely the shape of the superellipse approaches a rectangular contour.

Taking into account the method of forming of the surfaces under consideration, one can rank them among kinematic surfaces [8].

So far, superellipses and Lame curves have made possible to expand the range of solved geometric problems only in shipbuilding [9]. In architecture and construction, surfaces of congruent sections of the pendulum type with simple generating Lame curves in the form of a circle and an ellipse have been used [2; 3]. A paper [10] provides an example of using surface of congruent sections for cover of a bridge over the Kura River (Figure 2).

Figure 2. The glass Bridge of Peace, Tbilisi, Georgia (photo by I.A. Mamieva)

Figure 3. A shopping center, Khimki, Moscow region (photo by I.A. Mamieva)

There is also an example of a surface of congruent sections (Figure 3) in the city of Khimki (Moscow region). The need to construct an envelope of a family of congruent curves arises when surfaces of some technical products are formed [11].

[1] Weisstein E.W. Lamé Curve. *Wolfram MathWorld*. Available from: https://mathworld.wolfram.com/LameCurve.html (accessed: 30.05.2021).

Problem statement

Consider a right cylinder with a guiding superellipse, given in the form

$$z_0 = \pm T\left(1 - \frac{|y_0|^m}{W^m}\right)^{\frac{1}{n}}, \tag{1}$$

and a movable generatrix superellipse, given in the local coordinate system as

$$Z = \pm t\left(1 - \frac{|Y|^k}{\omega^k}\right)^{1/s}, \tag{2}$$

where n, m, k, s are constant non-negative numbers; the geometric parameters T, W, t, ω are shown in Figure 4.

In this case, the area, covered by the movement of the center of the movable superellipse (2) along the contour of the stationary superellipse (1), can be set according to Figure 4 by the equations:

$$y = y(y_0, Y) = y_0 + Y, z = z(y_0, Y) = |z_0| + Z = \left[T\left(1 - \frac{|y_0|^m}{W^m}\right)^{1/n} \pm t\left(1 - \frac{|Y|^k}{\omega^k}\right)^{1/s}\right]; \tag{3}$$

$$y = y(y_0, Y) = y_0 + Y, z = z(y_0, Y) = -|z_0| + Z = \left[-T\left(1 - \frac{|y_0|^m}{W^m}\right)^{1/n} \pm t\left(1 - \frac{|Y|^k}{\omega^k}\right)^{1/s}\right], \tag{4}$$

where $-W \le y_0 \le W$, $-\omega \le Y \le \omega$.

In formulas (3) and (4), y_0 and Y are independent variable parameters.

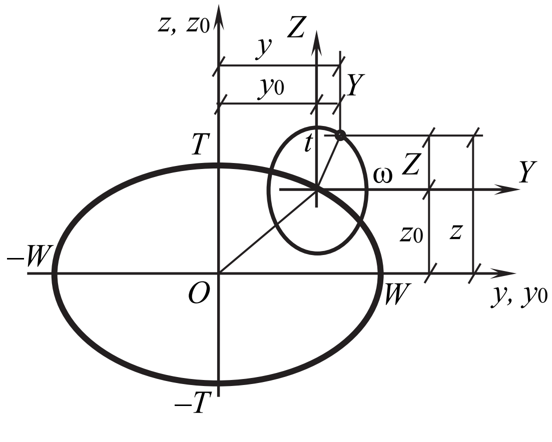

Figure 4. Scheme of formation of the surface of congruent sections

Considering that the movable superellipse performs oscillatory movements of the pendulum type and simultaneously moves uniformly along the x_0 axis (Figure 4), we can write:

$$y_0 = A\sin\frac{\pi x}{l}, \tag{5}$$

where A is the maximum deviation of the center of the moving superellipse from the Oz axis, that is, the amplitude of the sine wave in the horizontal plane xOy; l is the step of the half-wave of the sine wave.

In this case, the parametric equations of the surface of congruent sections of the pendulum type will have the form

$$x = x(x); \quad y = y(x, Y) = y_0 + Y; \tag{6}$$

$$z = z(x, Y) = |z_0| + Z = T\left(1 - \frac{|y_0|^m}{W^m}\right)^{1/n} \pm t\left(1 - \frac{|Y|^k}{\omega^k}\right)^{1/s}; \quad (7)$$

and

$$z = z(x, Y) = -|z_0| + Z = -T\left(1 - \frac{|y_0|^m}{W^m}\right)^{1/n} \pm t\left(1 - \frac{|Y|^k}{\omega^k}\right)^{1/s}. \quad (8)$$

Moreover, a formula (7) is used when constructing surface with a line of centers with $z_0 > 0$, and a formula (8) with $z_0 < 0$. The limits of the change in the parameter x are chosen arbitrarily, if necessary.

Example 1. Let formulas (1) and (2) define circles, that is, $m = n = k = s = 2$, and $T = W = 3$ m, $t = \omega = 1$ m, $A = 2$ m, $l = 2$ m, $-t \le Y \le t$; $0 \le x \le 4l$.

In this case, formulas (5)–(8) will take the form

$$x = x(x) = x; \quad y = y(x, Y) = y_0 + Y;$$

$$z = z(x, Y) = |z_0| + Z = (T^2 - y_0^2)^{1/2} \pm (t^2 - Y^2)^{1/2},$$

where

$$y_0 = A\sin(\pi x/l).$$

The surface is shown in Figure 5. This surface can be attributed to the subgroup of cyclic surfaces with a plane of parallelism from the class "Cyclic Surfaces." Some varieties of these surfaces are presented in [12].

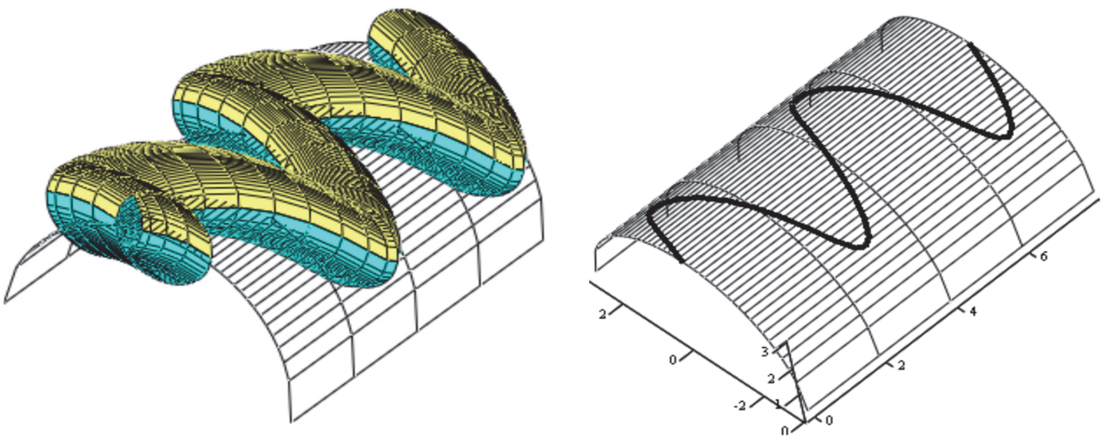

Figure 5. A cyclic surface on a circular cylinder and a line of centers of a movable circle on the cylinder

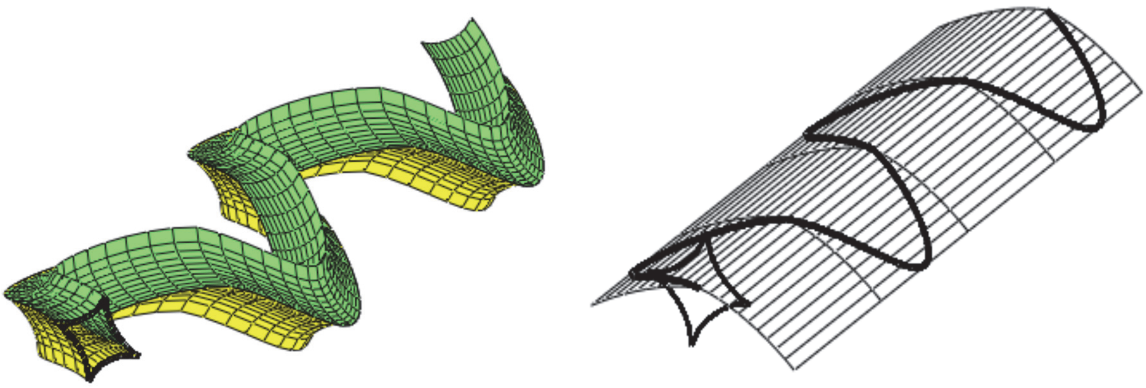

Figure 6. A congruent surface with a generative astroid on an oval cylinder and a line of centers of a movable astroid on the cylinder

Example 2. Let the cross section of a right cylinder has the form of a superellipse given by the formula (1), where T = 1 m, W = 1,5 m; m = n = 10/7, and the mobile superellipse has k = s = 2/3, t = ω = 0.5 m, that is, the mobile superellipse is an astroid [8]. In addition $-\omega \leq Y \leq \omega$; $0 \leq x \leq 4l$, l = 2 m, A = 1 m. Substituting the given values into formulas (5)–(8), we obtain parametric equations of the desired surface. The surface itself is shown in Figure 6.

***Example 3*.** Let formulas (1), (2) have the form

$$z_0 = \pm T\left(1 - \frac{|y_0|}{W}\right);\ \ Z = \pm t\left(1 - \frac{|Y|}{\omega}\right),$$

that is $m = n = k = s = 1$, and T = 1 m, W = 1.5 m; t = ω = 0.5 m, A = 1 m; $-t \leq Y \leq t$; $0 \leq x \leq 4l$, l = 1 m.

In this case, using formulas (5)–(8), it is possible to construct a box-shaped surface, shown in Figure 7. Box-shaped surfaces can be used in some sectors of the national economy. Various box-shaped surfaces with curved lines of centers are studied in [13].

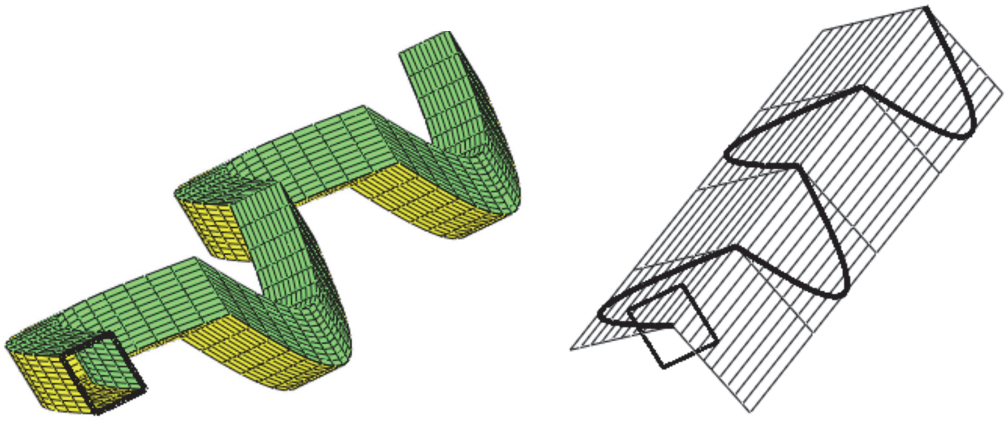

Figure 7. A congruent box-shaped surface on a box-shaped cylinder
and a line of centers of a movable quadrilateral on a box-shaped cylinder

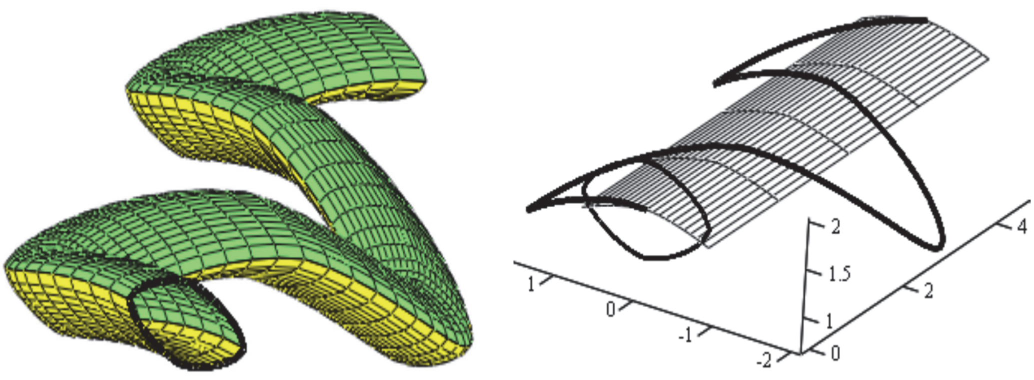

Figure 8. A congruent surface with generatrix ovals on an oval cylinder
and a line of centers of a movable oval

Example 4. Let the center of the movable oval (2) with k = s = 1.5; t = 0.5 m, ω = 0.8 m moves along a fixed oval (superellipse) (1) with m = n = 1.5; T = 1.5 m; W = 2.5 m and besides A = 2 m, l = 2 m, $-\omega \leq Y \leq \omega$; $0 \leq x \leq 3l$.

In this case, a formula (5) and parametric equations of the projected surface will take the form

$$x = x(x);\quad y = y(x, Y) = y_0 + Y;$$

$$z = z(x, Y) = |z_0| + Z = T\left(1 - \frac{|y_0|^{1,5}}{W^{1,5}}\right)^{1/1,5} \pm t\left(1 - \frac{|Y|^{1,5}}{\omega^{1,5}}\right)^{1/1,5};$$

$$y_0 = 2\sin(\pi x/2).$$

The surface is shown in Figure 8.

New problem statement

Superellipses (1), (2), taking into account that $m = n$ and $k = s$, can be represented as

$$y_0 = y_0(\beta) = W\cos^{2/m}\beta;\quad z_0 = z_0(\beta) = T\sin^{2/m}\beta; \tag{9}$$

$$Y = Y(\gamma) = \omega\cos^{2/k}\gamma;\quad Z = Z(\gamma) = t\sin^{2/k}\gamma, \tag{10}$$

then the equation of the surface of congruent sections of the pendulum type can be represented as

$$x = x(x);\quad y = y(x,\gamma) = y_0 + Y = A\sin(\pi x/l) + \omega\cos^{2/k}\gamma;$$

$$z = z(x, \gamma) = z_0 + Z = T\sin^{2/m}\beta + t\sin^{2/k}\gamma = T(1 - \cos^2\beta)^{1/m} + t\sin^{2/k}\gamma = T\,\{1 - |(A/W)\sin(\pi x/l)|^{m}\}^{1/m} + t\sin^{2/k}\gamma. \tag{11}$$

It should be borne in mind that $0 \le x \le C$; β, γ are the angles measured from the horizontal axis x or X (Figure 4), C is the required surface length,

$$\mathrm{tg} \propto = \frac{y_0}{z_0} = \frac{W}{T}\mathrm{ctg}^{\frac{2}{m}}\beta,$$

where α is the angle measured from the vertical axis Oz_0 clockwise (Figure 4).

Example 5. Let the center of the movable shaft (10) with $k = 1.5$; $t = 0.5$ m, $\omega = 0.8$ m moves along the stationary shaft (superellipse) (9) with $m = 1.5$; $T = 1.5$ m; $W = 2.5$ m and in addition $A = 2.5$ m, $l = 2$ m, $-\omega \le Y \le \omega$; $0 \le x \le 4l$, $0 \le \gamma \le 2\pi$.

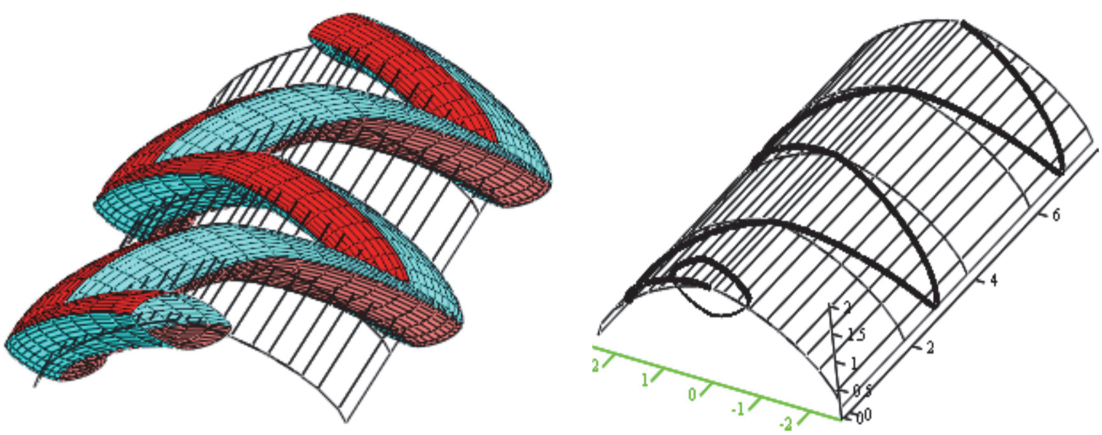

Figure 9. A congruent surface with generatrix ovals on an oval cylinder and a line of centers of a movable oval (10)

Substituting the above geometric parameters into the parametric equations of the surface (11), we obtain a pendulum-type surface with congruent curves, shown in Figure 9.

Results

Parametric equations of surfaces of congruent sections in the form of superellipses on right cylinders with guiding superellipses are obtained. The given method of constructing considered plane-and-parallel translation surfaces is illustrated by 5 examples. The four obtained surfaces are presented for the first time in Figures 5–8. With the help of parametric equations of general form obtained in this article, a large number of new surfaces of congruent sections of the pendulum type, as well as helical surfaces, can be constructed. Apparently, the surfaces of congruent sections of the pendulum type can be distinguished into a separate subgroup of the class “Surfaces of Congruent Sections.”

Conclusion

The article considers surfaces, formed by superellipses, that is, with $n = m$ and $s = k$. But the obtained parametric equations of the surface of the general form make it possible to consider the cases when $n \neq m$ and $s \neq k$. This will further expand the range of surfaces of congruent sections under consideration, since formulas (1), (2) can describe parabolas, hyperbolas and other open plane curves.

References

1. Krivoshapko S.N., Shambina S.L. On the question of surfaces of congruent sections of pendulum type on a circular cylinder. *Applied Geometry and Engineering Graphics*. 2011;(88):196–200. (In Russ.)

2. Krivoshapko S.N., Shambina S.L. The pendulum type surfaces with congruential cross sections. *Structural Mechanics of Engineering Constructions and Buildings.* 2021;17(2):165–174. https://doi.org/10.22363/1815-5235-2021-17-2-165-174

3. Grinko E.A. Surfaces of plane-parallel transfer of congruent curves. *Structural Mechanics and Analysis of Constructions.* 2021;3:71–77. (In Russ.) https://doi.org/10.37538/0039-2383.2021.3.71.77

4. Krivoshapko S.N., Ivanov V.N. Surfaces of congruent sections on cylinder. *Vestnik MGSU.* 2020;15(12):1620–1631. (In Russ.) https://doi.org/10.22227/1997-0935.2020.12.1620-1631

5. Kirillov S.V. Plane-parallel transfer surfaces. *Cybernetics of Graphics and Applied Geometry of Surfaces.* 1973;(10):21–25. (In Russ.)

6. Gbaguidi Aïssè G.L. Influence of the geometrical researches of surfaces of revolution and translation surfaces on design of unique structures. *Structural Mechanics of Engineering Constructions and Buildings*. 2019;15(4):308–314. https://doi.org/10.22363/1815-5235-2019-15-4-308-314

7. Méndez I., Casar B. A novel approach for the definition of small-field sizes using the concept of superellipse. *Radiation Physics and Chemistry.* 2021;189:109775. https://doi.org/10.1016/j.radphyschem.2021.109775

8. Abd-Ellah H.N., Abd-Rabo M.A. Kinematic surface generated by an equiform motion of astroid curve. *International Journal of Engineering Research and Science.* 2017;(3(7)):100–114. https://doi.org/10.25125/engineering-journal-IJO-ER-JUL-2017-13

9. Karnevich V.V. Hydrodynamic surfaces with midsection in the form of Lame curve. *RUDN Journal of Engineering Researches*. 2021;22(4):323–328. https://doi.org/10.22363/2312-8143-2021-22-4-323-328

10. Mamieva I.A. Analytical surfaces for parametrical architecture in contemporary buildings and structures. *Academia. Architecture and Construction*. 2020;1:150–165. (In Russ.)

11. Lyashkov A.A. Geometric and computer modeling of the main objects for shaping of technical products. *Omsk Scientific Bulletin. Series: Aviation-Rocket and Power Engineering*. 2017;1(2):9–16. (In Russ.)

12. Ivanov V.N. Geometry of the cyclic translation surfaces with generating circle and directrix meridians of the base sphere. *Structural Mechanics of Engineering Constructions and Buildings.* 2011;(2):3–8. (In Russ.)

13. Ivanov V.N. Geometry and forming of the polyhedral box type surfaces on base cyclic surface. *Structural Mechanics of Engineering Constructions and Buildings.* 2012;(2):3–10. (In Russ.)

Список литературы

1. *Кривошапко С.Н., Шамбина С.Л.* К вопросу о поверхностях конгруэнтных сечений маятникового типа на круговом цилиндре // Прикладна геометрія та інженерна графіка. Киев: КНУБА, 2011. Вип. 88. С. 196–200.

2. *Krivoshapko S.N.*, *Shambina S.L.* The pendulum type surfaces with congruential cross sections // Строительная механика инженерных конструкций и сооружений. 2021. Т. 17. № 2. С. 165–174. https://doi.org/10.22363/1815-5235-2021-17-2-165-174

3. *Гринько Е.А*. Поверхности плоскопараллельного переноса конгруэнтных кривых // Строительная механика и расчет сооружений. 2021. № 3. С. 71–77. https://doi.org/10.37538/0039-2383.2021.3.71.77

4. *Кривошапко С.Н.*, *Иванов В.Н.* Поверхности конгруэнтных сечений на цилиндрах // Вестник МГСУ. 2020. Т. 15. Вып. 12. С. 1620–163. https://doi.org/10.22227/1997-0935.2020.12.1620-1631

5. *Кириллов С.В.* Поверхности плоскопараллельного переноса // Кибернетика графики и прикладная геометрия поверхностей. М.: МАИ, 1973. Вып. 10. С. 21–25. (Труды Московского авиационного института имени С. Орджоникидзе. Вып. 268).

6. *Gbaguidi Aïssè G.L.* Influence of the geometrical researches of surfaces of revolution and translation surfaces on design of unique structures // Строительная механика инженерных конструкций и сооружений. 2019. Т. 15. № 4. С. 308–314. https://doi.org/10.22363/1815-5235-2019-15-4-308-314

7. *Méndez I., Casar B.* A novel approach for the definition of small-field sizes using the concept of superellipse // Radiation Physics and Chemistry. 2021. Vol. 189. 109775. https://doi.org/10.1016/j.radphyschem.2021.109775

8. *Abd-Ellah H.N., Abd-Rabo M.A.* Kinematic surface generated by an equiform motion of astroid curve // International Journal of Engineering Research and Science. 2017. No 3(7). Pp. 100–114. https://doi.org/10.25125/engineering-journal-IJO-ER-JUL-2017-13

9. *Карневич В.В.* Гидродинамические поверхности с мидель-шпагоутом в форме кривых Ламе // Вестник Российского университета дружбы народов. Серия: Инженерные исследования. 2021. Т. 22. № 4. С. 323–328. https://doi.org/10.22363/2312-8143-2021-22-4-323-328

10. *Мамиева И.А.* Аналитические поверхности для параметрической архитектуры в современных зданиях и сооружениях // Academia. Архитектура и строительство. 2020. № 1. С. 150–165.

11. *Ляшков А.А.* Геометрическое и компьютерное моделирование основных объектов формообразования технических изделий // Омский научный вестник. Серия: Авиационно-ракетное и энергетическое машиностроение. 2017. Т. 1. № 2. С. 9–16.

12. *Иванов В.Н.* Геометрия циклических оболочек переноса с образующей окружностью и направляющими меридианами базовой сферы // Строительная механика инженерных конструкций и сооружений. 2011. № 2. С. 3–8.

13. *Иванов В.Н.* Геометрия и формообразование многогранных коробчатых криволинейных поверхностей на базовой циклической поверхности // Строительная механика инженерных конструкций и сооружений. 2012. № 2. С. 3–10.