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РАСЧЕТ ТОНКИХ УПРУГИХ ОБОЛОЧЕК
ANALYSIS OF THIN ELASTIC SHELLS

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RESEARCH ARTICLE / НАУЧНАЯ СТАТЬЯ

Trial design of umbrella type shell structures

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Abstract. To create aesthetically expressive and functional small architectural forms, it is advisable to use reinforced concrete umbrella type shells in the shape of surfaces that can be specified in an analytical form. Hard landscaping is a suitable field of application for insufficiently studied and tested structures, in contrast to large structures of high importance class. The paper gives an example of a trial variant design of a small garden and park structure in the form of an umbrella type shell, during which different types of umbrella surfaces were analyzed and three variants were selected. Among the studied forms are the following surfaces: a paraboloid of rotation, an umbrella-type surface with a sinusoidal generator, an umbrella-type surface with radial waves based on cubic parabolas (with central flat point). The calculation of stress-strain state of three shells under their own weight was carried out using the finite element method and the peculiarities of working under load of each type of structures were revealed, recommendations are given when designing similar structures.

Keywords: analytical surfaces, trial design study, geometric modeling, umbrella type shell, thin shell structures

Предварительное вариантное проектирование конструкций в виде оболочек зонтичного типа

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Аннотация. Для создания эстетически выразительных и функциональных малых архитектурных форм целесообразно применение железобетонных или композитных оболочек зонтичного типа в виде поверхностей, которые могут быть заданы в аналитической форме. Разные аналитические поверх-

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ности визуально похожи, но при этом значительно отличаются в плане работы под нагрузкой. Малые архитектурные формы являются подходящей областью применения для недостаточно изученных и апробированных конструкций, в отличие от крупных ответственных сооружений. Приводится пример вариантного проектирования небольшого садово-паркового сооружения в виде оболочки зонтичного типа, в ходе которого были проанализированы разные виды зонтичных поверхностей и выбраны три варианта. В числе исследуемых форм такие поверхности, как параболоид вращения, поверхность зонтичного типа с синусоидальной образующей, поверхность зонтичного типа с радиальными волнами, образованная кубическими параболами (с центральной плоскостной точкой). Произведены расчет на прочность и исследование распределения напряжений для трех оболочек, шарнирно закрепленных по краям, при действии собственного веса при помощи метода конечных элементов и выявлены особенности работы под нагрузкой каждого вида конструкций, даны рекомендации при проектировании аналогичных сооружений.

Ключевые слова: аналитические поверхности, эскизное вариантное проектирование, геометрическое моделирование, оболочки зонтичного типа, тонкие оболочки

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Introduction

These days parametric and mathematical architecture has won recognition from architects and structural engineers all over the world. The masterpieces of Felix Candela and Eduardo Torroja, Santiago Kalatrava and other architects are familiar to everyone interested in the subject.

Fabrication of such structures becomes more and more easy, using numerical program control equipment, 3D printers and other innovative technologies [1; 2].

Academy of Engineering of RUDN University pays attention to shell structure form finding in architecture and has its' own traditions, which tend to analytical surfaces and mathematical architecture [3; 4]. The numerous students' and researchers' jobs are devoted to shell structures [5]. Most jobs consider geometrical modeling of such objects, like [6; 7]. New equations of umbrella type surfaces are arised in [8; 9].

Numerous up-to-date investigations are devoted to calculation of strength and stability of various shapes on the base of analytical surfaces, like rotation surfaces and umbrella-like surfaces [10–14], and especially domes [15–17].

The detailed overview of research and application of umbrella type shells is given in [18]. Considering directly strength analysis of umbrella type shells of building structures, the papers [19; 20] should be noted.

Meanwhile, one of the top challenges in shell architecture actually is shape optimization in all aspects [21–26]. Modern form finding is governed by efficient stress and strain distribution, mass minimization and other optimization criteria.

So, in this paper, an example is given with comparison of different shell shapes to choose the more reasonable and efficient. The umbrella type surfaces were chosen for trial design studies of dome-like structures.

Materials and methods

Umbrella dome is a cyclic spatial structure, composed of several identical elements, which crossing lines are the generatrixes of some rotational surface, which is called contour surface. Umbrella domes have increased stiffness, stability and aesthetical properties.

Umbrella-type shells are cyclical structures, composed of several identical elements, every of them is described by the same analytical equations that the whole surface is described. The equations can be explicit, implicit or parametric.

The main method for calculation umbrella-type shells is the finite element method, which is implemented in ANSYS APDL. The finite element method is a method for approximate numerical solutions of physical problems. It is based on two main ideas: the discretization of the object under study into a finite set of elements and the piecewise-element approximation of the function under study.

The reinforced concrete shell roofs of umbrella type were introduced earlier than parametric architecture appeared. Umbrella domes are well-known architectural elements, they are used in traditional church domes, long-span public buildings and some special structures for civil and military purposes.

The form finding of such structures usually is defined only by architect's arbitrary considerations, or, for military or industrial assistant engineering structures, only by practical considerations, based on ease of fabrication. The purpose of this article is to concentrate on more reasonable form-finding, its optimization, and diversification of variants. The way for optimization of design and calculation is introduced as mathematical approach to form-finding. Each new shape should have definite mathematical equation. Such an approach fits into actual trend of parametrical and 'digital' architecture. Some design proposals, recommended for implementation, are given in a present job.

Types of umbrella surfaces

1. *Paraboloid of rotation with radial waves.* Paraboloid of rotation with radial waves is formed by parabolic 2D curves, which have common central point of vertexes. Lines, tangent to parabolic curves, appertain in one plane. Parabolas lie at any section, are parallel to Oz axis.

Parametric equations are [9]

$$x(u,v) = u\cos(v),$$
$$y(u,v) = u\sin(v), \qquad (1)$$
$$z(u,v) = (A\sin(nv)+b)u^2,$$

where A is magnitude of the wave; n is number of wave vertexes; v is angular coordinate (Figures 1, 2).

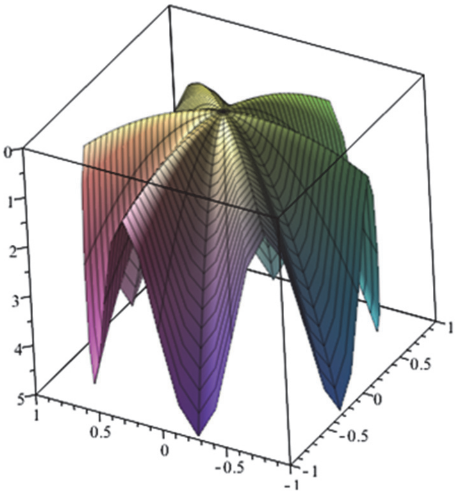

Figure 1. Paraboloid of rotation with radial waves with parameters $A = 1$, $n = 6$, $b = 1$, $uu = 0 \dots 30$, $v = 0 \dots 2\pi$

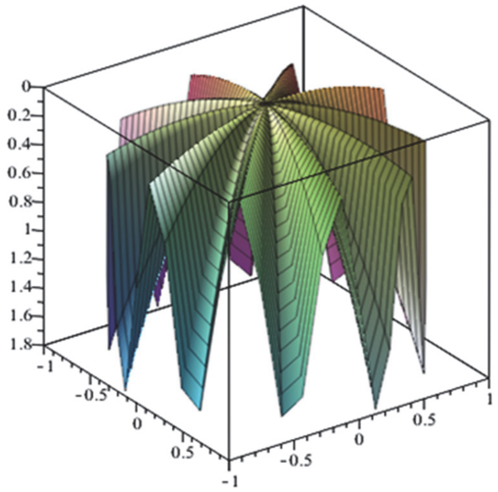

Figure 2. Paraboloid of rotation with radial waves with parameters $a = 0.8$, $n = 10$, $b = 1$, $uu = 0 \dots 1$, $v = 0 \dots 2\pi$

2. *Sphere with external cycloidal crimps* [18]:

a) type with epicycloid at base section (Figure 3):

$$x(u,\varphi) = ((R+r)\cos(\varphi) - r\cos((n+1)\varphi))\cos(u),$$
$$y(u,\varphi) = ((R+r)\sin(\varphi) - r\sin((n+1)\varphi))\cos(u), \qquad (2)$$
$$z(u,\varphi) = R\sin(u),$$

where R is radius of large circle; r is radius of small circle, small circle is rolling along the large one, arising epicycloid curve; n is number of outer vertexes of epicycloid; $r = R/n$; u, φ are coordinates;

b) type with hypocycloid at base section (Figure 4):

$$x(u,\varphi) = ((R-r)\cos(\varphi) - r\cos((n-1)\varphi))\cos(u),$$
$$y(u,\varphi) = ((R-r)\sin(\varphi) - r\sin((n-1)\varphi))\cos(u), \qquad (3)$$
$$z(u,\varphi) = R\sin(u).$$

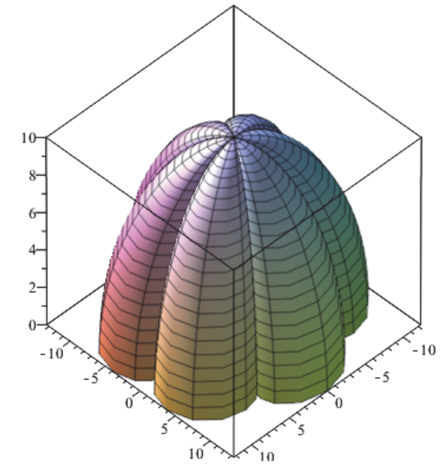

Figure 3. Sphere with external epicycloidal crimps with parameters $R = 10$, $n = 6$, $u = 0 \dots \pi/2$, $\varphi = 0 \dots 2\pi$

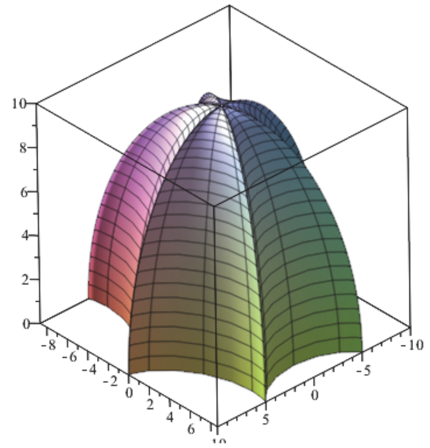

Figure 4. Sphere with external hypocycloidal crimps with parameters $R = 10$, $n = 6$, $u = 0 \dots \pi/2$, $\varphi = 0 \dots 2\pi$

3. *Corrugated surface of cubic parabolas* (Figure 5):

$$x(u,\varphi) = u^{1/3}((R+r)\cos(\varphi) - r\cos((n+1)\varphi)),$$
$$y(u,\varphi) = u^{1/3}((R+r)\sin(\varphi) - r\sin((n+1)\varphi)), \tag{4}$$
$$z(u,\varphi) = h(1-u),$$

where R and r – radii of large and small circles, which help to arise epicycloid, respectively; n – number of vertexes; h – height of the surface; u, φ are coordinates.

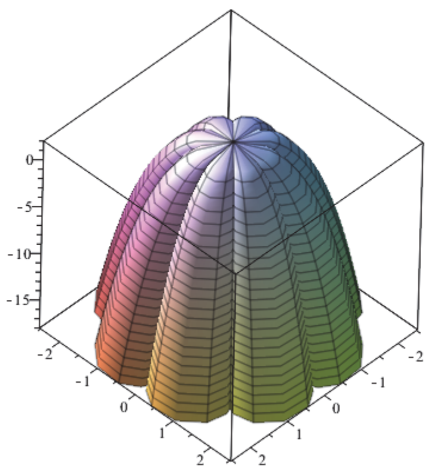

Figure 5. Corrugated surface of cubic parabolas $R = 1$, $n = 8$, $h = 2$, $r = R/n$

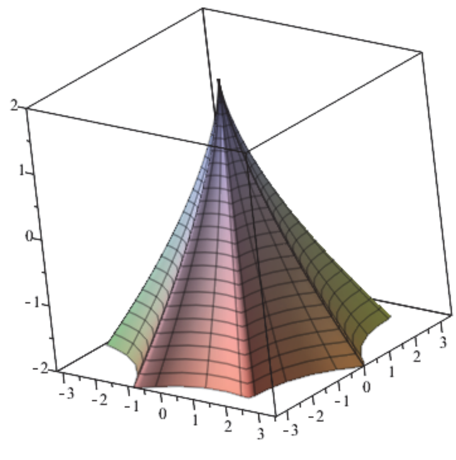

Figure 6. Corrugated surface of semi-cubic parabolas $R = 1$, $n = 3$, $h = 2$, $r = R/n$

4. *Corrugated surface of semi-cubic parabolas* (Figure 6) [18]:

$$x(u,\varphi) = u^{3/2}((R+r)\cos(\varphi) + r\cos((n+1)\varphi)),$$

$$y(u,\varphi) = u^{3/2}((R+r)\sin(\varphi) - r\sin((n+1)\varphi)), \tag{5}$$

$$z(u,\varphi) = h(1-u),$$

5. *Umbrella type surface with sinusoid generatrix* (Figure 7) [18]:

$$x(u,v) = u\cos(v),$$

$$y(u,\varphi) = u\sin(v), \tag{6}$$

$$z(u,\varphi) = a\sin(nv),$$

where a is the maximum magnitude of sinusoid generatrix; u, v are coordinates.

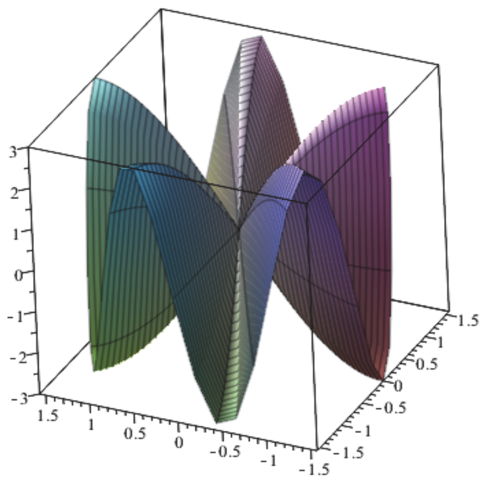

Figure 7. Umbrella type surface with sinusoid generatrix with parameters $n = 5$, $a = 3$, $u = 0 \dots \pi/2$, $v = 0 \dots 2\pi$

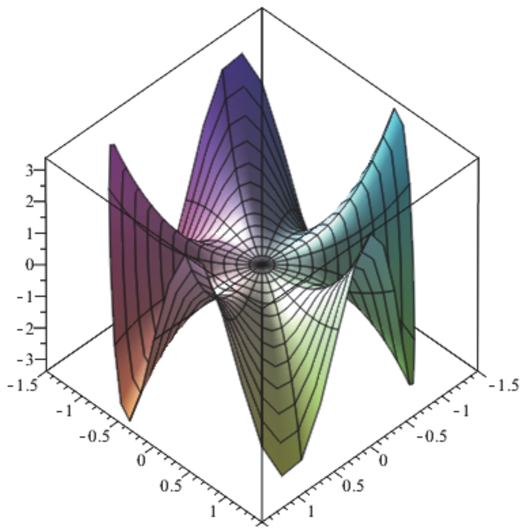

Figure 8. Umbrella type surface with flat central point with parameters $n = 5$, $a = 1$, $u = -1.5 \dots 1.5$, $v = 0 \dots 2\pi$

6. *Umbrella surface with radial waves, generated by cubic parabolas* (Figure 8) [18]. This surface has central flat point, the radial waves are decaying in this point.

$$x(u,v) = u\cos(v),$$

$$y(u,v) = u\sin(v), \tag{7}$$

$$z(u,v) = au^3\sin(nv),$$

where a is constant; n is waves number; u, v – coordinates.

Example of structure designing

While designing aesthetically attractive structures and buildings, several variants can be taken into consideration, the same span can be ceiled by shells of very close visually, but quite different mathematically shapes.

For example, let us sketch the roof for small exhibition hall or recreation facility like garden house, with circular plan and five sections. The diameter of this structure is 6 m, height is approximately 6 m. Three different umbrella surfaces are suitable for this purpose (see Figure 8): 1) paraboloid of rotation with radial waves, parameters $n = 5,\ b = 0,\ a = 0.35,\ u = 0\ ...\ 3,\ v = 0\ ...\ 2\pi$; 2) umbrella type surface with sinusoid generatrix, parameters $n = 5,\ b = 0,\ a = 3,\ u = 0\ ...\ \pi/2,\ v = 0\ ...\ 2\pi$; 3) umbrella surface with radial waves, generated by cubic parabolas, parameters $n = 5,\ a = 0.115,\ u = -3\ ...\ 3,\ v = 0\ ...\ \pi$.

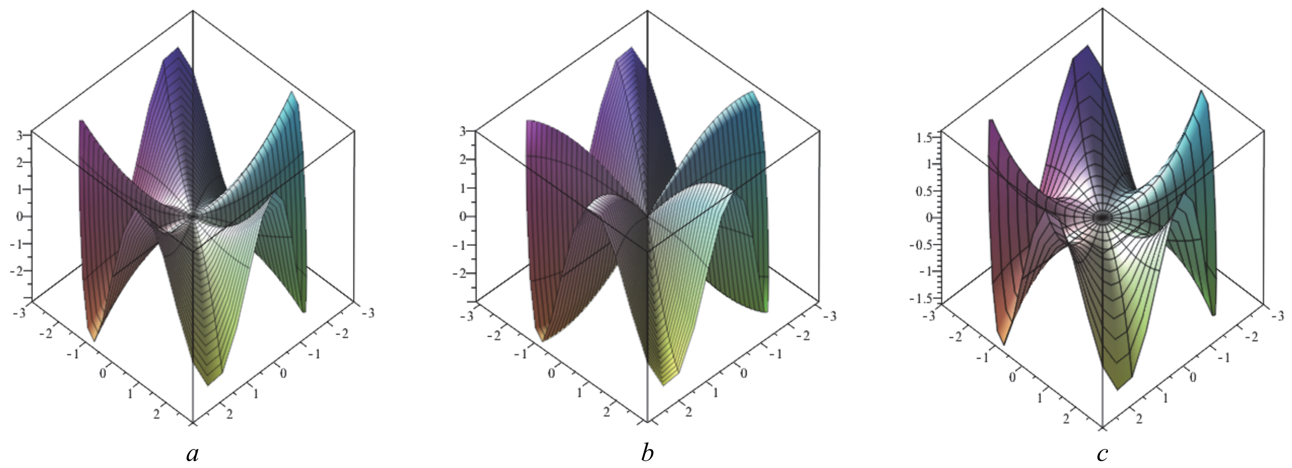

Figure 9. Three variants, visually similar in shape:
a – a paraboloid of rotation; *b* – an umbrella-type surface with a sinusoidal generator;
c – an umbrella-type surface with radial waves based on cubic parabolas (with central flat point)

It presents to be of interest to calculate stress-strain state of these shells to find more optimal one, which has the lowest values of bending moments, when material is mostly subjected to axial forces and particularly compression. The structure behavior can be predicted by analogy to corresponding arcs, but such an analysis doesn't take into account tridimensional behaviour of the shell. So, the aim of this job is a finite element analysis of the whole special structure. The three variants from Figure 9 were compared.

The aims of comparison were to reveal the features of each variant and give the recommendations for their application. Such recommendations should be taken into account while architectural sketching and desing, while creating finite element models.

Various software has special tools for creating surfaces by equations, but the results are sometimes not appropriate, so in this job it was decided to use Ansys APDL which needs directly writing special macros to construct geometry and finite element model.

1. *Paraboloid of rotation.* For creating this model the special macro was written to construct main nodes, which were defined as keypoints. The model consists of 72 splines, each spline has 20 basic keypoints, obtained by the surface equation. The problem occurred in the central point, which has singularity, so the central part was approximated by triangles. The very shell surface is got by skinning (special function for area creation) and meshed by size of 5 cm. The model is presented at the Figure 10.

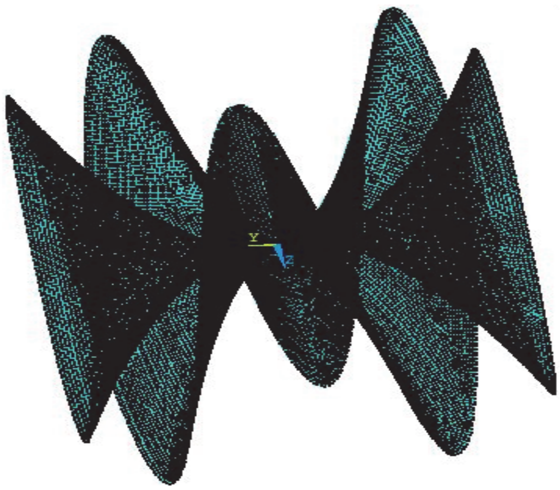

Figure 10. Finite element model of paraboloid of rotation

The boundary conditions were set as a hinged support along the edges, like this dome is supported freely on some supporting contour of load-bearing walls. The loading of dead weight is applied by density and gravity acceleration definition and their taking into account.

The thickness of the shell was defined as 8 cm, the material characteristics are: Young modulus is 325 MPa, Poisson's ratio is 0,17. The deflections of the model are presented at the Figure 11.

The equivalent stresses (von Mises) are presented at the Figure 12.

The maximum stresses occured at the edges, the maximum is 27 835 N/m^2. It can be concluded that it is necessary to strengthen the supporting contour on the edges.

2. *Umbrella type surface with sinusoid generatrix.* The model is analogous to the paraboloid of rotation model, but this one needed some refinement at the vawes tops. Finally, the appropriate results were obtained with meshing size of 2 cm. The central point, like that in paraboloid of rotation, has singularity, so it was filled by triangles like in the first example (Figures 13, 14).

For the preliminary analysis the model was considered sufficient and the calculation of deflection and equivalent stresses was conducted under dead weight loading.

The maximum deflection is $0.69 \cdot 10^{-5}$ m (Figure 15), the maximum stress is 99 764 N/m^2 (Figure 16). The maximum stresses occurred in the waves tops. Due to the stresses distribution, it will be rational to strengthen the structure with ribs.

3. *Umbrella surface with radial waves, generated by cubic parabolas.* The model has small difference from the first two models, specifically, the surface does not have singularity in the central point.

For the preliminary analysis the model was considered sufficient and the calculation of deflection and equivalent stresses was conducted under dead weight loading.

The results are presented at Figures 17–19. The maximum deflection occurred in the central point, it is $0.723 \cdot 10^{-5}$m. The maximum stresses are 36 785 N/m^2. The most dangerous sections are located closer to the middle, the maximum stresses are small compared to the maximum stresses, for example, in the second example. It can be recommended to arrange a support ring in the center, especially since the central point is flat. Comparison is given in Table.

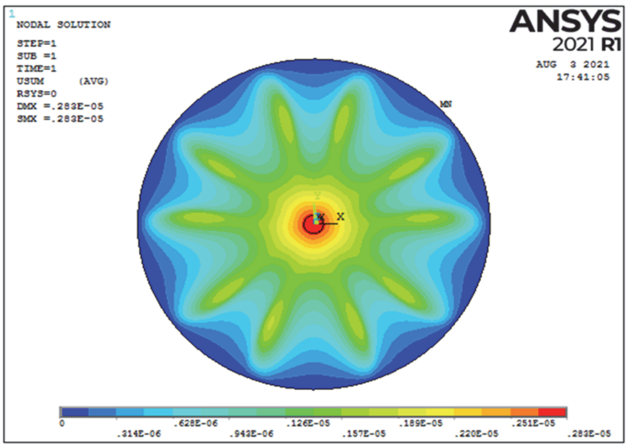

Figure 11. Vertical deflections u_z (isofields)

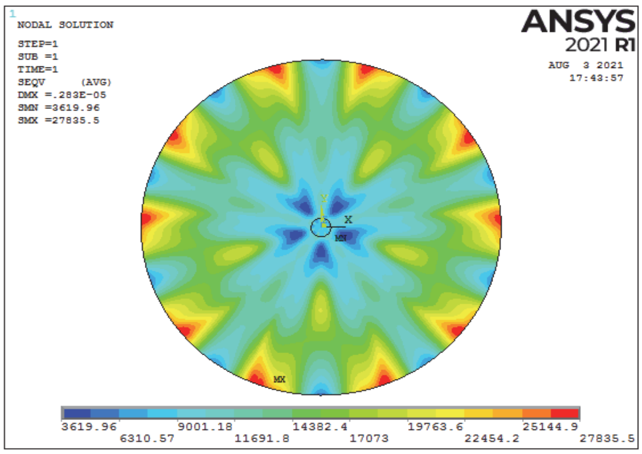

Figure 12. Equivalent stresses (isofields)

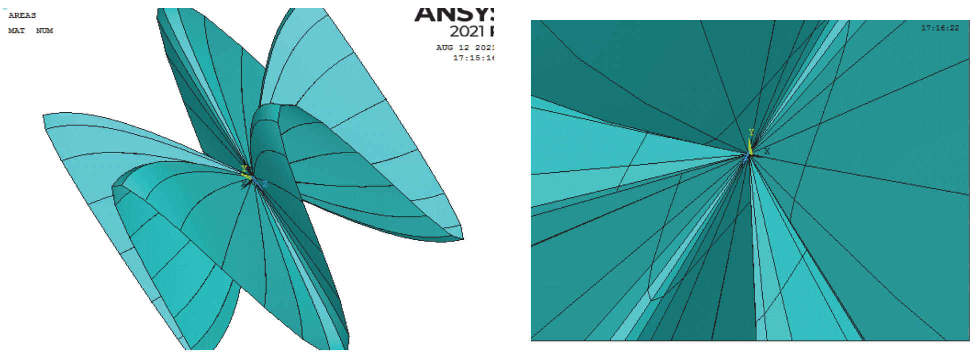

Figure 13. Creation of geometrical primitives

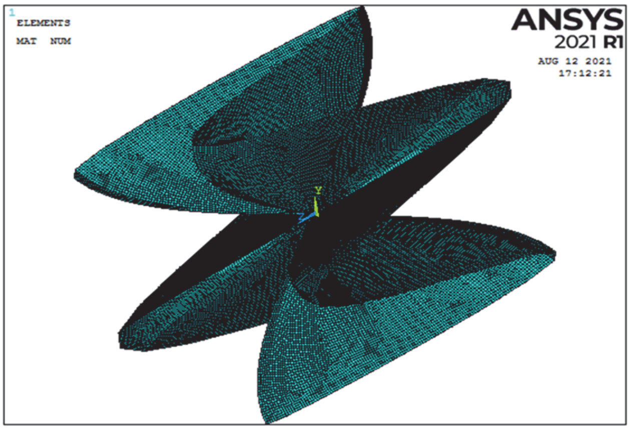

Figure 14. The meshed model of umbrella type surface with sinusoid generatrix

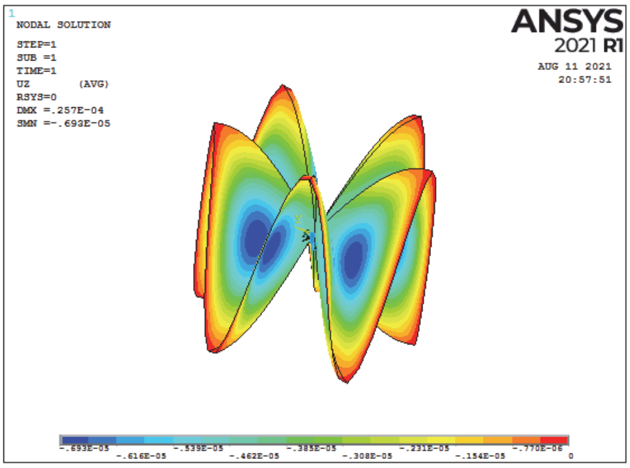

Figure 15. Vertical deflections u_z (isofields)

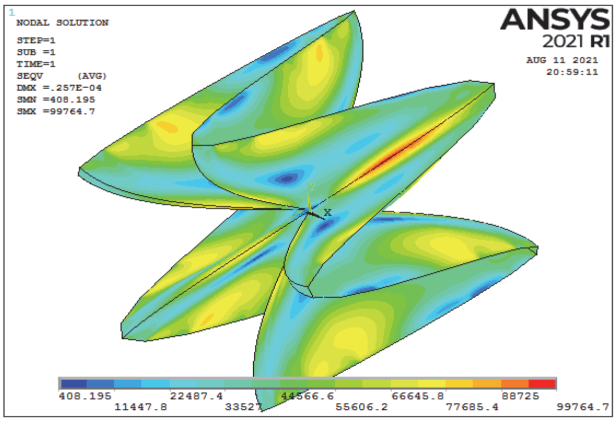

Figure 16. Equivalent stress (isofields)

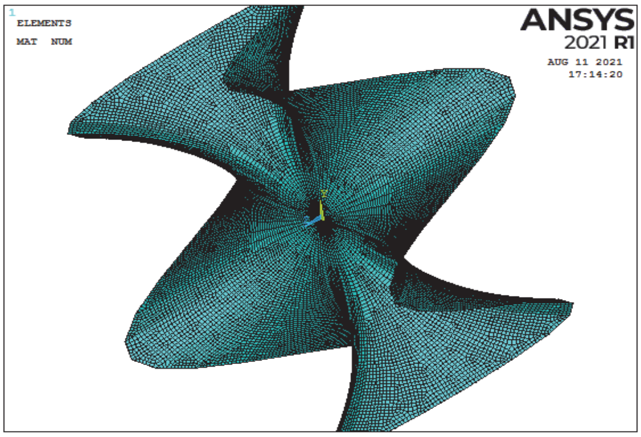

Figure 17. The meshed model of umbrella type surface

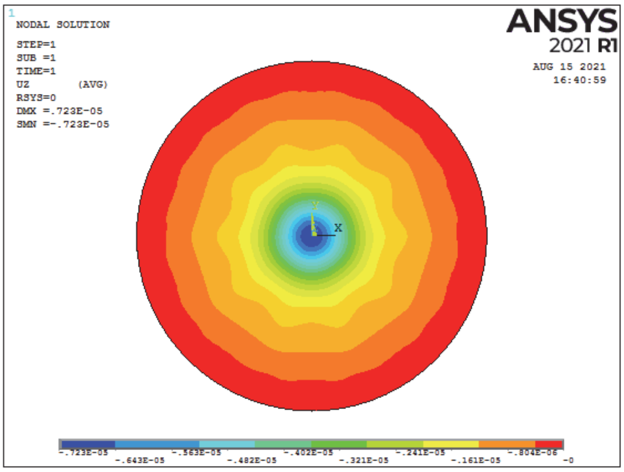

Figure 18. Vertical deflections u_z (isofields)

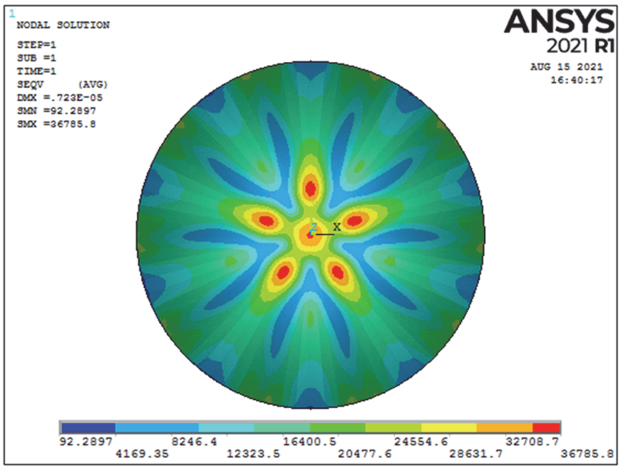

Figure 19. Equivalent stress (isofields)

Surface type	Max equivalent stress (von Mises), N/m^2	Max deflection, m	Max bending moment *My*, N·m/m	Max axial force *Fz*, N/m
1. Paraboloid of rotation	27 835	$0.281\cdot10^{-5}$	0.27	–5.41
2. Umbrella type surface with sinusoid generatrix	99 764	$0.69\cdot10^{-5}$	0.96	–5.26
3. Umbrella surface with radial waves, generated by cubic parabolas	36 785	$0.723\cdot10^{-5}$	–0.087	–0.98

The equivalent stress is chosen as a main parameter of comparison, because it is most clear in physical meaning, the bending moment and axial force enlisted in the table are calculated in cylindrical coordinate system (which does not fully corresponds to surface inner geometry), so it can be used only for approximate estimation of forces and moments. Compared to concrete compression and even tension strength, all these stresses are not significant, which means that in case of loadings of the same order as a dead weight, the reinforcement needed is minimal. The investigation of stability is necessary in prospects, the strengths evaluation is only the first stage of analysis. As the latest publications on the topic [18] show, the interest in umbrella shells and umbrella-type shells does not fade. Researchers are looking for new areas of application of the shells under consideration and not only in architecture and construction [12].

Conclusion

As results, the vertical deflection and equivalent stresses and their distribution were compared.

If consider the architect choosing from three different sketches, taking the revealed peculiarities into account present to be quite useful to conduct a technical and economic comparison of options.

Such an investigation is also be useful for classification and systematization of knowledge about shells in shape of analytical surfaces.

These results illustrate the idea [27] that visually similar shapes can have rather different strength characteristics and work under loading. The analytical shapes can be investigated to reveal some patterns and rules, in contrast to the free shapes, created only by architect's inspiration.

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