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Frame buckling

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SCIENTIFIC ARTICLE

Analysis of frame buckling without sidesway classification

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Abstract. The effective buckling length of a column in a steel frame depends on the sidesway of the frame. The classification sidesway – no sidesway of a frame depends on all members of the frame and is made on an empirical basis. A change of class leads to large changes in the effective column length, and thus affects the buckling load and the economy of the column design. In order to avoid the uncertainties of the empirical classification, it is proposed to determine the buckling load of the complete frame with a nonlinear analysis. The method is illustrated with an unbraced and a braced frame, which are analyzed for hinged as well as fixed columns at ground floor level. The forces in the columns at buckling of the frames are compared to the buckling loads of the single columns.

The design of high-rise steel frames against buckling by sidesway – no sidesway categorization has been compared to the buckling analysis of the frames as a whole with nonlinear models. The results confirm the large differences between the buckling loads of braced and unbraced high-rise frames, which are well known from analytical solutions for simple portal frames.

Keywords: high-rise building, column buckling, sidesway, effective length

1. Objective

Building codes of different countries stipulate that the effective length for the buckling of columns in a steel frame depends on the sidesway of the frame. The effective length factor of columns varies from 0.5 to 1.0 in a single bay portal frame without sidesway, but from 1.0 to infinity if there is sidesway. Frames are classified as frames with or without sidesway on an empirical basis, before separate alignment charts for effective length factors are applied for the two classes. Slight changes in the frame design, which change the class, can lead to unrealistic changes in the effective length factor.

Analytical solutions for axially loaded single columns with hinged and fixed ends [1] show, that the buckling load does not only depend on the rotational restraints at its nodes, but also on the restraint against relative lateral motion of the nodes. This lateral displacement is called sidesway.

The traditional analysis of column buckling in complete structures accounts explicitly for the bending stiffness of the adjacent members of a column. The stiffness factors are defined for both nodes of a column. Restraint against sidesway is not specified for the adjacent members, but for the structure as a whole. The classification is empirical according to rules specified in codes [2; 3]. There are only two classes of lateral restraint: sidesway and no sidesway. Intermediate degrees of restraint, which exist in the structure, are not considered in the buckling analysis.

A considerable amount of research is conducted in Russia in the area of mathematical and computer modeling of displacements and stability of 3-D rods subjected to compression, bending and torsion [4–9]. Numerical investigations with commercial software products are also being performed [10]. However, the determination of the effective length of columns in multistory frames still comprises a problem for design engineers.

A reliable method for the determination of the elastic buckling load of multi-storey and multi-bay steel frames is presented. Material nonlinearity due to yielding is not considered. In order to compare the proposed complete frame method to traditional single column design, the axial forces acting in the columns at buckling of the complete frame are determined. They are compared to the buckling loads of the single columns, which are restrained at their ends by the adjacent members of the frame.

Analytical solutions for axially loaded single columns with hinged and fixed ends [1] show, that the buckling load does not only depend on the rotational restraints at its nodes, but also on the restraint against relative lateral motion of the nodes. This lateral displacement is called sidesway.

A column *C*, which is part of a complete structure, is restrained laterally and rotationally at its nodes by the adjacent members of the structure. The degree of restraint depends not only on the properties of these members, but also on the stress resultants acting in the restraining members. If they are themselves near buckling, the adjacent members do not provide significant restraint against buckling of column *C*.

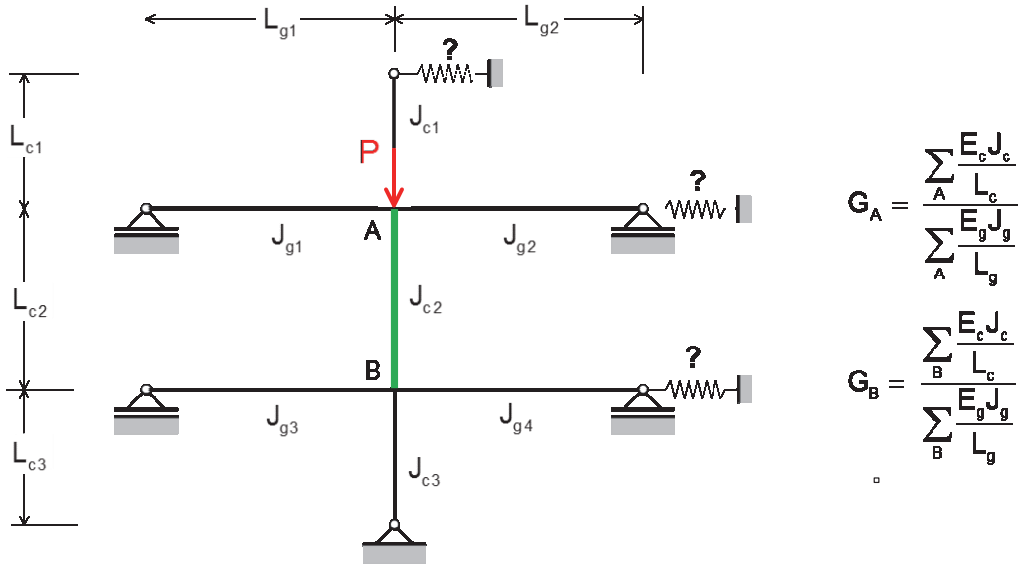

Figure 1. Traditional buckling analysis of columns in a frame

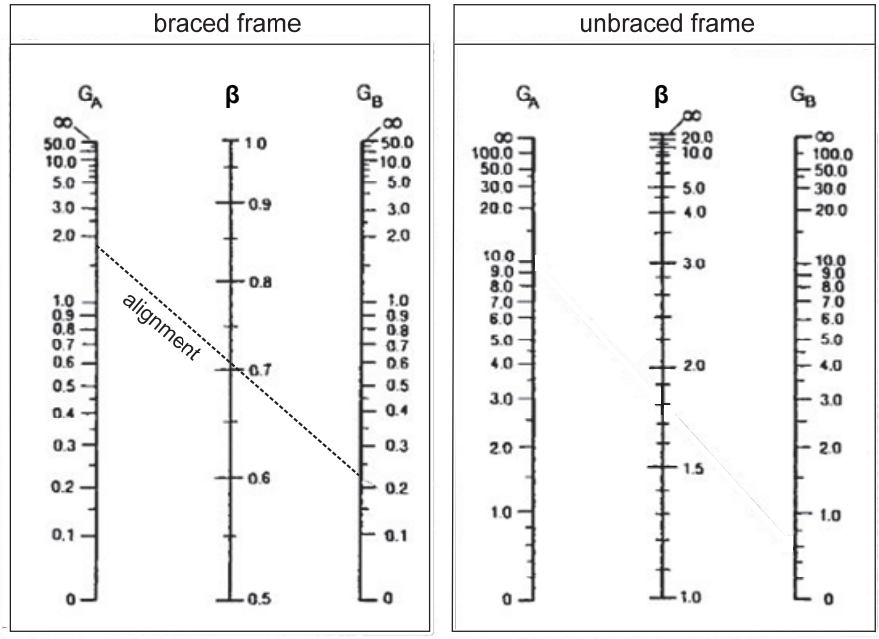

Figure 2. Alignment charts for the effective length factor β of columns

The traditional analysis of column buckling in complete structures accounts explicitly for the bending stiffness of the adjacent members of a column *C*. The stiffness factors G_A and G_B in figure 1 are defined for nodes *A* and *B* of column *C*. Restraint against sidesway is not specified for the adjacent members, but for the structure as a whole. The classification is empirical according to rules specified in codes. There are only two classes of lateral restraint: sidesway and no sidesway. Intermediate degrees of restraint, which exist in the structure, are not considered in the buckling analysis.

The influence of the restraints on the buckling load of column *C* is measured by means of the effective length factor β. The effective length factor of a simply supported column without sidesway equals 1 and its buckling load is given by the Euler formula. The buckling load P_{cr} of a column with general restraint is also computed with the Euler formula, but its true length *L* is replaced by the effective length βL. For given restraints, the effective length β is read in alignment charts [11] such as those shown in figure 2.

$$P_{cr} = \frac{\pi^2 EJ}{(\beta L)^2}. \qquad (1)$$

EJ – bending stiffness of the column.

The simplest frame is a portal frame, which consists of two equal columns connected by a horizontal girder. The analytical solutions for the effective length factor of portal frames with hinged and with fixed columns are shown in figure 3 for the classes sidesway and no sidesway. The stiffness ratio is $\varphi = G_A = (J_c L_g)/(J_g L_c)$.

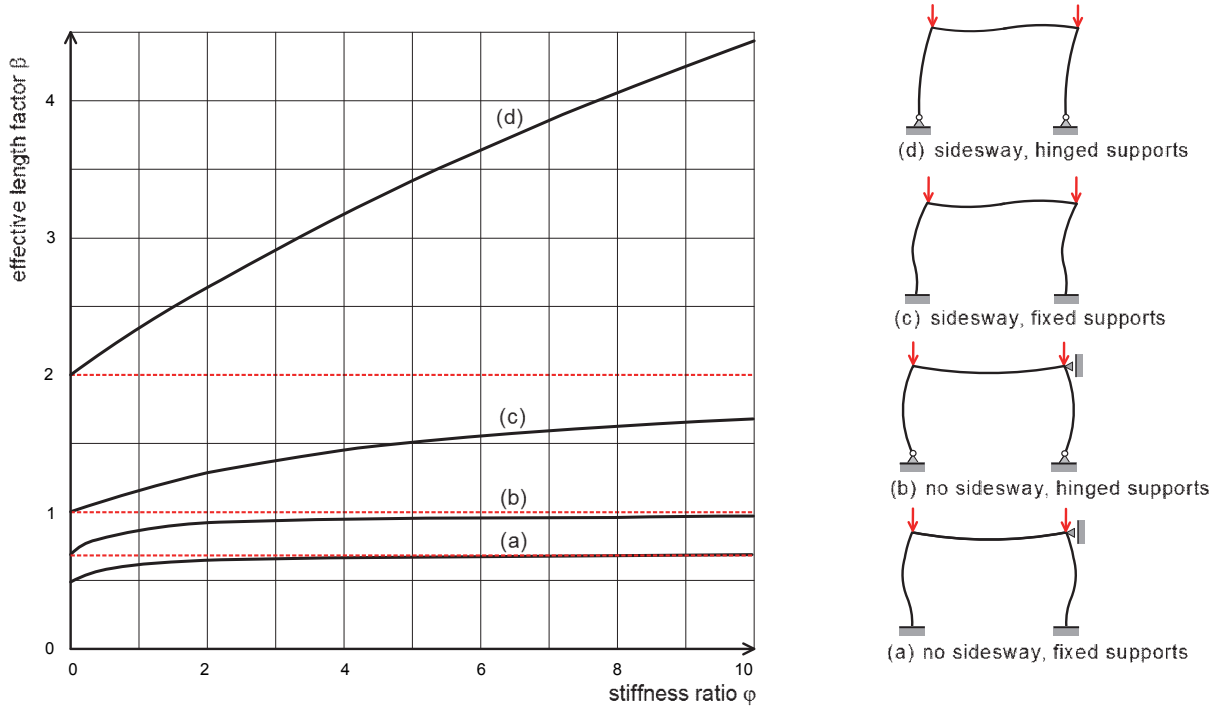

Figure 3. Buckling of portal frames

Figure 3 illustrates that the effective length factor β depends very strongly on the end restraint of the column. The buckling load in equation (1) depends on β^2. The economy of column design in engineering practice depends on the reliable determination of the effective length factor. To avoid the uncertainties associated with the empirical classification sidesway and no sidesway, which has a dominant effect on the effective length, and to account for the state of the adjacent members, which provide the buckling restraint for a column, a new approach is followed by basing the buckling design on a nonlinear analysis of the structure as a whole. The objective of the reported research is to compare this approach to the traditional column design method.

2. Nonlinear Analysis of Frames

In order to account for the true stiffness of the elastic frame in the buckling of columns, a geometrically nonlinear analysis of the frame as a whole

is performed. The attributes of the frame and a load pattern are prescribed. The applied load is the product of the load pattern and a load factor. The finite element method is used to formulate the governing equations [12]. The equilibrium conditions are satisfied for the instant configuration of the structure and the nonlinear terms of the strain-displacement equations are taken into account. The nonlinear governing equations are solved with a stepwise iterative method. The step size is controlled by keeping the arc increment constant. The displacement increments in the steps are summed to yield the total displacements.

In each step of the analysis, the tangent stiffness matrix K of the current frame configuration is decomposed into the product of a left triangular matrix L with unit diagonal elements, a diagonal matrix D with diagonal coefficients d_i and a right triangular matrix L^T. The product $d_1 d_2 ... d_n$ of the diagonal coefficients of D equals the determinant of the tangent stiffness matrix K of the frame in the current load step.

$$K = L D L^T. \tag{2}$$

$$\det K = d_1 d_2 ... d_n. \tag{3}$$

The diagonal coefficients d_i are monitored. If the sign of at least one coefficient d_i changes from positive to negative in a load step, this coefficient has the value null for a load factor λ_c within the load step. The tangent stiffness matrix K becomes singular for this load factor, and the frame buckles.

After the load step has been determined in which the frame configuration becomes singular, the value of the critical load factor λ_c is determined by solving a general eigenvalue problem. The formulation of this eigenvalue problem is also treated in [3] and implemented in a software platform. The following examples have been analyzed with this platform.

3. Test Cases

The buckling load of plane test frames with the geometry and loading shown in figure 4 is determined by nonlinear analysis. The test frame consists of 4 bays with equal widths of 6.0 m and 12 storeys with equal heights of 4.0 m.

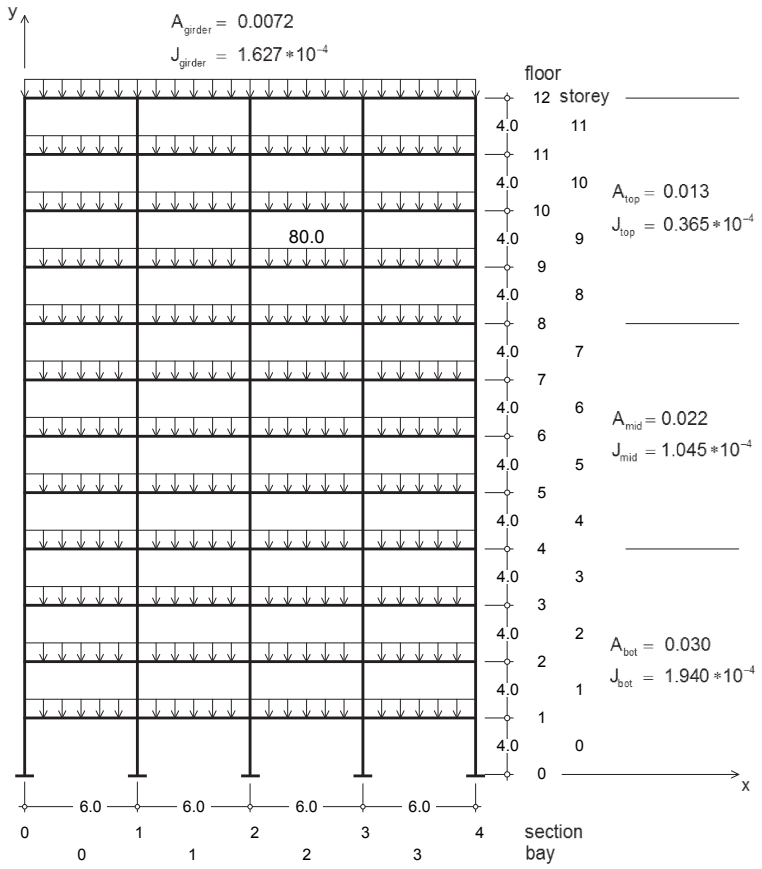

Figure 4. Multi-storey steel frame subjected to uniform floor load

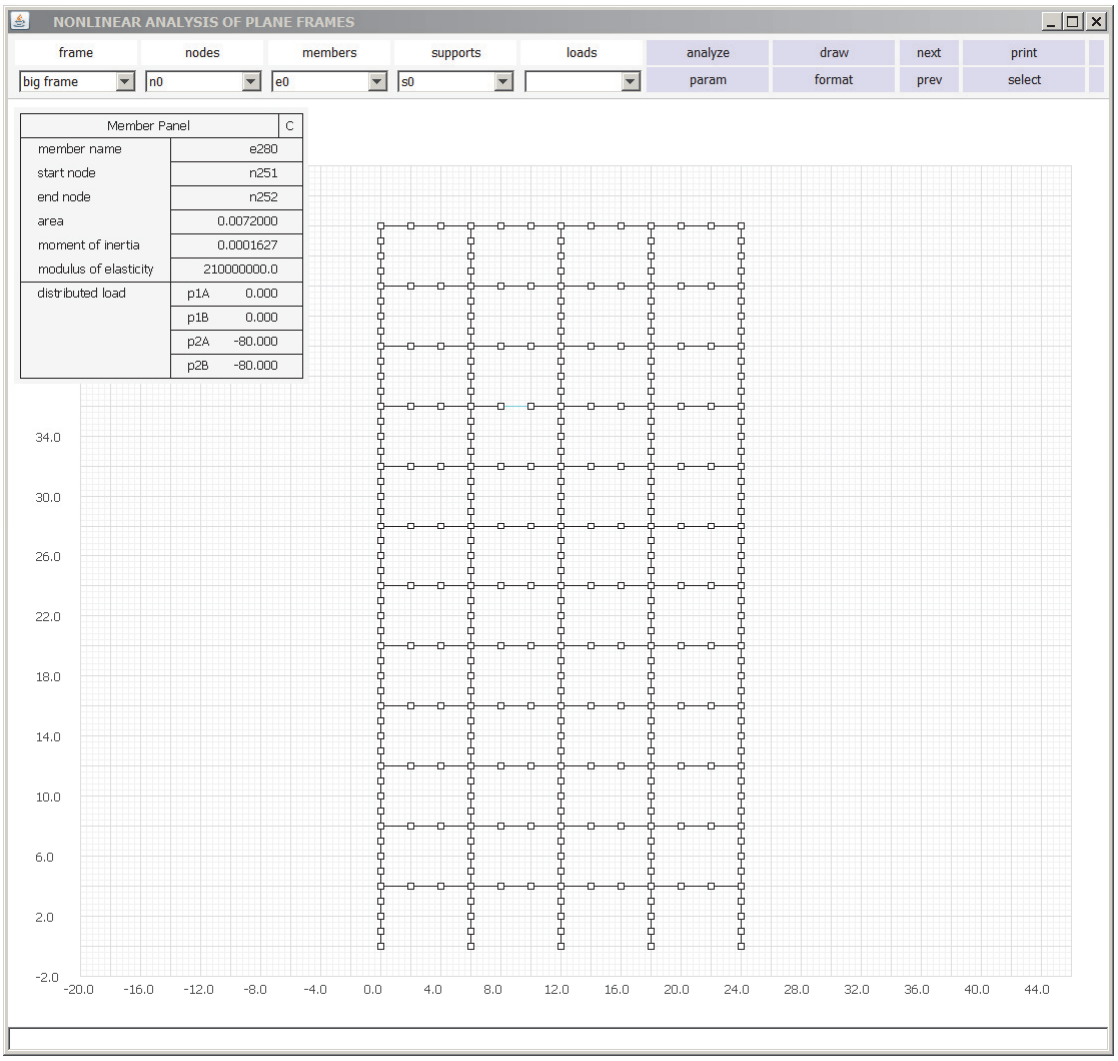

Figure 5. Graphic display with finite element model and member attribute panel

The bays are numbered consecutively from left to right, starting at 0. The vertical lines containing the columns are called sections. The sections are also numbered consecutively from left to right, starting at 0. Bay k starts at section k and ends at section $k + 1$. The storeys are numbered consecutively, starting at 0. The horizontal lines containing the girders are called floors. The floors are also numbered con-secutively, starting at 0. Storey k starts at floor k and ends at floor $k + 1$.

The cross-section of the girders is constant over the height of the frame. The cross-section of the columns is constant in floors 0 to 3, 4 to 7 and 8 to 11. The section properties are shown in the figure. Areas are specified in m^2, moments of inertia in m^4. All members of the frame have a modulus of elasticity of $2.1 * 10^8$ kN/m^2. The girders carry a uniformly distributed load of 80.0 kN / m.

The four analyses of the test frame are identified as case 1 to case 4. In cases 1 and 3 the columns of the lowest floor 0 are hinged at the foundations, in cases 2 and 4 they are fixed. In all cases the columns are fixed against translation at the foundations.

Different degrees of restraint against side-sway are provided by means of bracing in bay 0. In cases 1 and 2 the frame is unbraced. In cases 3 and 4 the frame is braced. A range of bracing stiffness is studied in both cases by varying the area of the braces from 0.0005 to 0.0020 m^2.

The test frame is mapped by a parameterized generator to a finite element model. The finite elements for bending in the nonlinear frame analysis do not account for the influence of the axial force on the bending moments due to the curvature of the deformed axis of the finite element. In order to achieve adequate accuracy of the buckling loads in a stability analysis, it is therefore not sufficient to model the column between two floors of the frame with a single finite element. Each column of the frame is mapped to 4 members with a length of 1.0 m in the finite element model. The girders of the frame are not subject

to large axial forces. Each girder of the frame is therefore mapped to 3 members with a length of 2.0 m in the finite element model.

Figure 5 shows the digital display of the software platform in which the nonlinear analysis has been implemented. The upper two rows contain buttons and combo-boxes for the control of the functions of the platform and the identification of nodes, members, loads and supports of the finite element model. The screenshot shows the finite element model for the frame in figure 4. Also shown is the panel with the attributes of the member which is marked with the color cyan in the frame elevation. At other stages of the analysis, the computed results are displayed in the graphic panel.

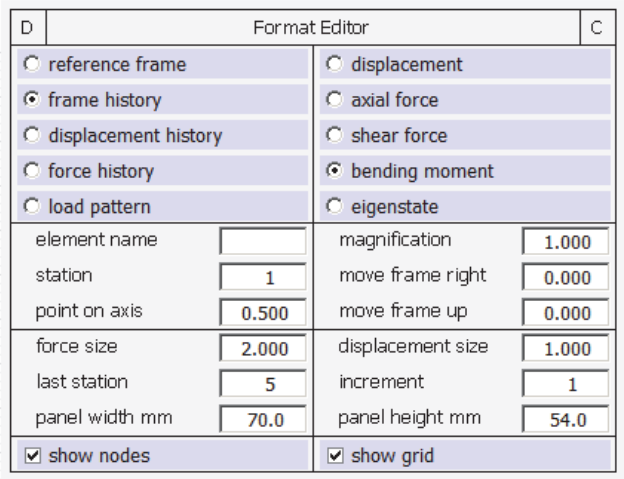

Figure 6. Format editor of the graphic user interface

The output of the nonlinear analysis is controlled with the format editor in figure 6.

Case 1. Unbraced frame with hinged supports

The load is applied in 10 steps. The frame reaches a singular state for load factor 0.9628. The displacements, bending moments and axial forces in the frame at the buckling load are shown in figure 7. Also shown is the buckled shape of the frame. The upper 10 storeys remain essentially undeformed at buckling and displace laterally due to bending deformations of the columns of the lowest two storeys.

There is no lateral displacement until buckling occurs. The vertical displacement of the topmost left node is 14.2 mm that of the neighboring node on the same floor is 30.0 mm. Bending of the inner columns is negligible. The bending moments in the outer columns reach 105 kN*m. The bending moments of -240 kN*m at the end points of the inner girders are nearly equal to $-qL^2/12 = -0.9628*80*6^2/12 = -231$ kN*m. The bending moment of 130 kN*m at mid-span of the outer girders exceeds $qL^2/24 = 116$ kN*m significantly.

The total load at buckling is 22182 kN. The axial forces in the columns in sections 0 to 4 of floor 0 are 2692, 5617, 5560, 5617, 2692 kN. The buckling loads of the single columns, as determined with the alignment charts, are 2165 kN for the outer columns and 3718 kN for the inner columns. The total capacity of the columns in floor 0 is $2*2165+3*3718 = 15484$ kN, which is 69.8 percent of the buckling load of the frame.

Case 2. Unbraced frame with fixed supports

The load is applied in 10 steps. The frame reaches a singular state for load factor 2.442. The displacements, bending moments and axial forces in the frame at the buckling load are shown in figure 8. Also shown is the buckled shape of the frame. Unlike case 1, the columns bend significantly in storeys 0 to 6 due to the fixed supports. The building undergoes shear deformation after buckling.

There is no lateral displacement until buckling occurs. The vertical displacement of the topmost left node is 36.3 mm, that of the neighboring node on the same floor is 76.4 mm. The increase relative to case 1 is proportional to the increase in the load factor. Bending of the inner columns is negligible. The bending moments in the outer columns reach 289 kN*m. The bending moments of -578 kN*m at the end points of the inner girders are nearly equal to $-qL^2/12 = -2.442*80*6^2/12 = -586$ kN*m. The bending moment of 430 kN*m at mid-span of the outer girders exceeds the value $qL^2/24 = 293$ kN*m significantly.

The total load at buckling is 56235 kN. The axial forces in the columns in sections 0 to 4 of floor 0 are 6823, 14247, 14095, 14247, 6823 kN. The buckling loads of the single columns, as determined with the alignment charts, are 12822 kN for the outer columns and 16084 kN for the inner columns. The total capacity of the columns in floor 0 is $2*12822+3*16084 = 73896$ kN, which is 131 percent of the buckling load of the frame, as compared to 69.8 percent in case 1.

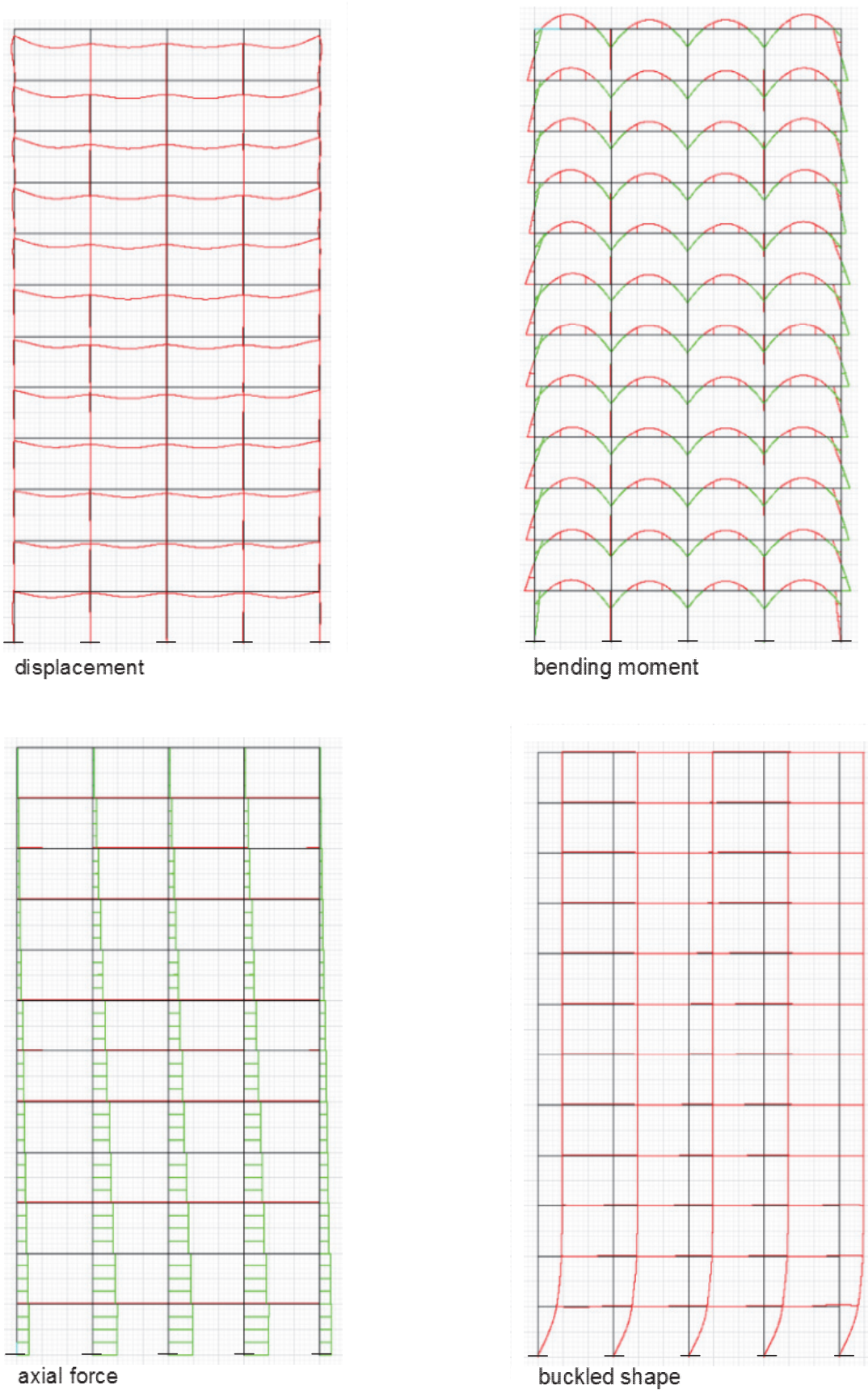

Figure 7. Singular state in case 1 (unbraced frame with hinged supports)

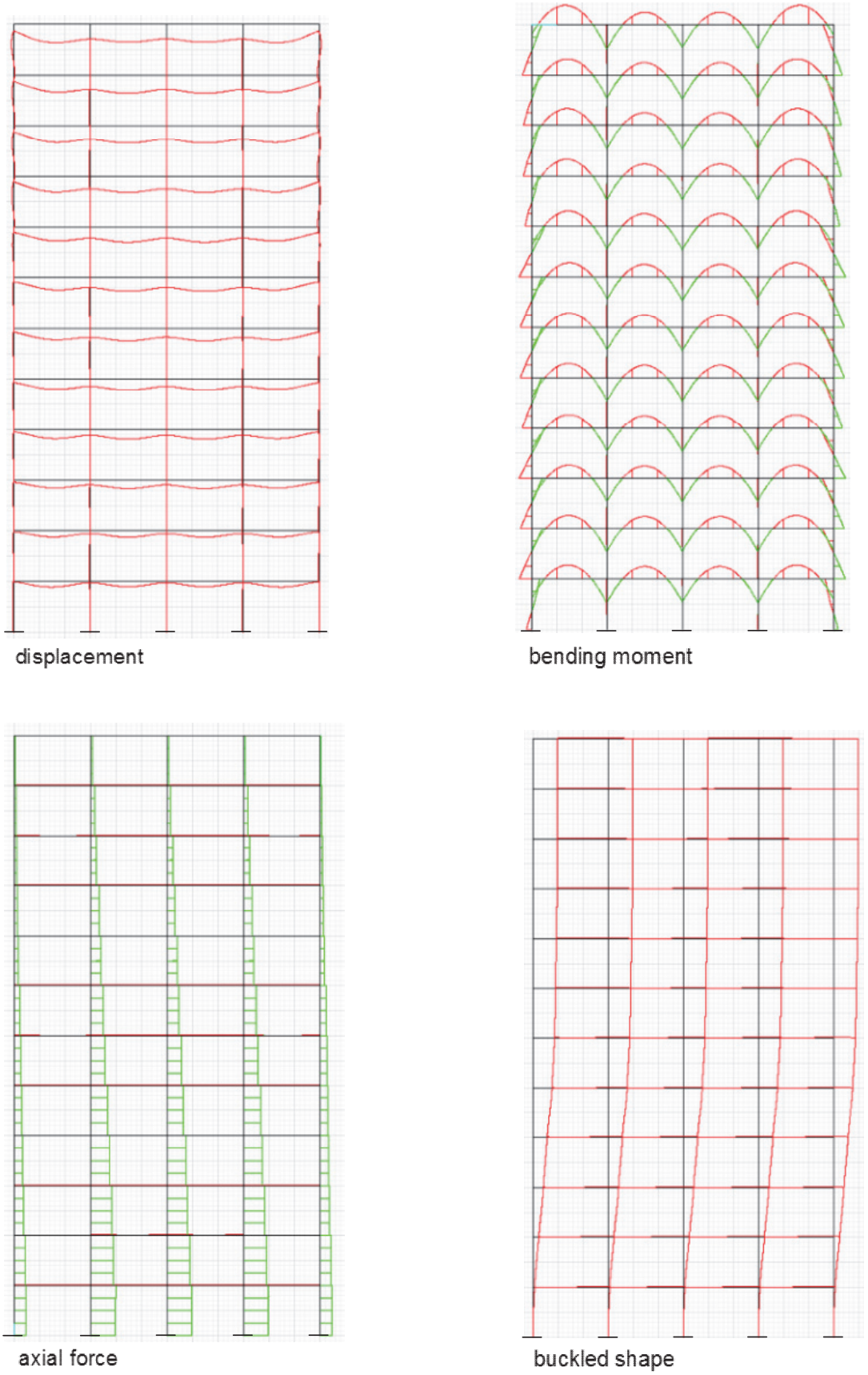

Figure 8. Singular state in case 2 (unbraced frame with fixed supports)

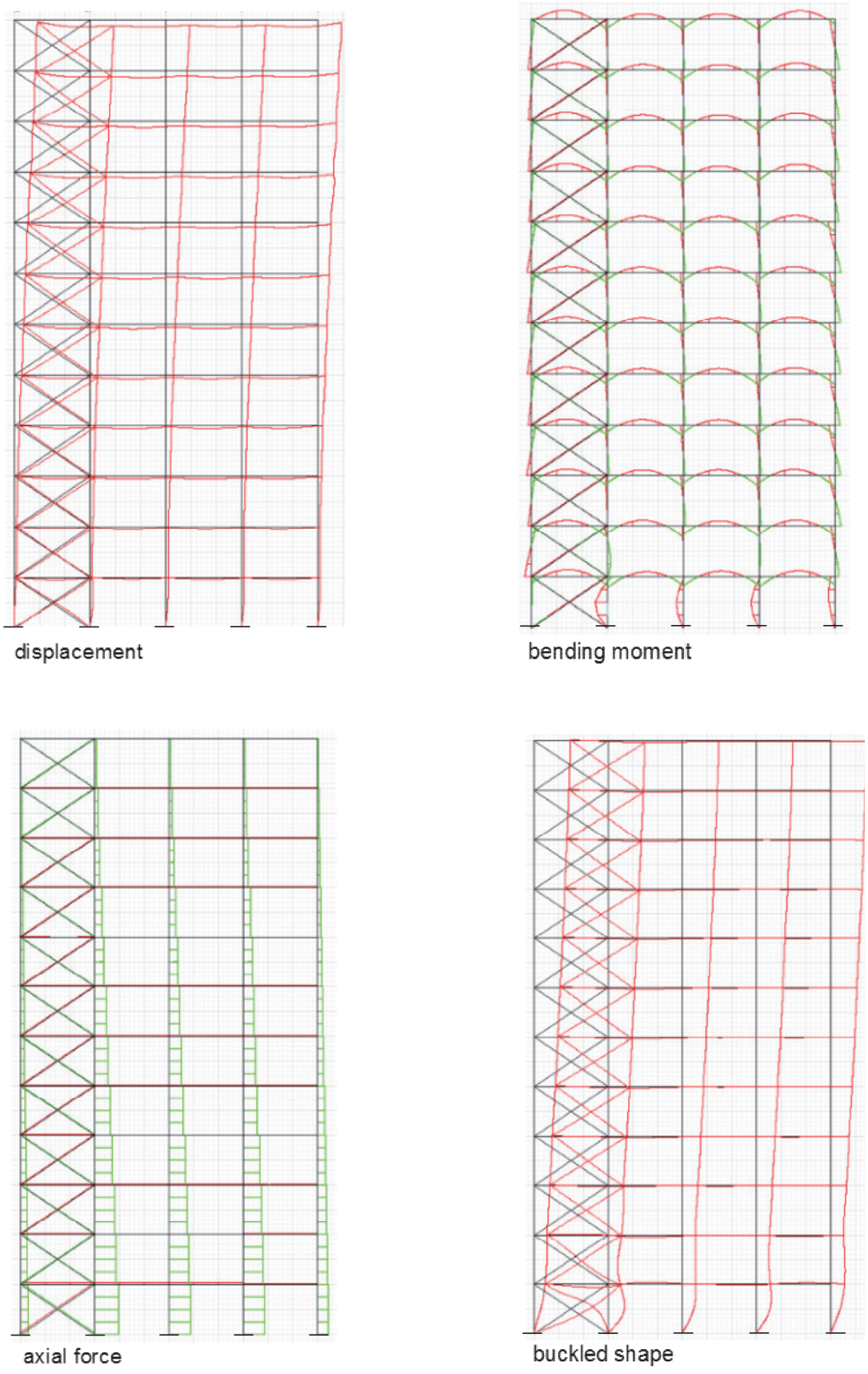

Figure 9. Singular state in case 3 (braced frame with hinged supports), $A_{brace} = 0.001$

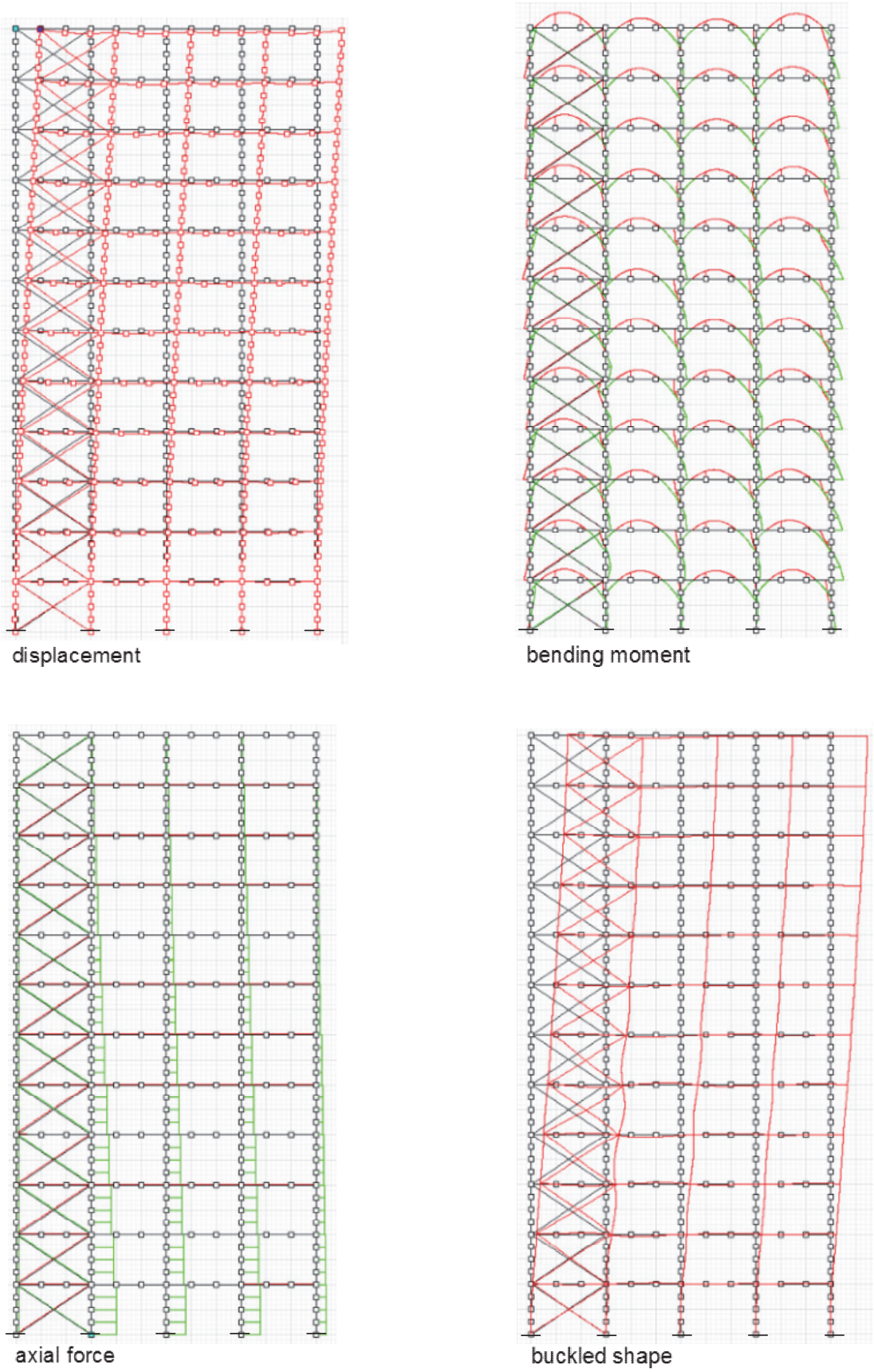

Figure 10. Singular state in case 4 (braced frame with fixed supports), $A_{brace} = 0.001$

Case 3. Braced frame with hinged supports

The load is applied in 10 steps. For a brace area of 0.001, the frame reaches a singular state for load factor 5.3533. The displacements, bending moments and axial forces in the frame at the buckling load are shown in figure 9. Also shown is the buckled shape of the frame. The buckled shape is a shear deformation of frame with strong deformation in storey 0.

The frame displaces laterally before buckling. This is due to the unsymmetrical bracing. The vertical displacement of the topmost left node is 7.0 mm, its lateral displacement is 737 mm. If the maximum lateral displacement of a frame of height H is limited to $H/100$, the maximum permitted lateral displacement is 480 mm. The frame therefore cannot be loaded up to the singular state. Because the load factor-displacement diagram of the left topmost node is highly nonlinear, the maximum permitted load factor of 4.62 is read in the diagram.

Due to the bracing, the bending moments in the columns are lower than in cases 1 and 2, but the moments in the columns of storey 0 are significantly higher than those in the other storeys of the frame.

The total load at buckling is 123370 kN. The axial forces in the columns in sections 0 to 4 of floor 0 are 9709, 35733, 31091, 31127, 15710 kN. The buckling loads of the single columns, as determined with the alignment charts, are 28320 kN for the outer and 30615 kN for the inner columns. The total capacity of the single columns in floor 0 is $2 * 28320 + 3 * 30615 = 148485$ kN or 120 percent of the buckling load of the frame.

Case 4. Braced frame with fixed supports

The load is applied in 10 steps. For a brace area of 0.001, the frame reaches a singular state for load factor 6.9384. The displacements, bending moments and axial forces in the frame at the buckling load are shown in figure 10. Also shown is the buckled shape of the frame. The buckled shape is a shear deformation of the frame. The deformation in storey 0 is much less than in case 3, but the bending deformation of the columns in section 1 has become large in storeys 2 to 6.

The frame displaces laterally before buckling. The vertical displacement of the topmost left node is 81.0 mm, its lateral displacement is 1486 mm. If the maximum lateral displacement is limited to H/100, the maximum permitted lateral displacement is 480 mm. Because the load factor-displacement diagram of the left topmost node is highly nonlinear, the maximum permitted load factor of 4.70 is read in the diagram.

The total load at buckling is 159860 kN. The axial forces in the columns in sections 0 to 4 of floor 0 are 5880, 51937, 40668, 40185, 21193 kN. The buckling loads of the single columns, as determined with the alignment charts, are 47615 kN for the outer and 59480 kN for the inner columns. The total capacity of the single columns in floor 0 is $2 * 47615 + 3 * 59480 = 273670$ kN or 171.2 percent of the buckling load of the frame.

Stiffness of the braces

Table 1 and figure 11 show the influence of the area of the cross-bracing on the critical load factor of the frame. The sensitivity of the buckling load to the stiffness of the bracing is not reflected in the alignment charts in figure 2.

Table 1. **Influence of the brace area on the load factor *LF* for buckling**

Area of a brace	*LF* for hinged supports	*LF* for fixed supports
0.0005	3.7236	5.6305
0.0006	4.1440	6.1045
0.0007	4.5093	6.5058
0.0008	4.8573	6.7105
0.0009	5.1534	6.8433
0.0010	5.3533	6.9384
0.0015	5.8648	7.1724
0.0020	5.9944	7.2583

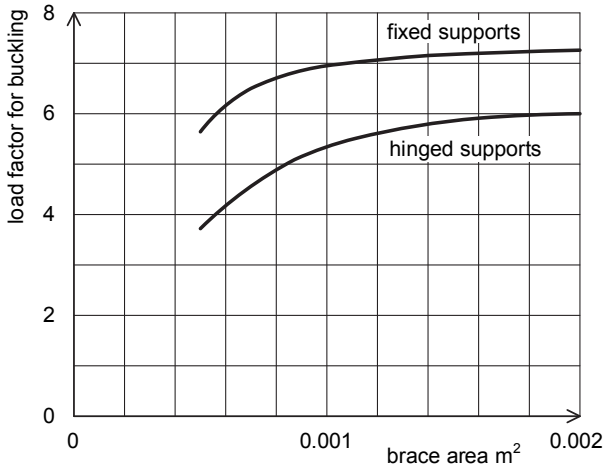

Figure 11. Influence of the brace area on the buckling load of the braced frame

4. Conclusions and recommendations

The design of high-rise steel frames against buckling by sidesway – no sidesway categorization, combined with the use of alignment tables for effective

length factors of single columns, has been compared to the buckling analysis of the frames as a whole with nonlinear models. Both methods confirm the large differences between the buckling loads of braced and unbraced high-rise frames, which are well known from analytical solutions for simple portal frames. Sidesway is a very important factor influencing the stability of the frames.

Four test frames have been analyzed to show that the differences between the results of the two methods are significant. The two-class sidesway categorization does not account for the stiffness of the bracing. Either there is no bracing, or the restraint against sidesway is rigid. The nonlinear analysis shows that the stiffness of the restraint has a strong influence on the magnitude of the buckling load of the braced frame. It is therefore expected that the nonlinear analysis leads to more economical designs than the two-category method. The nonlinear analysis also improves safety because the "no sidesway" condition of the two-category method cannot be implemented in the built structure, such that the buckling load is less than the value computed with that method.

Before the nonlinear method can be recommended for general use in the buckling design of frames, additional investigations are required. For example, the buckling loads for general load patterns should be studied in addition to the uniformly distributed load on all beams used in the examples of this paper. Broader ranges of frame geometry and member stiffness should be investigated. The influence of elastic-plastic behavior must be considered. For general structures, a wide range of three-dimensional nonlinear analyses must be performed and evaluated relative to the traditional method of design.

An additional fundamental theoretical issue must also be addressed. Several finite elements have been used in this study to model one member of the frame. This subdivision is necessary because the element stiffness matrices in the applied method (and in many commercial packages) do not account for the influence of axial force on the bending stiffness of the individual member. If each member of a frame can be mapped to a single element in the model, which buckles at the Euler load corresponding to its restraint condition in the frame, the required storage and computational capacity for the nonlinear analysis is reduced very significantly. This reduction is highly desirable if the method is considered for general use.

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НАУЧНАЯ СТАТЬЯ

Анализ устойчивости рам без учета классификации по возможности поперечных смещений

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Расчетные длины колонн при расчете стальных рам определяются в зависимости от типа рамы – с возможностью поперечного смещения или при отсутствии такового. Классификация рам по этому признаку зависит от жесткости конструкции рамы и условий ее закрепления и выполняется эмпирически. Изменение типа рамы в соответствии с этой классификацией ведет к значительному изменению расчетных длин ее колонн, что влечет за собой изменение критической нагрузки, влияет на размер сечения колонн и общую материалоемкость конструкции рамы. Для того чтобы избежать неопределенности эмпирической классификации, предлагается определять критическую нагрузку рамы при помощи нелинейного расчета, а расчетные длины колонн уточнять, исходя из формы потери устойчивости. Предлагаемый метод проиллюстрирован примерами расчета жесткой и связевой рам. Полученные усилия в колоннах первого этажа сравнены с критическими нагрузками отдельно стоящих колонн. Выполнено сравнение расчетов по методике норм США с использованием классификации рам и предлагаемому нелинейному методу. Результаты сравнения подтверждают значительные расхождения в критической нагрузке для связевых и жестких многоэтажных рам.

Ключевые слова: высотное здание, потеря устойчивости колонны, продольный прогиб, расчетная длина

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