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Epihypocurves and epihypocyclic surfaces with arbitrary base curve

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Abstract. If a circle rolls around another motionless circle then a point bind with the rolling circle forms a curve. It is called epicycloid, if a circle is rolling outside the motionless circle; it is called hypocycloid if the circle is rolling inside the motionless circle. The point bind to the rolling circle forms a space curve if the rolling circle has the constant incline to the plane of the motionless circle. The cycloid curve is formed when the circle is rolling along a straight line. The geometry of the curves formed by the point bind to the circle rolling along some base curve is investigated at this study. The geometry of the surfaces formed when the circle there is rolling along some curve and rotates around the tangent to the curve is considered as well. Since when the circle rotates in the normal plane of the base curve, a point rigidly connected to the rotating circle arises the circle, then an epihypocycloidal cyclic surface is formed. The vector equations of the epihypocycloid curve and epihypocycloid cycle surfaces with any base curve are established. The figures of the epihypocycloids with base curves of ellipse and sinus are got on the base of the equations obtained. These figures demonstrate the opportunities of form finding of the surfaces arised by the cycle rolling along different base curves. Unlike epihypocycloidal curves and surfaces with a base circle, the shape of epihypocycloidal curves and surfaces with a base curve other than a circle depends on the initial rolling point of the circle on the base curve.

Keywords: geometry of the curves, geometry of the surfaces, base curve, epihypocycloids, epihypocycloid cycle surfaces

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Эпигипоциклоиды и эпигипоциклические поверхности с произвольной базовой кривой

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Аннотация. При качении окружности по другой неподвижной окружности точка, жестко связанная с подвижной окружностью, образует кривую: при качении неподвижной окружности – эпициклоиду, при качении по внутренней стороне неподвижной окружности – гипоциклоиду. При качении окружно-

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сти при постоянном наклоне к плоскости неподвижной окружности точка, жестко связанная с подвижной окружностью, описывает пространственную кривую. Циклоидой называется кривая, образованная точкой подвижной окружности, катящейся по прямой. Рассматривается геометрия кривых, образуемых точкой, жестко связанной с окружностью, катящейся по произвольной базовой кривой, а также геометрия поверхностей, образованных при одновременном качении окружности по базовой кривой и вращении окружности вокруг касательной к базовой кривой. Так как при вращении окружности в нормальной плоскости базовой кривой точка, жестко связанная с вращающейся окружностью, описывает окружность, то образуется эпигипоциклоидальная циклическая поверхность. Получено векторное уравнение эпигипоциклоид и эпигипоциклоидальных циклических поверхностей с произвольной базовой кривой. На основе векторных уравнений с использованием программного комплекса MathCad построены графики эпигипоциклоидальных кривых с базовым эллипсом и синусоидой. Приведены рисунки эпигипоциклоидальных циклических поверхностей с базовым эллипсом. Они показывают большие возможности формообразования новых видов поверхностей при качении окружности по различным базовым кривым. В отличие от эпигипоциклоидальных кривых и поверхностей с базовой окружностью форма эпигипоциклоидальных кривых и поверхностей с базовой кривой, отличной от окружности, зависит от начальной точки качения окружности на базовой кривой.

Ключевые слова: геометрия кривых, геометрия поверхностей, базовая кривая, эпигипоциклоиды, эпигипоциклоидальные циклические поверхности

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Introduction

Geometry of epi- and hypocycloids formed by the point bind with the circle which is rolling inside or outside the motionless circle is given in various textbooks, references books, monographs [1–9]. If the moving circle is moving along another motionless circle with some constant slope, then the point bind with the moving circle forms the space curve. If the moving circle is rotating around the tangent at the point of contact of the circles, the point bind with moving circle is arising a circle normal the tangent line. With rotation at 180° the point of epicycloid goes to point of hypocycloid and if the moving circle is rolling around the motionless circle and rotating at 180° the epicycloid curve goes to the hypocycloid curve. The space curves formed with constant slope of moving circle can be called the space epihypocycloids. If the moving circle is rolling along the motionless circle with rotating around tangents, the cyclic surfaces are formed. The geometry of the epihypocycloids and epihypocycloidal cycle surfaces there were analyzed in [10; 11]. It is shown in this paper that the epihypocycloid cycle surfaces are the canal surfaces [3; 12–15]. This article discusses the geometry of the epihypocycloid and epihypocycloidal surfaces formed when rolling a circle along an arbitrary base curve and cyclic surfaces formed when rolling and rotating a circle.

Geometry of the epihypocycloids of the common type

Let us consider the directrix base line $\boldsymbol{r}_{н}(u)$ along which the circle of radius a is rolling (Figure 1). The point d which is linked with the moving circle is arising the curve $\boldsymbol{r}(u)$. To obtain the equation of the curve it's necessary to determine the length of the base line, covered by the moving circle from the initial point u_0 to the contact point u of the motionless cycle: $S_{н}(u)=\int_{u_0}^{u} s'_{н}du$, где $s'_{н}=\left|\frac{\partial \boldsymbol{r}_{н}}{\partial u}\right|$. The radius of the moving circle will be rotated relative the normal of the base curve at the angle $\theta(u)=\frac{S_{н}(u)}{a}$.

For many curves the integral of the length doesn't have an analytical solution, but the integrals can be calculated by numerical methods, for example using MathCad software.

The vector equation of the formed curve can be written as

$$\boldsymbol{r}(u)=\boldsymbol{r}_{н}(u)-a(1-\mu\cos\theta)\boldsymbol{\nu}_{н}-a\mu\sin\theta\boldsymbol{\tau}_{н}, \tag{1}$$

where $\boldsymbol{\tau}_{н}$, $\boldsymbol{\nu}_{н}$ – the vectors of the tangent and normal of the base curve.

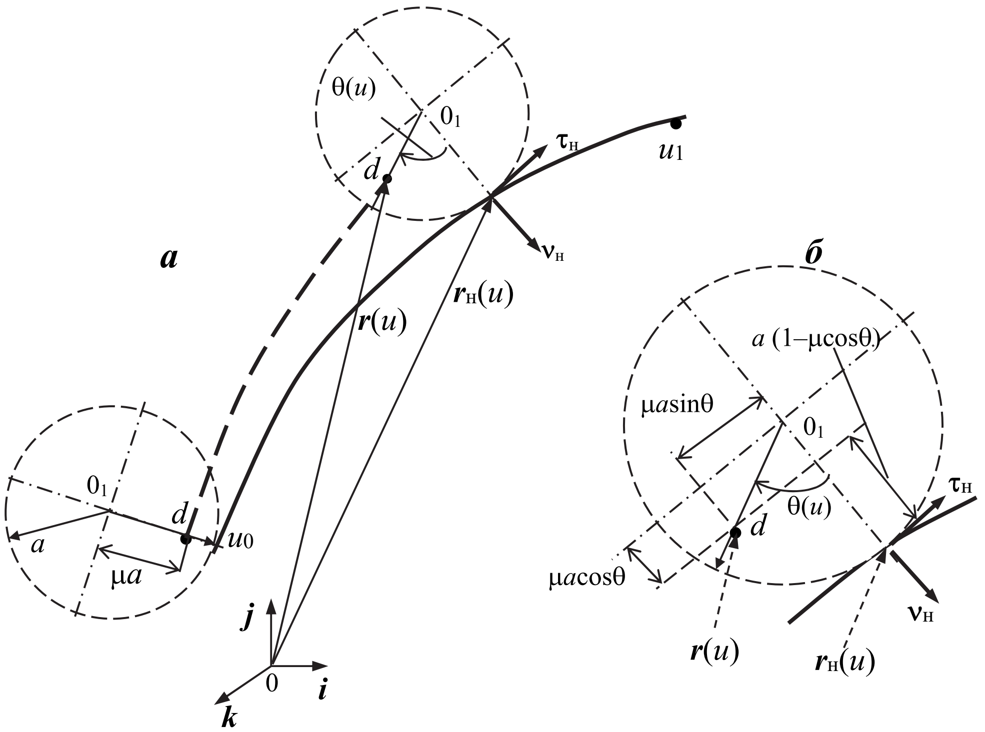

Figure 1. Rolling of the circle along base directrix

If the there is rolling along the base curve at the angle v to the plane of the base curve, then the point d forms the space curve, the vector equation of which can be obtained as following:

$$\boldsymbol{r}(u,v)=\boldsymbol{r}_{\text{н}}(u)-a\mu\sin\theta\boldsymbol{\tau}_{\text{н}}+a(1-\mu\cos\theta)\boldsymbol{e}(u,v), \tag{2}$$

where $\boldsymbol{e}(u,v)=\boldsymbol{\nu}\cos v+\boldsymbol{\beta}\sin v$, $\boldsymbol{\beta}$ – binormal of the base curve.

If the circle rolls from point u_0 towards point u_1 of the base curve and the radius of the rolling circle is determined as $a=\dfrac{S_{01}}{2k\pi}$, where $S_{12}=\int_{u_0}^{u_1}s'_{\text{н}}du$, then the moving circle makes k complete turns of 2π.

For plane base curve $\boldsymbol{\beta}=\boldsymbol{k}$ is an ort of rectangular coordinate system.

If angle v is 0, then the circle moves at concave side of the base curve, the hypocycloid is arised, if v = π, epicycloid is formed at the convex side of the base curve.

If parameter v changes from 0 to 2π, then point d describes a circle of radius $R=a(1-\mu\cos\theta)$ around the tangent to the base curve. If parameter v changes from 0 to π then point of the hypocycloid transforms into the point of the epicycloid.

If the moving circle is rolling along a base curve and rotating v = (0 ÷ 2π) the cycle surface is forming at all points of the contacts with the base curve.

$$\boldsymbol{\rho}(u,v)=\boldsymbol{r}_{\text{н}}(u)_{\text{н}}-a\mu\sin\theta\boldsymbol{\tau}_{\text{н}}+a(1-\mu\cos\theta)\boldsymbol{e}(u,v). \tag{3}$$

The equations (2), (3) are correct for forming cycloid surfaces by rolling the circle around both plane and space curve.

Epihypocycloids

At the Figure 2 there is shown the figures of the epihypocycloids with the base ellipse, given by the equation $\boldsymbol{r}_{\text{н}}(u)=b\cos u+c\sin u$, with parameters b = 2, c = 1 with point d on the moving circle (μ = 1) and with different radius of the moving circle and initial point u_0 at the base curve.

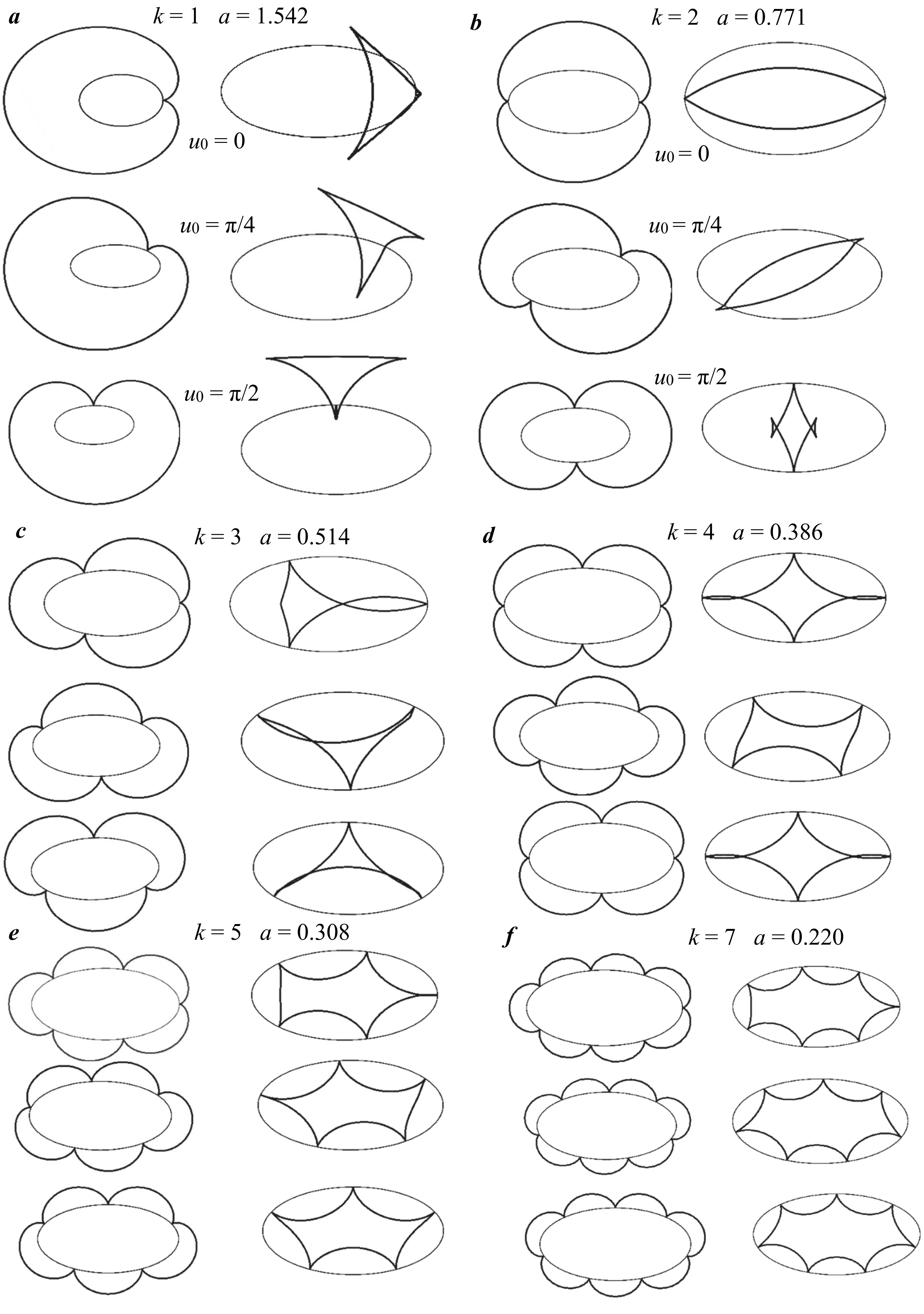

Figure 2. Epihypocycloids with the base ellipse $b = 2$, $c = 1$ formatting point is on the circle, $\mu = 1$

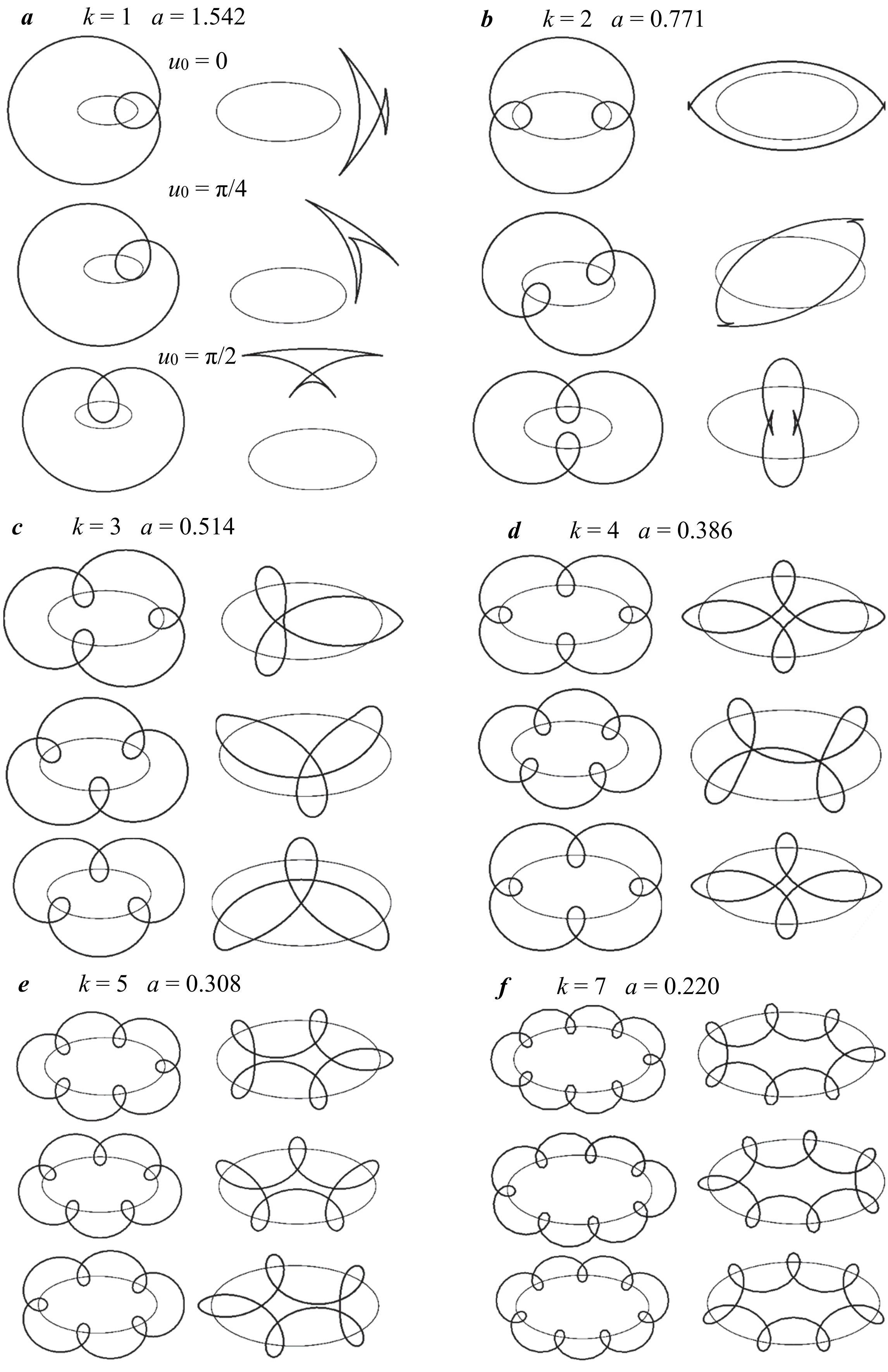

Figure 3. Epihypocycloids with the base ellipse $b = 2$, $c = 1$ formatting point is out of the circle, $\mu = 2$

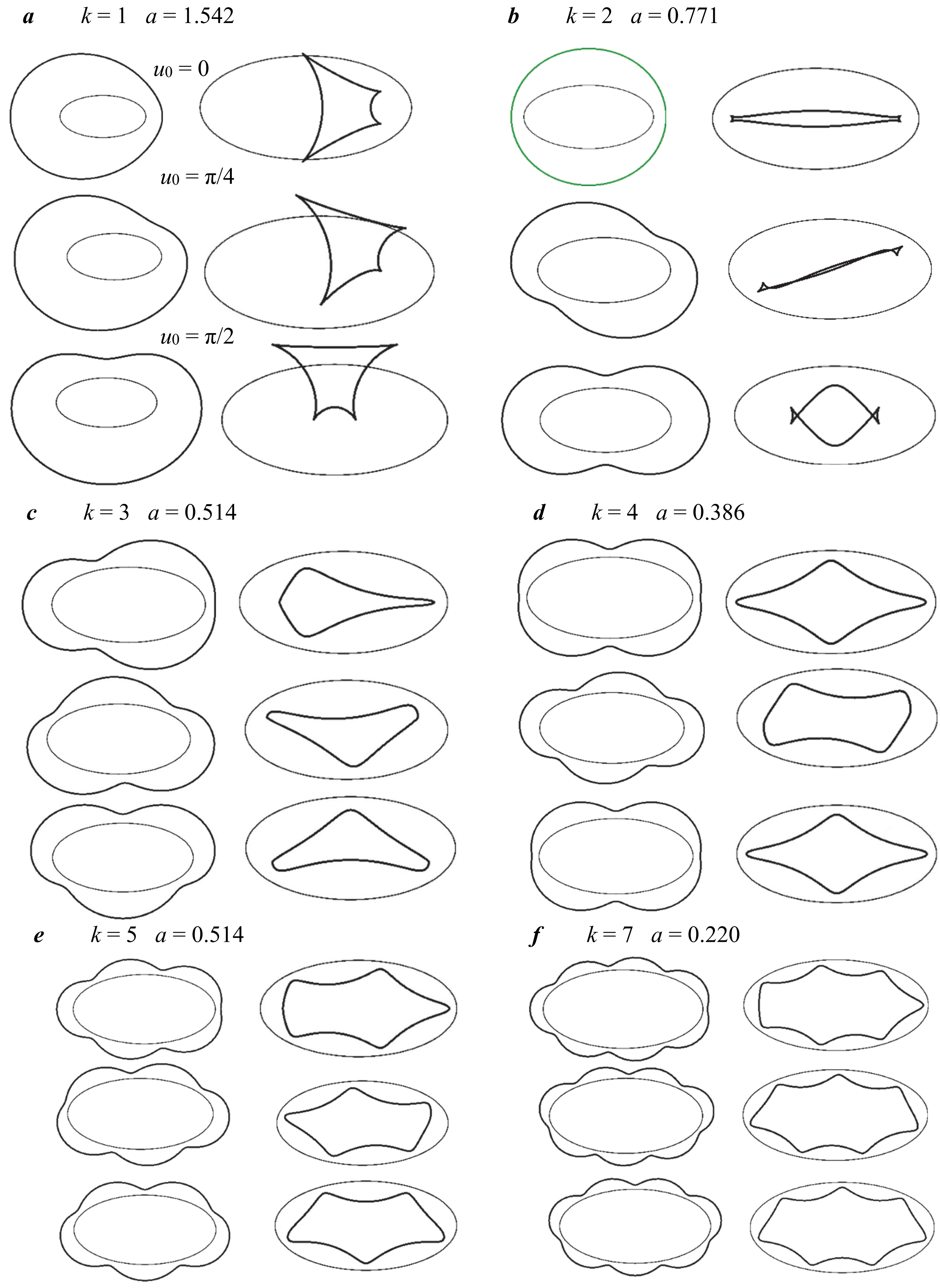

Figure 4. Epihypocycloids with the base ellipse $b = 2$, $c = 1$ formatting point is inside the circle, $\mu = 0.5$

As it's seen from the Figure 2 the shapes of the epi- and hypocycloids depend on the initial point u_0 on the base curve.

At the Figure 3 the figures of the epihypocycloids are shown, with the on the base ellipse with parameters $b = 2$, $c = 1$ and the forming point d outside the rolling circle ($\mu = 2$) and with different parameters of rolling circle.

At the Figure 4 the figures of the epihypocycloids are shown, with the base ellipse with parameters $b = 2$, $c = 1$ and the forming point d inside the rolling circle ($\mu = 0.5$) and with different parameters of rolling circle.

At the Figure 5 the figures of the epihypocycloids are shown with the base sine curve $y = a\sin\pi x/L$, $a = 0.65$, $L = 1$, $x = 0 \div 2L$; with various parameter of μ: a) $\mu = 1$; b) $\mu = 0.5$; c) $\mu = 2$.

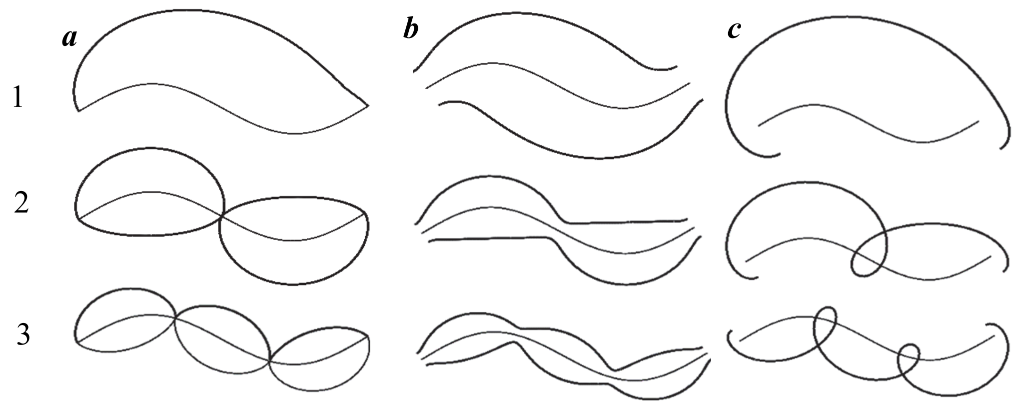

Figure 5. Epihypocycloids with the base sinus curve on two halfwaves:
a – $\mu = 1$; *b* – $\mu = 0.5$; *c* – $\mu = 2$; *1* – top row $\kappa = 1$; *2* – middle row $\kappa = 2$; *3* – lower row $\kappa = 3$

At the Figures 5, *a*1, *b*1, *c*2 the hypocycloids are shown. At another figures epi- and hypocycloids together are shown. It's seen from the combined figures that hypocycloids on the base sine are antisymmetrical to the epicycloids.

Epihypocycloid cycle surfaces

The figures of epihypocycloid cycle surfaces on the base ellipse are given with parameters $\kappa = 1$ (Figure 6), $\kappa = 2$ (Figure 7), $\kappa = 3$ (Figure 8).

At the Figure 9 there are shown epihypocycloid cycle surfaces on base sine with different numbers of half-waves.

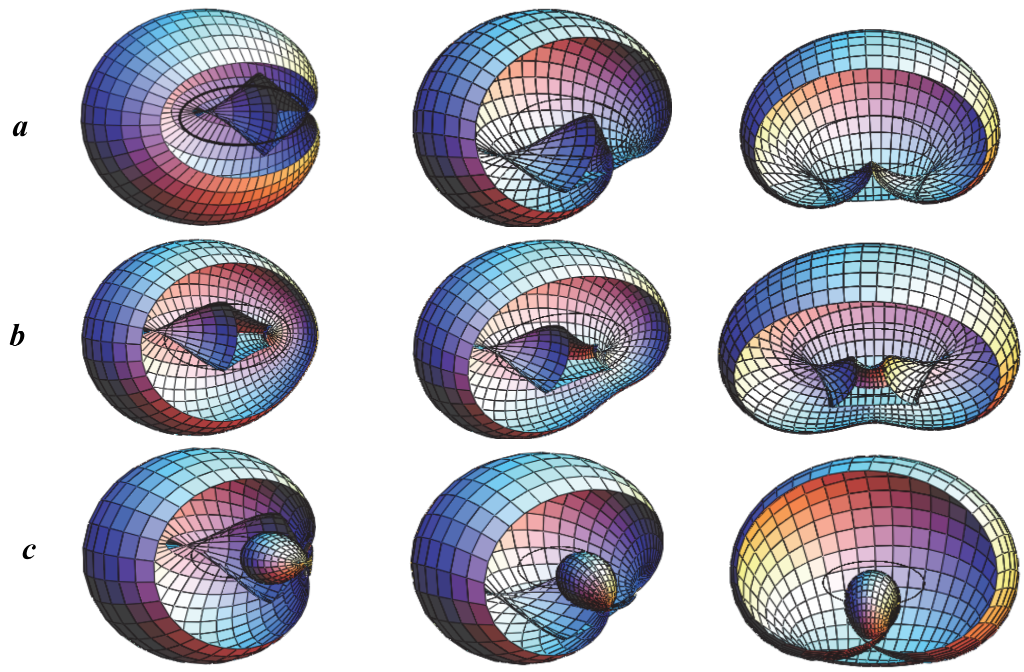

Figure 6. Epihypocycloid cycle surfaces with the base ellipse, $k = 1$:
a – $\mu = 1$; *b* – $\mu = 0.5$; *c* – $\mu = 2$

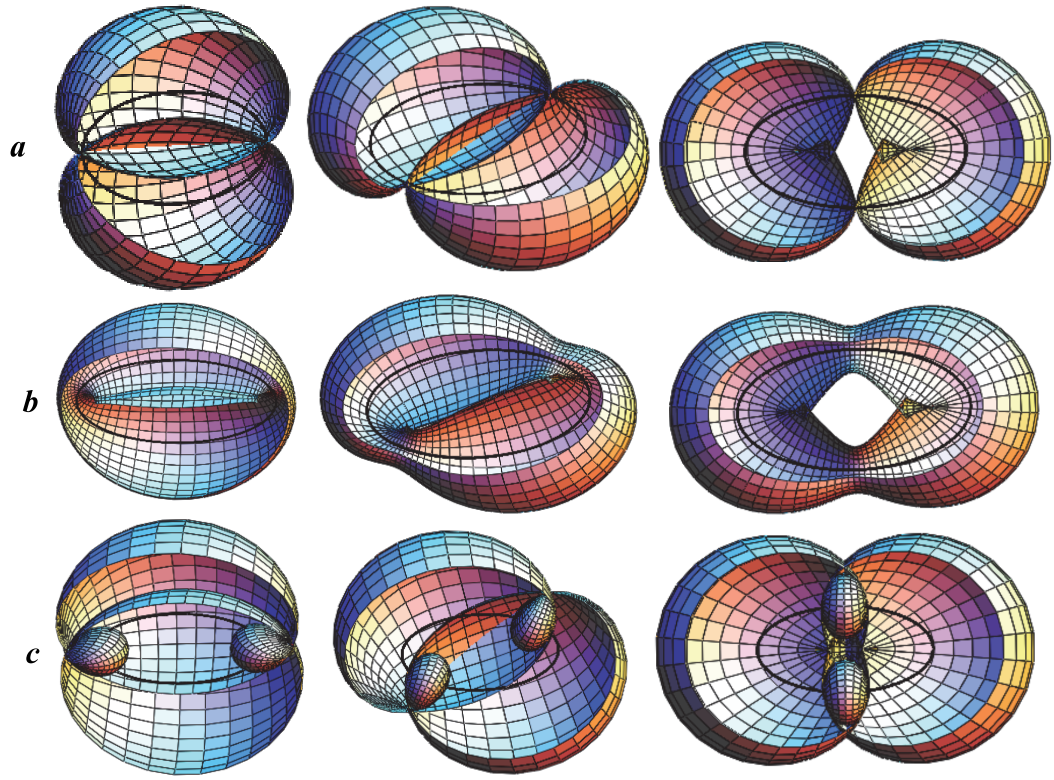

Figure 7. Epihypocycloid cycle surfaces with the base ellipse, $k = 2$:
$a - \mu = 1$; $b - \mu = 0.5$; $c - \mu = 2$

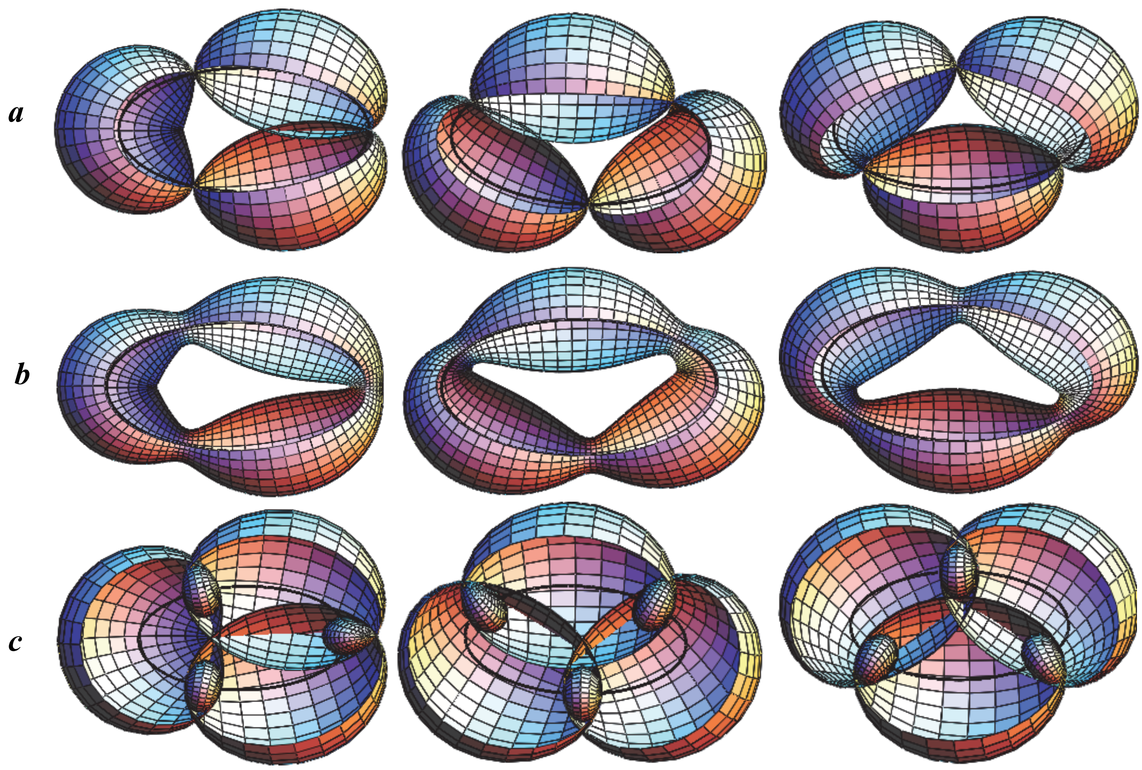

Figure 8. Epihypocycloid cycle surfaces with the base ellipse, $k = 3$:
$a - \mu = 1$; $b - \mu = 0.5$; $c - \mu = 2$

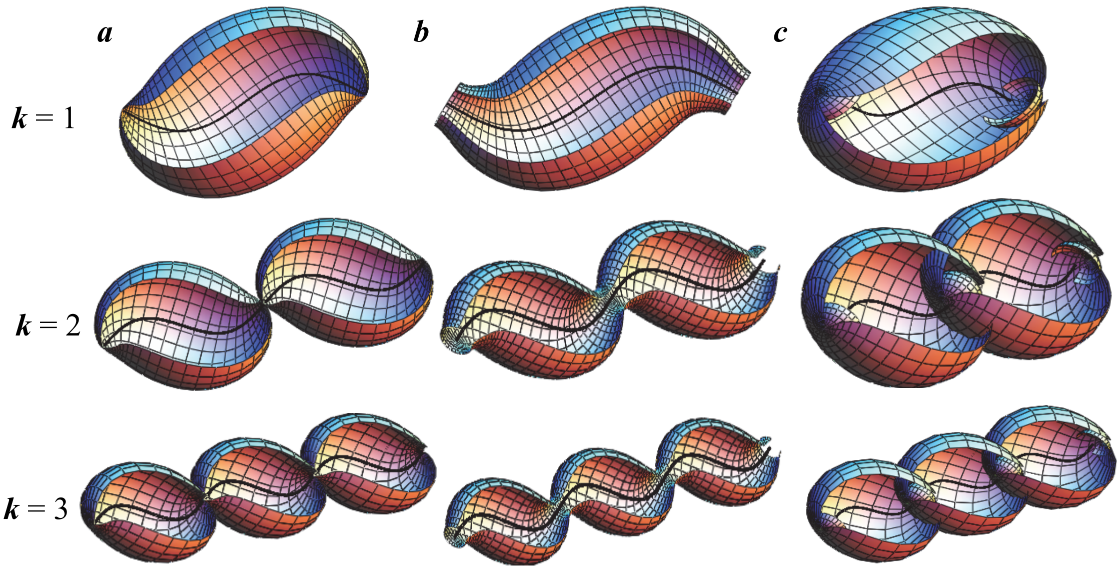

Figure 9. Epihypocycloid cyclic surfaces with the base sine:
a – μ = 1; b – μ = 0.5; c – μ = 2

Conclusion

The article shows the possibility of constructing generalized epihypocycloidal curves, formed by the cycle with bind point rolling along any base curve, and the epihypocycloid cyclic surfaces, formed by rotating the moving circle around the tangent of the base curve.

The figures are shown of the variants of the epihypocycloids and epihypocycloid cycle surfaces with rolling cycle at the base ellipse and sine. The figures of curves and surfaces are made with the help of MathCad software, on the base of the vector equations (1), (3). Using the equations with the different base curves including space ones it is possible to create various shapes of cyclic surfaces.

The given examples of surface visualization show great opportunities for creating new forms of spatial structures and their use in architecture and modern urban planning.

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