2019. 15(3). 201–209

СТРОИТЕЛЬНАЯ МЕХАНИКА ИНЖЕНЕРНЫХ КОНСТРУКЦИЙ И СООРУЖЕНИЙ

STRUCTURAL MECHANICS OF ENGINEERING CONSTRUCTIONS AND BUILDINGS

HTTP://JOURNALS.RUDN.RU/STRUCTURAL-MECHANICS

THEORY OF THIN ELASTIC SHELLS

UDC 539.3:624.04
DOI: 10.22363/1815-5235-2019-15-3-201-209

REVIEW PAPER

Optimal shells of revolution and main optimizations

Sergey N. Krivoshapko[1]

Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya St., Moscow, 117198, Russian Federation

Keywords:
optimization;
optimization problem;
shell of revolution;
dome;
minimum-weight shell

Article history:
Received: May 3, 2019
Revised: June 3, 2019
Accepted: June 10, 2019

For citation
Krivoshapko S.N. (2019). Optimal shells of revolution and main optimizations. *Structural Mechanics of Engineering Constructions and Buildings, 15*(3), 201–209. http://dx.doi.org/10.22363/1815-5235-2019-15-3-201-209

Abstract

Introduction. Optimization is a criterion, on the ground of which, comparative estimation of possible alternatives and selection of the best decisions is carried out. Cost of a shell, its minimum weight, absence of bending moments and tensile normal stresses, given stress state for acting external load, given bearing capacity when optimal shallowness, maximum external load, minimum weight under limitation on the value of natural frequencies of vibration and maximum displacements, absence of bending moments with taking into account internal pressure, dead weight, and centrifugal forces; maximum of critical force and something else can be criterion of selection of optimal shape of shell of revolution.

Methods. The main criteria of optimality for shells of revolution and information sources for the 1970–2019 periods are presented in a paper. It will help to study previous results devoted to using optimizations and to set about further investigation. But there is no single approach to the definition of optimal shell of revolution and obviously will not be, because own optimizations are necessary for every concrete case of loading, or distribution of stresses along the thickness, or under the demands to the ratio of the volume and area of considered shell, or with due regard for different kind of expenses, and other demands.

Results. For the first time, 24 criteria of optimality only for shells of revolution were discovered. The names of scientists offered presented criteria of optimality and the 45 references dealing with this question are pointed out. It is shown that principles put in the basis of optimal design and criteria of optimality must be given with the help of language quite naturally for computer. Having used optimizations presented in the paper, designers can choose the criterion for their own design of optimal shell shape. Study of the prerequisites of the structural solutions in building and machine-building, the history of the development and perfecting of technologies of erection of shells of revolution will permit to generalize the experience accumulated by designers and to develop new fundamental solutions. Otherwise, architects, structural engineers, and designers will be repeating the achieved solutions in building, architecture, and machine-building.

Introduction

It is well known that shells of revolution are the most widely-distributed type of shells. They are used in different fields of human activity viz in architecture, building, machine-building, and so on. Large number of scientific-and-technical publications [1–3] is devoted to analysis of these shells on strength, stability, dynamic, and to their application and classification.

Now, not only problem of shell analysis is one of actual problems, but a problem of finding of optimal shells of revolution with given in advance characteristics and required constraints is also an important task. Optimization is the selection of a best element with regard to some criteria from some set of

Acknowledgments. The publication has been prepared with the support of the "RUDN University Program 5-100".
Conflict of interest. The author confirms that this article content has no conflict of interest.
[1] ***Sergey N. Krivoshapko***, DSc, Professor, Department of Civil Engineering, krivoshapko-sn@rudn.ru

available alternatives. In the simplest case, an optimization problem consists of search of minimum or maximum value of a real function by choosing it from within an allowed set [4]. W. Stadler and V. Krishnan [5] have written that about 5,000 scientific papers and more than 100 monographs published since earnest 1970 until 1990 were devoted to problems of optimization of forms of shells of revolution. N.I. Abramov and V.T. Alekcandrov [6] noted: "Algorithms and program means, in general, are devoted to solution of classical optimization problems and do not take into consideration the real design requirements, constructive restrictions, and regulations of building standards". Principles put in the basis of optimal design and criteria of optimality must be given with the help of language quite naturally for computer [7].

Consider some of the most important criteria of optimality that are adapted for practical conditions or already have found application under investigation of shells of revolution.

Criteria of optimality of shells of revolution

Maximum of ratio of the volume of inner shell space to the area of the surface. V.E. Mihailenko, V.S. Obuhova, A.L. Podgorniy [8] introduced into practice the special criterion

$$n = V/S, \tag{1}$$

where S is the minimum area of surface of revolution covering the maximum volume V.

The more criterion n the better designed midsurface of the shell of revolution is with geometrical point of view. The scientists presented this criterion approved it in a class of the second order surfaces of revolution and obtained the optimal criteria $n_{\max}$ for every group of surfaces of revolution with variable rise of the surface.

So, they determined optimal rise of a surface of revolution in every group of the surfaces with the rest constant values of the geometric parameters.

But having used the method of investigation presented in a book [8], it was shown that the results obtained in a paper [9] differed in the main from the results given in [8]. It means that no one in the considered surfaces of revolution does not have maximum for the introduced criterion n, i.e. optimization of chosen geometric shape of surface of revolution under the criterion (1) does not make sense.

Minimum volume of inner space of shell with given area of its middle surface. This problem can be formulated by the following way: it is necessary to design a shell of revolution of the nodoid type with minimum area of surface under given volume in advance [10]. It means to find the plane curve, i.e. the meridian, when its rotation forms the necessary middle surface of the shell of revolution of alternating gauss curvature. This criterion is recommended to use for designing atomic installations [11].

A condition of equi-strength. A condition of equistrength of thin-walled shell of reservoir is assumed as a basis of analysis of drop-shaped reservoir for the liquid products [12]. Geometry of the middle surface of a shell is chosen on condition that tensile meridional and circular forces will be equal to each other and constant ($N_1 = N_2 = N =$ const) under an action of designed load. It means that a condition

$$1/R_1 + 1/R_2 = \gamma(h + y)/N = pN$$

must be satisfied. This equation follows from the condition of equilibrium of a shell element (Laplace formula). Here R_1 and R_2 are radiuses of principle curvatures correspondingly in meridional and circular directions. The key designed load (inner pressure)

$$p = \gamma(h + y)$$

is a sum of hydrostatical pressure of liquid and uniform redundant pressure; y is the distance the peak from a considered point of the shell in the vertical direction; γ is a density of the product; h is a height of designed column of liquid.

This criterion was used, for instance, for a membrane chaotically reinforced composite shell [13]. The meridian of the shell is built through the points.

Forming of the single surface of revolution with two given parallels and having the given magnitudes of coefficients of the first fundamental form in the theory of surfaces. E. Annaberdyev [14] offers a method of selection of the single surface of revolution passing through given parallels and having the given magnitudes of coefficients of the first fundamental form in the theory of surfaces

$$ds^2 = Edu^2 + Gdv^2.$$

A surface of revolution cannot be designed when a finite number of its parallels is taken. A smooth meridian of the middle surface of shell of revolution is formed as a curve passing through two points and having the common tangents at the joints of the parallels for maintaining smoothness of the meridian.

Minimum-weight shell. This criterion is used rather often. For instance, comparison of the results of a strength analysis of membrane shells of revolution with an ellipsoid of revolution of variable and constant thickness is given in a paper [15]. Comparative analysis showed that the shells of minimum weight of variable thickness loaded by uniform inner pressure have some advantages.

Minimum weight of a shell with given in advance angles of inclination of tangents to the meridian on circle edges. Efforts to develop low-mass, high-strength aerospace structures have produced many efficient structural concepts. This criterion is used too in aerospace field. As a result, they derive optimal shape of the meridian and distribution of the thickness along the meridian. Several examples illustrating an application of this criterion are presented in a report [16]. The method makes use of linear membrane theory. Stroud W. Jefferson supposed that membrane theory greatly simplifies the calculations and appears to be sufficiently accurate for the type of shell problems considered. The method uses the strain energy of distortion yield condition of Von Mises to relate the stress resultant to the thickness. The mass as a function of the shape is expressed with the help of an integral. The integral is minimized by a Ritz procedure together with nonlinear mathematical programing method. Two types of shell shapes were considered: transition sections, which are open at both ends, and pressure-vessel heads, which are open at one end and closed at the other.

Minimum weight with step-function distribution of shell thickness. A methodology of optimal design of shells with step-function distribution of thickness, i.e. ribbed shells and shells with a discrete inner layer, is used for designing of shells of revolution of minimum weight with simultaneous guaranteeing strength, stability, and rigidity. This problem is solved in two stages [17]. At the first stage, the optimization problem consists in determination of the general dimensional parameters of shell, i.e. shape and constant thickness of the shell of minimum weight. The second stage gives a possibility to detail dimensional parameters of the shell. These design parameters describe the location and size of the ribs, the reinforcement parameters, etc. It is supposed that the thickness is changed from the center to the edges linearly, but is not changed along the parallels. The solution of the second stage is based on the results of the first stage calculation. A methodology of solution of the problem of the first stage for ribbed shells working in the geometrically nonlinear deformation stage uses the Galerkin method in finite element formulation.

In a paper [17], the effectiveness of the presented method of optimization is shown in comparison with methods offered by other researchers earlier. Such methods as the multidimensional continuous search or a method of the grid, the steepest descent method with a variable step, and a method based on a combination of the gradient method and the random search method have been chosen for a comparative analysis in [17].

Optimal distribution of thickness. In a paper [18], a problem on optimal distribution of thickness in an elastic shell loaded by external pressure and by self-weight under loss of bearing capacity restriction is studied. The same criterion is used by M. Serra [19] for membrane shells.

Optimal distribution of variable thickness in combined shells of revolution similar to equi-strength shells. There is a method of solution of problems of optimal distribution of constructional materials in combined elastic shells with taken geometrical nonlinearity into consideration when stationary value of the functional of additional energy of strain is criterion of quality. This criterion can be used for a non-thin shell under non-axisymmetric loading. In that case, the method of solution of problems of optimal distribution of constructional materials in a non-thin elastic orthotropic shell of revolution with given volume of material comes to the guarantee of minimum value of the functional of energy of strain. This criterion was illustrated on concrete examples in a dissertation [20]. The criterion can be used for the solution of important applied problems of analysis and optimization of elements of aircrafts and articles of constructive optics.

Minimum volume and expected strength under given load. Those who use this criterion raise a problem: it is necessary to find a shape of axisymmetric shell of revolution having minimum volume and this shell must be durable when bending moments are absent under action of inner pressure, dead weight, or centrifugal forces [21]. Methods of calculus of variations, the criterion of strength of Tresca, and expansion of the solution into a power series of the independent variable are used in the time of solution of the problem.

The absence of bending moments and tensile normal forces. Designers must secure the absence of bending moments and tensile normal forces in projected tensionless masonry domes. For these cases, the criterion used by M. Farshad [22] should be rather helpful. As the result of this study the meridional shape and thickness variation of a tensionless masonry dome was obtained.

Minimum weight of shell from quasi-brittle material with taking into account appearance and progress of cracks as a result of action of given minimum value of cycles. The most important task is a problem of optimization of shells from brittle material with due regard for initial defects and accumulation of damages [23]. But the most part of investigations on theory of optimal design is fulfilled without taking into consideration initial damages and assumptions of possibilities of appearance and progress of cracks. At first, problems of optimization with taking into account appearance of cracks with given parameters in advance were studied but N.V. Banichuk with colleagues [23] already introduces a factor of incompleteness of information. In this paper, the authors [23] study the problem of finding axisymmetric shape of thin-walled momentless shell of revolution possessing minimum mass and answering the geometrical constraints, the constraints on the permissible number of cycles before destruction, if axisymmetric external actions are applied. The problem of finding of optimal shape of shell consists in determination of radii of parallels of the shell of revolution of constant thickness.

The given bearing capacity for optimal slope. When it is necessary to ensure the given bearing capacity for optimal slope of shell, designer can avail oneself of the results presented in a paper [24] where this criterion is considered.

L.Yu. Stupishin [25] used the same criterion of optimality but he solved a problem on the basis of the principle of maximum of L.S. Pontryagin under buckling constraints.

Minimum volume of concrete and minimum weight of reinforcement of ellipsoidal shell of revolution with optimal rise (slope). This criterion practically coincides with the foregoing but it is used only for reinforced concrete shells. K.M. Gmirach with his colleagues [26] proposes to find an optimal shell rise under constant value of the radius of the foundation, i.e. an optimal ratio of the rise of an elliptical reinforced concrete shell of revolution to the radius of the foundation may be taken as the criterion of optimality. In a paper [26], three domes with different rise were analyzed. The stresses in the support ring, the total area of reinforcing, and the total expense of concrete were calculated. The results of investigations proved that application of elliptical reinforced concrete domes with a rise less than 11.5 m and with a 27 m diameter of the foundation is not rational, because the dome begins to work on tension instead of compression.

Given bearing capacity of combined shell under condition of dividing of its meridian into 5 sections that are approximated by circles of different radii. A meridian of a shell of revolution is divided into five section, every section is substituted for a curve outlined along a circle of R_i ($i = 1 - 5$) in radius. A numerical analysis of the shell is fulfilled with the application of standard formulae of a theory of thin shells. Having studied the obtained results, one can change one or several radii and repeat the analysis. These analyses are fulfilled several times until acceptable result will be. This method is described in a paper [27].

Minimum weight of the combined shell with given in advance bearing capacity. V.V. Toropov [28] solved a problem of minimization of weight of the combined axisymmetric shell structures with the application of the principle of stage parametrical optimization. The analysis of the structure under given set of the operated parameters of thickness of separate components of the shells was carried out by the finite difference energy method. A system of the interacting restrictions for stresses, displacements, and stability was approved.

On the basis of a complex search in a dissertation of V.G. Malakhov (2003) and in his paper [29], an algorithm of finding of a combined shell of revolution with optimal weight under strength constraint was devised when the thicknesses of the shell structures and the parameters of supporting rings are unknown design variables.

Weight constraint for a shell of conjugation of two shells of revolution. This criterion coincides practically with the foregoing criterion. In a paper [30], a numerical procedure of optimization of a shell of revolution connecting a cylinder to a sphere under internal pressure is offered. A procedure based on a direct variational method. An optimal shape of the shell of conjugation is determined under condition that its meridian is given by a function differentiable at least twice. The design is set up with respect to volume restriction, weight constraint and governing yield criterion. The shell of conjugation can have variable thickness and has the same strength as that of an unpierced sphere subjected to internal pressure.

An approach to problem, presented in a paper [30], can be applied for shells of smooth conjugation with conic and cylindrical midsurfaces, brought in a paper [31], where meridians are taken in the form of the section of the sinusoid.

The minimization of the volume of the shell material, the maximization of the fundamental natural frequency, the minimization of the maximum stresses, and the minimization of the maximum displacement. This criterion of optimality is given in a paper "Sensitivity analysis and optimal design of thin shells of revolution" [32]. Stress-strain state of axisymmetric shells of revolution under action of arbitrary loads is determined in classical formulation of Love – Kirchhoff with the help of FEM. The criterion was approved for conic, cylindrical, and compound cylindrical-and-conical shells. The objective of the design is the minimization of the volume of the shell material, the maximization of the fundamental natural frequency, the minimization of the maximum stresses, or the minimization of the maximum displacement. The constraint functions are the displacements, stresses, enclosed volume of the structure, volume of shell material or the natural frequency of a specified mode shape.

G.I. Belikov, A.A. Tarasov [33] solved a problem of increasing of the lowest frequency of natural oscillations with the help of change of the geometrical parameters and the form of meridian of hyperbolic cooling towers with taking into consideration influence of self-weight on the stress-strain state of the shell.

Solving the problems of optimization, they used a method of relaxation with correction of the vector of solution for every iteration. By the way, G.I. Belikov derived that demands to shell about frequency of natural vibrations are well coordinated with strength constraint [34].

The same criterion is used in a dissertation [35] where problems of optimization of a shell of minimum mass are considered with constraints on maximum stresses or displacements, on minimal natural frequencies and on the geometrical parameters when the natural frequencies depend on preliminary loading.

Maximum of the buckling load capacity when a volume of the barrel-shaped shell is given in advance. In a paper [36], barrel-shaped shells with fixed opposite edges subjected to axial compression are examined under buckling constraints. For this case, optimal curvature of the meridian of the middle surface of shell of revolution under action of maximum critical load is found when its volume is given in advance

Minimum normal stresses in shells of revolution with the same overall dimensions, boundary conditions, and external load. Several shells of revolution, the middle surfaces of which are formed by convex or concave in one side meridians, are taken for consideration. Then, it is necessary to choose the constants in the parametrical equations of these surfaces in order to receive the same overall dimensions of the shells. Then, they obtain the parameters of stress-strain state of the selected shells; make proper graphic representations of displacements, bending moments, normal and shearing forces, and compute maximum normal stresses in appointed sections. At the last stage, they find minimum values from maximum values of normal stresses and that shell is considered as optimal one. In a paper [37], this criterion was approved after the example of five shells of revolution of negative curvature (Figure).

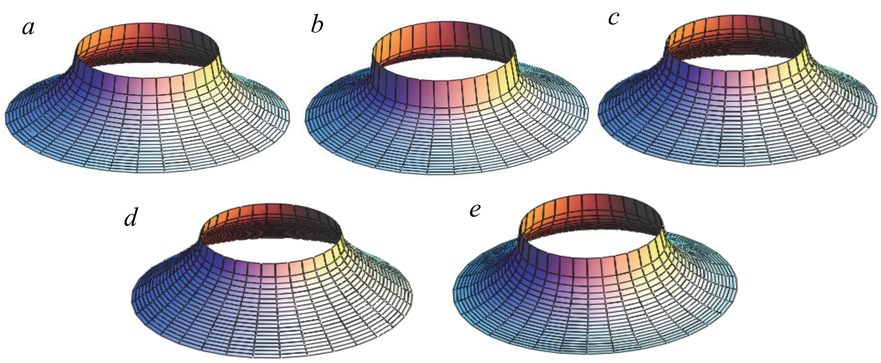

Figure. Five types of surfaces of revolution:
a – a catenoid; *b* – a surface of revolution of the fourth order parabola;
c – a surface of revolution of the second order parabola; *d* – a one sheet hyperboloid of revolution; *e* – a globoid

The investigation of B. Nick [38], conducted with 5 types of domes (spherical, elliptical, parabolic, hyperbolic domes, and combined domes with middle surfaces, consisting of hyperbolic and parabolic surfaces), was realized with the application of the same criterion. He insists that a dome formed from the lower hyperbolic dome and the upper paraboloid of revolution is the best of all.

Maximum of inside blast pressure. Now, the protection of erections from the terrorist attacks and, thus, search of effective structural shapes to mitigate the blast energy is very important problem. The influence of inside blast pressure is studied for six shapes of domes of the same weight and thickness in a paper [39]. The study shows that the parabolic and bowl shape of domes could withstand the blast load with least top displacement.

Minimum expenditures for making, transport, assembling, and exploitation of combined reinforced concrete shells. This criterion was presented in a paper [40]. The height of a combined reinforced concrete dome and the parameter describing the form of combined shells were taken as the modified guided parameters. A shell made of unified plates with the dimensions of 3×6 m in the form of spherical surface was considered as an example.

Minimum cost under minimal values of variable rise and thicknesses in the apex and at the support. This algorithm of optimization is *a method of* *selection* which gives minimum cost of building of shell structure under strength constraints.

A method of selection is decrease of permissible initial thicknesses until stresses and deformations in shell do not achieve the allowable tensile stresses and the allowable deformation under compression. This process is put into effect with changing shell rise from maximum height till minimal height. The value of decided parameters giving the minimum cost will be taken as optimal solution.

A cost restriction is total cost of reinforced concrete shell, cost of external and inner finishing layers. This minimization must be limited by technical demands of safety. A cost restriction is represented by three group restrictions that limit the minimization of design variables that are strength constraints of material, minimum thickness restriction, and restriction of the minimum height which must be functionally required.

In a dissertation [41], the computer program for optimization of a spherical shell is presented and approved. Data generation includes the constant shell span (D) and variable parameters of a process of optimization, i.e. the shell rise (h), the initial shell thickness (t_c) at the peak and at the support (t_s). The program of optimal analysis consists of two parts [41]. The first part contains data generation. This part non-linearly analyses a shell with taking in consideration the self-weight and live load (snow and wind load). The second part uses the method of selection. This part offers optimal analysis of a shell, i.e. it finds optimal thickness at the shell peak and at the support that are variables in the cost restriction. Finally, the cost of the shell of revolution is calculated.

The results of the second part are used again as the data generation for the first part but with new geometrical parameters (h, t_c, t_s) and again it is necessary to revert to the second part, and so on.

The total cost of a reinforced concrete dome with minimum normal stresses and minimum constant thickness. This criterion is introduced in a paper [4], where a dome with reinforcement in both meridional direction and hoop direction is under consideration. The criterion is illustrated for three spherical domes with diameters equal to 6 m, 20 m, and 45 m. The total cost consists of the cost of reinforcing steel and concrete. For optimization of dome structure dynamic programming method is used.

Conclusions

V.V. Novozhilov was one of the first who began to seek for a shell of revolution with the most advantageous indices of stress-strain state. In particular, examining four different domes viz spherical, parabolic, half-elliptical, and the lesser part of elliptical, he determined that a dome in the form of the lesser part of ellipsoid of revolution was the most advantageous one because it can work as momentless shell with comparatively slight rigidity of the support contour. Membrane strength theory of shells subjected to dead load was used.

V.G. Malakhov [20] shows that yearly number of publications devoted to optimal design of shells remains rather great during the last ten years. It points at stable interest of researchers in problems of optimal designing. Considerably less works are devoted to optimal designing of thick shells.

After the first results of shell optimization since 1960 till present time, interest in establishment of criteria of optimality and in solution of optimization problems increases under conditions of regularly increasing requirements for creation of optimal shells of revolution for different branches of human activity. Before the beginning of subsequent investigations, it is necessary to study previous results of using of criteria of optimality. This review paper can help in studying of existing criteria.

In this paper, criteria of optimality for composite shells of revolution are not presented. Those who are interested in this problem can begin with a book [42] with 73 references, where criteria of optimality of shell structures from reinforced composite materials are enumerated or with more late works [43; 44] and with a work [45] containing 88 references, where laminated composite and sandwich plates and shells are considered.

References

1. Noor A.K. (1990). Bibliography of books and surveys on shells. *AMR, 43*(9), 223–234.

2. Zarutzkiy V.A., Sivak V.F. (1999). Experimental investigations of dynamic of shells of revolution. *Prikl. Meh. (Kiev), 35*(3), 3–11 (In Russ.)

3. Krivoshapko S.N., Mamieva I.A. (2012). *Analiticheskie poverhnosti v arhitekture zdaniy, konstruktziy i izdeliy* [*Analytical Surfaces in Architecture of Buildings, Structures, and Products*]. Moscow: LIBROCOM Publ. (In Russ.)

4. Prabhavati P., Vankudre S.B., Varur Veeresh. (2014). Optimization of RCC Dome. *International Journal of Engineering Research & Technology (IJERT), 3*(6), 1515–1519.

5. Stadler W., Krishnan V. (March 1989). Natural structural shapes for shells of revolution in the membrane theory of shells. *Structural Optimization, 1*(1), 19–27.

6. Abramov N.I., Aleksandrov V.T. (1989). On using mathematical methods of optimization in designing. *Stroit. Meh. i Raschet Soor.*, (4), 40–41. (In Russ.)

7. Furunjiev R.I., Guglya V.A., Furunjiev R.I. (1987). *SAPR ili kak EVM pomogaet konstruktoru* [*System of automated design or the help of computer to designer*]. Minsk: Vysheishaya shkola Publ. (In Russ.)

8. Mihailenko V.E., Obuhova V.S., Podgorniy A.L. (1972). *Formoobrazovanie obolochek v arhitekture* [*Shell Forming in Architecture*]. Kiev. (In Russ.)

9. Krivoshapko S.N., Emel'yanova Yu.V. (2006). On a problem of surface of revolution with geometrically optimal rise. *Montazh. i Spetz. Raboty v Stroit., 2*, 11–14. (In Russ.)

10. Kreychman M.M. (1982). *Research of stress-strain state of shells of nodoid type loaded by non-axisymmetrical load quickly changing.* Kazan: Kazan. un-t. (In Russ.)

11. Gorodov G.F., Gagarin Yu.A., Mitenkov F.M., Pichkov S.N. (2000). The application of nodoid and unduloid shells for the design of atomic installations. *Prikl. Probl. Prochnosti i Plastichnosti*, (61), 61–63 (In Russ.)

12. Krivoshapko S.N. (1998). Drop-shaped, catenoidal and pseudo-spherical shells. *Mont. i Spetz. Raboty v Stroitelstve*, (11–12), 28–32. (In Russ.)

13. Alehin V.V. (1979). Construction of equal strength shells of revolution. *Mat. Modeli i Vychisl. Metody Meh. Sploshn. Sredy*, 77–84. (In Russ.)

14. Annaberdyev E. (1971). On one method of determination of the single surface of revolution passing through two given circles. *Kibernetika Grafiki i Prikl. Geom. Poverhnostey, VIII*(231), 47–48. (In Russ.)

15. Stolyarchuk V.A. (1977). The determination of form of certain class of shells of revolution of minimal weight loaded by inner uniform pressure. *Prikl. Problemy Prochnosti i Plastichnosti*, (7), 104–108 (In Russ.)

16. Stroud W.J. (February 1971). *Minimum-mass isotropic shells of revolution subjected to uniform pressure and axial load* (Report No. NASA TN D-6121). NASA Langley Research Center Hampton.

17. Stupishin L., Nikitin K., Kolesnikov A., Altuhov F. (2017). Optimal design of shells with step-function distribution of thickness. *International Journal of Optimization in Civil Engineering, 7*(2), 231–240. Retrieved from http://ijoce.iust.ac.ir/article-1-295-en.html

18. Kosmodemianskiy A.S., Tataranova O.P. (1981). Natural vibrations of complex shell system. *Dokl. AN USSR, 5*, 50–53. (In Russ.)

19. Serra M. (2010). Design of Membrane Shells of Revolution with Optimal Stiffness. *Mechanics Based Design of Structures and Machines, 38*(4), 403–416.

20. Malakhov V.G. (2003). *Optimization of Shells of Revolution* (Dis. kand. fiz.-mat. nauk). Kazan: IMM KazNTz RAN. (In Russ.)

21. Kruzelecki J. (1979). Pewne problemy ksztaltowania powlok osiowo-symetrycznych w stanie blonowym. *Mechanica teoretyczna i stosowana, 17*(1), 75–92. (In Polish.)

22. Farshad M. (1977). On the shape of momentless tensionless masonry domes. *Build. and Environ., 12*(2), 81–85.

23. Banichuk N.V., Ivanova S.Yu., Makeev E.B., Sinitzin A.V. (2005). Some problems of optimal design of shells with paying attention to accumulation of damages. *Problemy Prochnosti i Plastichnosti, 67*, 46–59. (In Russ.)

24. Dehtyar A.S. (1975). The optimal shell of revolution. *Stroit. Meh. i Raschet Soor.*, (2), 11–15. (In Russ.)

25. Stupishin L.Yu. (1993). *Investigation of optimal forms of shallow shells of revolution with the help of the principle of maximum of L.S. Pontryagin*. Kursk: KPI Publ. (In Russ.)

26. Gmirach K.M., Kozlov A.V., Proskurov R.A. (2017). Selection of optimal parameters of elliptical reinforced concrete shell of revolution. *International Research Journal*, (2–56–3), 100–104. (In Russ.)

27. Polivanov A.A. (2009). Calculation of optimal geometrical characteristics of shell structures under static loading. *Sovremennie Problemy Nauki i Obrazovaniya*, (6–3). Retrieved from http://www.science-education.ru/ru/article/view?id=1442. (In Russ.)

28. Toropov V.V. (1979). Weight optimization of combined shells of revolution on condition of strength, rigidity, and stability. *Prikl. Problemi Prochnosti i Plastichnosti: Vses. Mezhvuz. Sb., 13*, 122–127. (In Russ.)

29. Malakhov V.G. (1999). Search of optimal thickness of non-thin shell of revolution. *Mehanika obolochek i plastin: sb. dokl. XIX Mezhd. konf. po teorii obolochek i plastin*, 135–140. (In Russ.)

30. Diallo Boubacar, Ellyin Fernand. (February 1983). Optimization of connecting shell. Journal of Engineering Mechanics, *109*(1). https://doi.org/10.1061/(ASCE)0733-9399(1983)109:1(111)

31. Krivoshapko S.N. (2005). Model surfaces of connecting fragments of two pipe lines. *Montazhn. i spetz. raboty v stroitelstve*, (10), 25–29. (In Russ.)

32. Mota Soares C.M., Mota Soares C.A., Barbosa J. Infante. (1992). Sensitivity analysis and optimal design of thin shells of revolution. *4th AIAA/USAF/NASA/OAI Symp. Multidiscip. Anal. and Optimiz., Cleveland, Ohio, Sept. 21–23, 1992: Collect. Techn. Pap.*, (2), 701–709.

33. Belikov G.I., Tarasov A.A. (1982). Optimization of geometrical parameters of hyperbolic cooling towers when natural vibration. *Stroit. Meh. i Raschet Soor.*, (4), 12–15. (In Russ.)

34. Belikov G.I. (2012). Optimization of topology of a hyperboloid of rotation on condition of strength and rigidity. *Vestnik VolgGASU. Ser. Stroitelstvo i Arhitektura*, (29), 110–114. (In Russ.)

35. Ermolaev N.V. (1984). *Compound shells of revolution of minimal mass with limitation for natural frequencies, stresses, and displacements* (Dis. kand. tekh. nauk). Gorkiy. (In Russ.)

36. Blachut J. (1987). Optimal barrel-shaped shells under buckling constraints. *AIAA Journal, 25*(1), 186–188.

37. Krivoshapko S.N., Ivanov V.N. (2018). Catenoidal shells. *Promyshlennoe i Grazhdanskoe Stroitel'stvo*, (12), 7–13. (In Russ.)

38. Nick B. (2017). Search for dome. *3D Warehouse.* Trimble Inc., The Netherlands.

39. Ram Ranjan Sahu, Pramod Kumar Gupta. (2015). Blast Diffusion by Different Shapes of Domes. *Defense Science Journal, 65*(1), 77–82.

40. Skladnev N.N., Zhukovskiy E.Z. Sharshukova L.M. (1989). Optimization of shells on a base of system analysis and numerical methods. *Stroit. Meh. i Raschet Soor.*, (1), 9–13. (In Russ.)

41. Al-Khattab Salim Abdul-Razzak. (2005). Optimization of shells of revolution from materials having different mechanical characteristics for tension and pressure (Dis. kand. tekh. nauk). Moscow. (In Russ.)

42. Obraztzov I.F., Vasil'ev V.V., Bunakov V.A. (1977). *Optimal'noe armirovanie obolochek vrascheeniya iz kompozitzionnih materialov* [*Optimal Reinforcement of Shells of Revolution from Composite Materials*]. Moscow: Mashinostroenie Publ. (In Russ.)

43. Karpov Ya.S., Gagauz P.M. (2010). *Proektirovanie obolochek vrascheeniya iz kompozitzionnih materialov* [*Designing of Shells of Revolution from Composite Materials*]. Kharkov: Natz. aero-kosm. un-t "Kharkov. aviatz. in-t". (In Russ.)

44. Elsayed Fathallah. (May 2019). Finite element modelling and multi-objective optimization of composite submarine pressure hull subjected to hydrostatic pressure. *Materials Science Forum, 953*, 53–58. doi: 10.4028/www.scientific.net/MSF.953.53

45. Kanak Kalita, Tanmoy Mukhopadhyay, Partha Dey, Salil Haldar. (May 2019). Genetic programming assisted multi-scale optimization for multi-objective dynamic performance of laminated composites: the advantage of more elementary-level analyses. *Neural Computing and Applications.*

ОБЗОРНАЯ СТАТЬЯ

Оптимальные оболочки вращения и основные критерии оптимальности

С.Н. Кривошапко[1]

Российский университет дружбы народов, Российская Федерация, 117198, Москва, ул. Миклухо-Маклая, 6

Ключевые слова:
критерий оптимальности;
оптимизационная задача;
оболочка вращения;
купол;
оболочка минимального веса

История статьи:
Поступила в редакцию: 3 мая 2019 г.
Доработана: 3 июня 2019 г.
Принята к публикации: 10 июня 2019 г.

Для цитирования
Krivoshapko S.N. Optimal shells of revolution and main optimizations (Оптимальные оболочки вращения и основные критерии оптимальности) // Строительная механика инженерных конструкций и сооружений. 2019. Т. 15. № 3. С. 201–209. http://dx.doi.org/10.22363/1815-5235-2019-15-3-201-209

Аннотация

Цели. Критерий оптимальности – признак, на основании которого производится сравнительная оценка возможных альтернатив и выбор наилучшего решения. Критерием выбора оптимальной формы оболочки вращения может быть ее стоимость, минимальный вес, отсутствие изгибающих моментов и растягивающих нормальных усилий, заданное напряженное состояние для действующей внешней нагрузки, заданная несущая способность при оптимальной пологости, максимальная внешняя нагрузка, минимальный вес при ограничениях на значения собственной частоты колебаний и максимальных перемещений, отсутствие изгибающих моментов при учете внутреннего давления, собственного веса и центробежных сил, максимум критической нагрузки и многое другое. Выбрать приемлемый критерий оптимальности оболочки вращения – цель настоящего исследования.

Методы. В статье представлены основные критерии оптимальности для оболочек вращения и источники получения информации за период с 1970 по 2019 г., что поможет изучить предшествующие результаты по использованию критериев оптимальности и приступить к дальнейшим изысканиям. Однако единого подхода к определению оптимальной оболочки вращения нет и, по-видимому, не будет. Для каждого конкретного случая нагружения, или распределения напряжений по толщине, или требований к отношению объема и площади поверхности рассматриваемой оболочки, к учету различного вида расходов и других требований необходимы свои критерии оптимальности.

Результаты. Впервые представлены 24 критерия оптимальности, применяемые для оболочек вращения. Указаны ученые, предложившие эти критерии, и даны соответствующие ссылки на 45 источников информации, в которых описываются рассматриваемые критерии. Показано, что принципы, положенные в основу оптимального проектирования, должны быть изложены с помощью языка, понятного компьютерам. Используя материалы статьи, проектировщики могут выбрать критерии для своего собственного проекта оптимальной формы оболочки.

Список литературы

1. *Noor A.K.* Bibliography of books and surveys on shells // AMR. 1990. Vol. 43. No. 9. Pp. 223–234.
2. *Заруцкий В.А., Сивак В.Ф.* Экспериментальные исследования динамики оболочек вращения // Прикл. мех. (Киев). 1999. Т. 35. № 3. С. 3–11.
3. *Кривошапко С.Н., Мамиева И.А.* Аналитические поверхности в архитектуре зданий, конструкций и изделий: монография. М.: ЛИБРОКОМ, 2012. 328 с.
4. *Prabhavati P., Vankudre S.B., Varur Veeresh*. Optimization of RCC Dome // International Journal of Engineering Research & Technology (IJERT). 2014. Vol. 3. Issue 6. Pp. 1515–1519.
5. *Stadler W., Krishnan V.* Natural structural shapes for shells of revolution in the membrane theory of shells // Structural Optimization. March 1989. Vol. 1. Issue 1. Pp. 19–27.
6. *Абрамов Н.И., Александров В.Т.* Об использовании математических методов оптимизации в проектировании // Строительная механика и расчет сооружений. 1989. № 4. С. 40–41.
7. *Фурунжиев Р.И., Гугля В.А., Фурунжиев Р.И.* САПР, или как ЭВМ помогает конструктору. Минск: Вышэйшая школа, 1987. 208 с.
8. *Михайленко В.Е., Обухова В.С., Подгорный А.Л.* Формообразование оболочек в архитектуре. Киев, 1972. 208 с.
9. *Кривошапко С.Н., Емельянова Ю.В.* К вопросу о поверхности вращения с геометрически оптимальной стрелой подъема // Монтажные и специальные работы в строительстве. 2006. № 2. С. 11–14.
10. *Крейчман М.М.* Исследование НДС оболочек нодоидного типа, нагруженных несимметричной быстро изменяющейся нагрузкой. Казань: Казан. ун-т, 1982. 15 с.
11. *Городов Г.Ф., Гагарин Ю.А., Митенков Ф.М., Пичков С.Н.* Использование нодоидных и ундулоидных

Благодарности. Публикация подготовлена при поддержке Программы РУДН «5-100».

Конфликт интересов. Автор подтверждает, что содержание этой статьи не имеет конфликта интересов.

[1] ***Кривошапко Сергей Николаевич***, доктор технических наук, профессор, департамент строительства, Инженерная академия, krivoshapko-sn@rudn.ru

оболочек при проектировании ядерных установок // Прикл. пробл. прочности и пластичности. 2000. № 61. С. 61–63.

12. *Кривошапко С.Н.* Каплевидные, катеноидальные и псевдосферические оболочки // Монтажные и специальные работы в строительстве. 1998. № 11–12. С. 28–32.

13. *Алехин В.В.* Проектирование равнопрочной безмоментной оболочки вращения // Мат. модели и вычисл. методы мех. сплош. среды. Красноярск: КГУ, 1979. С. 77–84.

14. *Аннабердыев Э.* Об одном способе выделения единственной поверхности вращения среди всевозможных, проходящих через данные окружности // Кибернетика графики и прикл. геометрия поверхностей. 1971. Т. VIII. Вып. 231. С. 47–48.

15. *Столярчук В.А.* Определение формы некоторого класса оболочек вращения минимального веса, нагруженных внутренним равномерным давлением // Прикл. проблемы прочности и пластичности. 1977. № 7. С. 104–108.

16. *Stroud W.J.* Minimum-mass isotropic shells of revolution subjected to uniform pressure and axial load: Report № NASA TN D-6121 / NASA Langley Research Center Hampton. February 1971. 43 p.

17. *Stupishin L., Nikitin K., Kolesnikov A., Altuhov F.* Optimal design of shells with step-function distribution of thickness // International Journal of Optimization in Civil Engineering. 2017. Vol. 7. No. 2. Pp. 231–240.

18. *Космодемианский А.С., Татаранова О.П.* Собственные колебания сложной оболочечной системы // Докл. АН УССР. 1981. № 5. С. 50–53.

19. *Serra M.* Design of Membrane Shells of Revolution with Optimal Stiffness // Mechanics Based Design of Structures and Machines. 2010. Vol. 38. Issue 4. Pp. 403–416.

20. *Малахов В.Г.* Оптимизация оболочек вращения: дис. … канд. физ.-мат. наук. Казань: ИММ КазНЦ РАН, 2003.

21. *Kruzelecki J.* Pewne problemy ksztaltowania powlok osiowo-symetrycznych w stanie blonowym // Mechanica teoretyczna i stosowana. 1979. T. 17. № 1. S. 75–92.

22. *Farshad M.* On the shape of momentless tensionless masonry domes // Build. and Environ. 1977. Vol. 12. No. 2. Pp. 81–85.

23. *Баничук Н.В., Иванова С.Ю., Макеев Е.В., Синицын А.В.* Некоторые задачи оптимального проектирования оболочек с учетом накопления поврежденностей // Проблемы прочности и пластичности. 2005. Вып. 67. С. 46–59.

24. *Дехтярь А.С.* Оптимальная оболочка вращения // Строительная механика и расчет сооружений. 1975. № 2. С. 11–15.

25. *Ступишин Л.Ю.* Исследование оптимальных форм пологих оболочек вращения с помощью принципа максимума Л.С. Понтрягина. Курск: КПИ, 1993. 14 с.

26. *Гмирач К.М., Козлов А.В., Проскуров Р.А.* Подбор оптимальных параметров эллипсоидной железобетонной оболочки вращения // Международный научно-исследовательский журнал. 2017. № 2 (56). Ч. 3. С. 100–104.

27. *Поливанов А.А.* Расчет оптимальных геометрических характеристик оболочечных конструкций при статическом нагружении // Современные проблемы науки и образования. 2009. № 6–3. URL: http://www.science-education.ru/ru/article/view?id=1442 (дата обращения: 17.04.2019).

28. *Торопов В.В.* Весовая оптимизация составных оболочек вращения из условий прочности, жесткости и устойчивости // Прикл. проблемы прочности и пластичности: Всес. межвуз. сб. Горьк. ун-т, 1979. Вып. 13. С. 122–127.

29. *Малахов В.Г.* Поиск оптимальной толщины нетонкой оболочки вращения // Механика оболочек и пластин: сборник докл. XIX Межд. конф. по теории оболочек и пластин. Нижний Новгород: Изд-во ННГУ, 1999. С. 135–140.

30. *Diallo Boubacar, Ellyin Fernand.* Optimization of connecting shell // Journal of Engineering Mechanics. February 1983. Vol. 109. Issue 1. https://doi.org/10.1061/(ASCE)0733-9399(1983)109:1(111)

31. *Кривошапко С.Н.* Модельные поверхности соединительных участков двух трубопроводов // Монтажные и специальные работы в строительстве. 2005. № 10. С. 25–29.

32. *Mota Soares C.M., Mota Soares C.A., Barbosa J. Infante.* Sensitivity analysis and optimal design of thin shells of revolution // 4th AIAA/USAF/NASA/OAI Symp. Multidiscip. Anal. and Optimiz., Cleveland, Ohio, Sept. 21–23, 1992: Collect. Techn. Pap. Pt 2. Washington (D.P.), 1992. Pp. 701–709.

33. *Беликов Г.И., Тарасов А.А.* Оптимизация геометрических параметров гиперболических градирен при собственных колебаниях // Строительная механика и расчет сооружений. 1982. № 4. С. 12–15.

34. *Беликов Г.И.* Оптимизация топологии гиперболоида вращения по условиям прочности и жесткости // Вестник ВолгГАСУ. Серия: Строительство и архитектура. 2012. № 29. С. 110–114.

35. *Ермолаев Н.*В. Составные оболочки вращения минимальной массы с ограничениями на собственные частоты, напряжения и перемещения: дис. … канд. техн. наук. Горький, 1984. 193 с.

36. *Blachut J.* Optimal barrel-shaped shells under buckling constraints // AIAA Journal. 1987. Vol. 25. No. 1. Pp. 186–188.

37. *Кривошапко С.Н., Иванов В.Н.* Катеноидные оболочки // Промышленное и гражданское строительство. 2018. № 12 . С. 7–13.

38. *Nick B.* Search for dome // 3D Warehouse. The Netherlands: Trimble Inc., 2017.

39. *Ram Ranjan Sahu, Pramod Kumar Gupta.* Blast Diffusion by Different Shapes of Domes // Defense Science Journal. 2015. Vol. 65. No 1. Pp. 77–82.

40. *Складнев Н.Н., Жуковский Э.З., Шаршукова Л.М.* Оптимизация оболочек на основе системного анализа и численных методов // Строительная механика и расчет сооружений. 1989. № 1. С. 9–13.

41. *Аль-Кхаттаб Салим Абдул-Раззак.* Оптимизация оболочек вращения из материалов, по-разному сопротивляющихся растяжению и сжатию: дис. ... канд. техн. наук. М., 2005, 153 с.

42. *Образцов И.Ф., Васильев В.В., Бунаков В.А.* Оптимальное армирование оболочек вращения из композиционных материалов. М.: Машиностроение, 1977. 144 с.

43. *Карпов Я.С., Гагауз П.М.* Проектирование оболочек вращения из композиционных материалов: учеб. пособие. Харьков: Нац. аэро-косм. ун-т «Харьк. авиац. ин-т», 2010. 64 с.

44. *Elsayed Fathallah.* Finite element modelling and multi-objective optimization of composite submarine pres sure hull subjected to hydrostatic pressure // Materials Science Forum. May 2019. Vol. 953. Pp. 53–58. doi: 10.4028/www.scientific.net/MSF.953.53

45. *Kanak Kalita, Tanmoy Mukhopadhyay, Partha Dey, Salil Haldar.* Genetic programming assisted multi-scale optimization for multi-objective dynamic performance of laminated composites: The advantage of more elementary-level analyses // Neural Computing and Applications. May 2019. 45 p.