Механика жидкости

MODELING THE HYDRAULIC CALCULATION OF THE CONTROLLING RESERVES IN DRINKING WATER SUPPLY SYSTEMS

Taofic BACHAROU*, Gossou HOUINOU, Gérard A. GBAGUIDI,
Clément AHOUANNOU, Ezéchias KANHO, Ruben AZA-GNANDJI
Polytechnic School of Abomey-Calavi, University of Abomey-Calavi,
Laboratory of Energetics and Applied Mechanics
01 BP 2016 Cotonou, Benin
** Correspondence, e-mail : btaofic@yahoo.fr*

This study is part of the development of a model of hydraulic calculation of controlling reserves control in drinking water supply systems. To that end, the general expressions of the cumulative curves of gravity water supply and pumped water supply as well as the ones of the distribution over time were formulated. It appears from this study that for a peak coefficient Kp between 1.2 and 1.4, the cumulative consumption evolves linearly and for Kp ranging from 1.45 to 2.5 the cumulative consumption follows a polynomial curve of degree 3. Then, there is a relationship between the respective totals of gravity water supply and pumped water supply and the total of the cumulative consumption, which superimposed have enabled the establishment of the mathematical model for determining the volume of the controlling reserves.

Keywords: Drinking water supply - Modeling - Water supply - Consumption - Reservoir - Volume – Program.

1. INTRODUCTION

In drinking water supply systems, drinking water reservoirs are needed in cities where spontaneous urban settlements with middle class standing are important. The satisfaction of the population with respect to the water demand is ensured by the continuous service offered by the structures and equipment of the water supply system throughout the day. This performance is largely ensured by water supply reserves (such as cistern, water towers etc.) that serve as a buffer between the production which is fairly constant and the distribution which remains highly variable with time. Poor design of these reservoirs in water networks causes severe problems on the performance and even on the cost of the entire water supply network [4]. The various reserve components (regulation, fire, emergency and maintenance) of the reservoir volume show that the storage capacity varies depending on the controlling reserves since the others are estimated with specific standards or guidelines for each region. Controlling reserves for a given day represent about 20% of the daily consumption, the maximum volume of control being calculated from the hourly consumption of the day of the highest consumption [5]. However, studies have shown that according to guidelines, the reservoir capacity can range from less than 25% to 100% of the daily consumption peak taken for the project horizon [2.9]. Indeed, the present study focuses on the development of a rational approach for the hydraulic calculation of the controlling reserves to reduce differences on the basis of the analysis of design approaches currently used by field engineers.

2. METHODOLOGY

1.1 Materials

The cumulative water supply has been modelled using the Laplace transform which is a widely used tool for mathematical modelling of physical phenomena (inflow, consumption pattern, etc.). The Maple 13 software enabled the adjustments of curves of the cumulative consumption for each peak coefficient. MATLAB software is used for the treatment of matrix components of the pumped water flow rate and the pumping time.

2.2 Methods

The most used method to estimate the controlling volume allows viewing offs between the periods of low consumption and those of high consumption in order to adjust the pumping periods to minimize the risk of rupture of supply during hours of high consumption [5]. In fact, we fix the time for daily pumping, the pumping periods and the pumping rate and we successively represent per day the water supply and the consumption, simplified in hourly slot, the curves of the accumulated previous flow rates and the superposition of the curves of accumulated flow rates. A parallel translation of the supply curve for covering the distribution curve allows the visualization of both maximum fluctuations. The sum of these two fluctuations indicates the volume of the controlling reserves. These methods have been generally replaced by extended period simulation models which can be more flexible than the graphical methods.

Reservoirs operation Analysis showed that the capacity of the reservation distribution is function of the supply flow rate and the fluctuations in the distribution flow rate [8]. Let $r_1(t)$ the hourly flow rate of water supply, $r_2(t)$ the hourly flow rate of distribution and $x(t)$ the flow rate of controlling reserves, then:

$$x(t) = r_1(t) - r_2(t). \tag{1}$$

2.2.1 Modelling of the water supply

Pumped water supply

The function $r_1(t)$ reflects the pumped flow rate at time t, defines as [10]:

$$r_1(t) = \begin{cases} Q_1 & if \ t \in [a_0, a_1[\\ Q_2 & if \ t \in [a_1, a_2[\\ Q_3 & if \ t \in [a_2, a_3[\\ Q_4 & if \ t \in [a_3, a_4[\\ Q_5 & if \ t \in [a_4, a_5[\\ \vdots & \vdots \\ Q_n & if \ t \in [a_{n-1}, a_n[\end{cases},$$

where: t is the pumping time, $a_n - a_0 = 24\,h$, Q_n is the pumped flow rate in the n^{th} time interval, n is the number of intervals $(n, \dots, \geqslant 1)$.

To obtain the Laplace transform [7] of the function $r_1(t)$ which allows for the accumulation of supply, it is necessary to express it in the form of linear combination of unit step functions:

$$r_1(t) = Q_1 + (Q_2 - Q_1)ua_1(t) + (Q_3 - Q_2)ua_2(t) + (Q_4 - Q_3)ua_3(t) + (Q_5 - Q_4)ua_4(t) + \dots + (Q_n - Q_{n-1})ua_{n-1}(t),$$

hence :

$$r_1(t) = \sum_{n=1}^{+\infty}(Q_n - Q_{n-1})ua_{n-1}(t), \tag{2}$$

With $ua_{n-1}(t)$- the Heaviside function defines as:

$$ua_{n-1}(t) = \begin{cases} 0 & if\ \ t < a_{n-1} \\ 1 & if\ \ t \geqslant a_{n-1} \end{cases}$$

Let $R_1\,(t)$ be the cumulative flow rates of supply, $R_1\,(t) = \sum r_1\,(t)\,dt$.

Using the theorem of the Laplace derivations [7], it is established that:

$$\mathcal{L}[\![r_1\,(t)\,]\!] = s\mathcal{L}[\![R_1\,(t)\,]\!] - R_1\,(0)\,. \tag{3}$$

Whereas $R_1\,(0) = 0$, since there is no pumping at time $t = 0$

$$\mathcal{L}[\![r_1\,(t)\,]\!] = s\mathcal{L}[\![R_1\,(t)\,]\!] \qquad ==> \quad \mathcal{L}[\![R_1\,(t)]\!] = \frac{\mathcal{L}[\![r_1\,(t)\,]\!]}{s}\,. \tag{4}$$

From equation (2), we have:

$$\mathcal{L}[\![r_1\,(t)\,]\!] = \textstyle\sum_{n=1}^{+\infty}(Q_n - Q_{n-1})\mathcal{L}[\![ua_{n-1}\,(t)\,]\!]\,avec\;(Q_n - Q_{n-1}) = constante.$$

Whereas the Laplace transform of the Heaviside function gives:

$$\mathcal{L}[\![ua_{n-1}\,(t)\,]\!] = \frac{e^{-a_{n-1}s}}{s}\ \ with\ \ s > 0\,. \tag{5}$$

From equations (4) and (5), we obtain the Laplace transform of $R_1\,(t)$:

$$\mathcal{L}[\![R_1\,(t)\,]\!] = \sum_{n=1}^{+\infty}(Q_n - Q_{n-1})\frac{e^{-a_{n-1}s}}{s^2} \quad with\,s > 0. \tag{6}$$

The inverse Laplace transform can enable to express $R_1\,(t)$ as

$$R_1\,(t) \quad = \quad \mathcal{L}^{-1}[\![R_1\,(t)]\!] =$$

$$= \sum_{n=1}^{+\infty}(Q_n - Q_{n-1})\,\mathcal{L}^{-1}\left[\!\!\left[\frac{e^{-a_{n-1}s}}{s^2}\right]\!\!\right]\ with\,s > 0. \tag{7}$$

Whereas $\mathcal{L}^{-1}\left[\!\!\left[\frac{e^{-a_{n-1}s}}{s^2}\right]\!\!\right] = \mathcal{L}^{-1}\left[\!\!\left[\frac{1}{s^2}\ .\,e^{-a_{n-1}s}\right]\!\!\right] = \mathcal{L}^{-1}\left[\!\!\left[\frac{1!}{s^{1+1}}\ .\,e^{-a_{n-1}s}\right]\!\!\right] =$

$$= \mathcal{L}^{-1}[\![F(s).\,e^{-a_{n-1}s}\,]\!]. \tag{8}$$

With $F(s) = \mathcal{L}[f]$, where $f(t) = t^{\alpha}$, here the exponent α is equal to 1 according to the equation (8) then $f\,(t) = t$, and then:

$$\mathcal{L}^{-1}\left[\!\!\left[\frac{e^{-a_{n-1}s}}{s^2}\right]\!\!\right] \ = \ f(t - a_{n-1})\,ua_{n-1}\,(t) = (t - a_{n-1})\,ua_{n-1}\,(t) \tag{9}$$

From equations (7) and (9), we obtain the accumulation of supply as follows:

$$R_1\,(t) = \sum_{n=1}^{+\infty}(Q_n - Q_{n-1})(t - a_{n-1})\,ua_{n-1}\,(t) \qquad with\ Q_0 = 0 \tag{10}$$

The expression (10) is the model of accumulation of pumped water supply over time.

Gravity water supply

The function $r_1\,(t)$ reflects the flow rate of gravity water supply at time t, defines as:

$$r_1\,(t) = \left\{Q_p = \frac{Q_{jmax}}{24} \right. \qquad\qquad if\ t\epsilon\,[0;24]$$

We have: $n = 1$ (n being the number of intervals)

$$R_1\,(t) = \textstyle\sum_{n=1}^{1}(Q_p\ - 0)\,(t - a_0)\,ua_0(t), \quad a_0 = 0\,;\,a_1 = 24\,;$$

$$ua_0(t) = \begin{cases} 0 & if\ t < 0 \\ 1 & if\ t \geqslant 0 \end{cases},\ \ R_1\,(t) = \frac{Q_{jmax}}{24}(t - 0).\,1,$$

$$R_1\,(\mathrm{t}) = \frac{Q_{jmax}}{24}\mathrm{t}\ . \tag{11}$$

The expression (11) shows that the accumulation of water supply is linear with respect to time.

2 Modelling consumption

Modelling the consumption pattern is based on the hourly distribution of the maximum daily consumption according to the hourly peak coefficient Kp ranging from 1.2 to 2.5 [2] (Fig. 1).

The consumption curves (Figure 2) were used to calculate the cumulative consumption based on the hourly peak coefficients Kp. It is observed that the cumulative consumption varies linearly for Kp between 1.2 and 1.4, and varies in the form of polynomial curve of degree 3 for Kp ranging from 1.45 to 2.5.

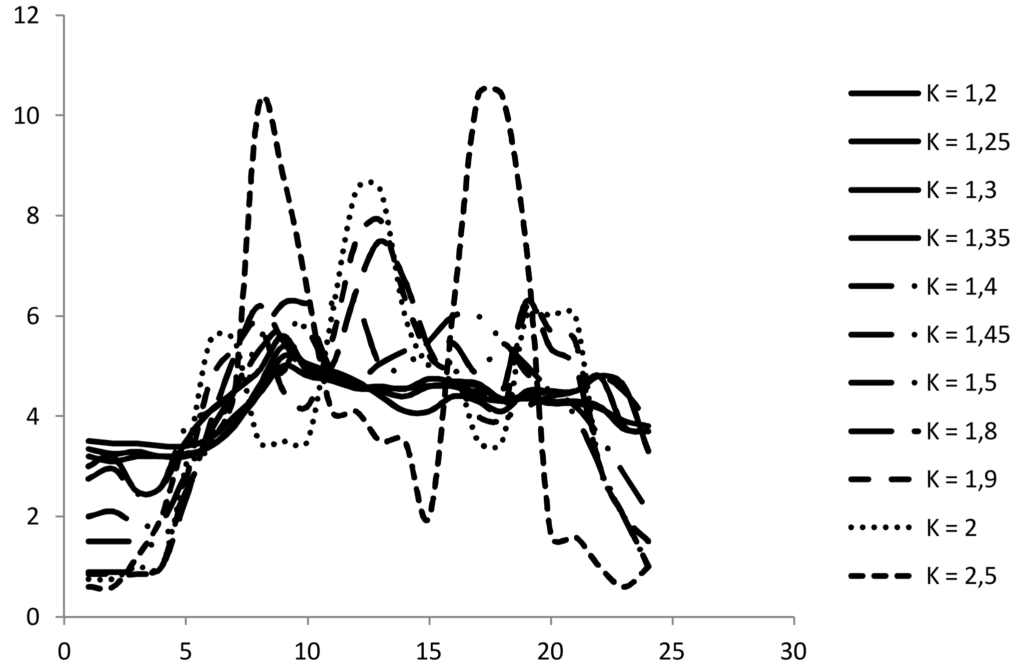
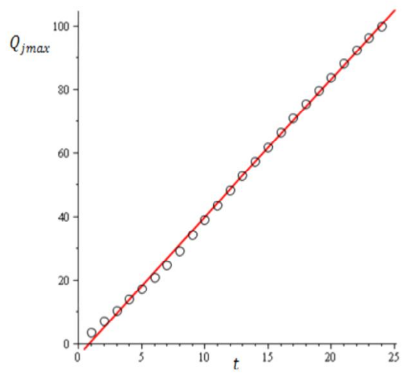
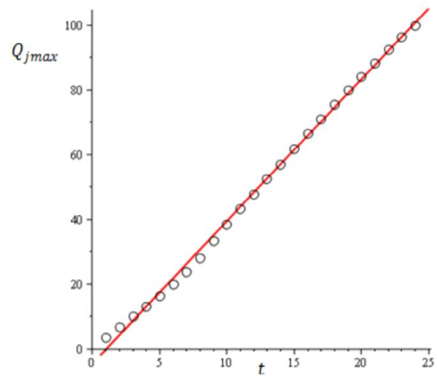


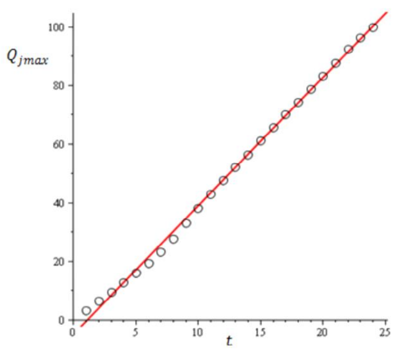
Figure 1: Consumption Curves over 24 hours (in % of the maximum daily consumption)



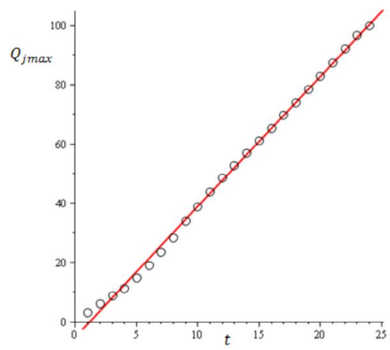
Cumulative consumption curve for K=1,2



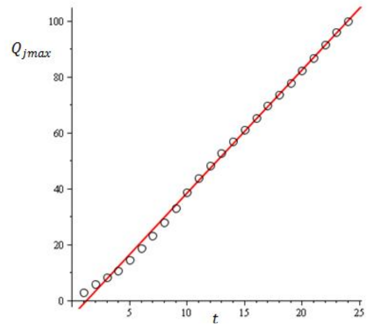
Cumulative consumption curve for K=1,25



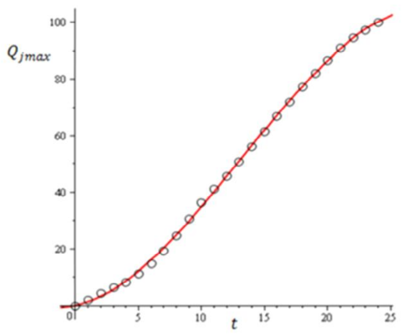
Cumulative consumption curve for K=1,3



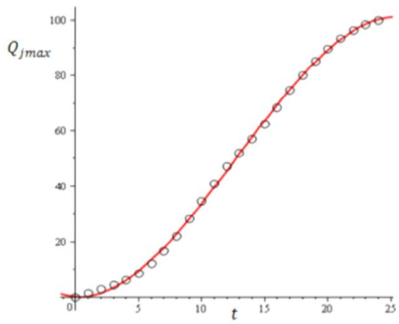
Cumulative consumption curve for K=1,35



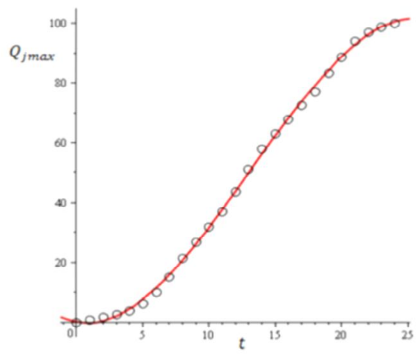
Cumulative consumption curve for K=1,4



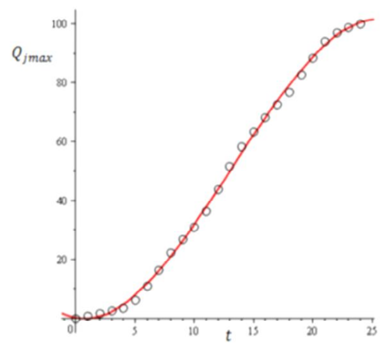
Cumulative consumption curve for K=1,45



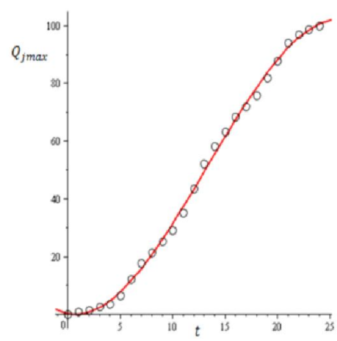
Cumulative consumption curve for K=1,5



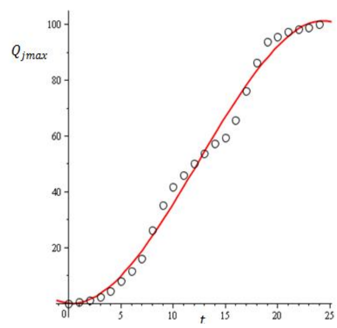
Cumulative consumption curve for K=1,8



Cumulative consumption curve for K=1,9



Cumulative consumption curve for K=2



Cumulative consumption curve for K=2,5

Figure 2: Evolution of the cumulative consumption for different hourly peak coefficients according to the time

The expression of the accumulated consumption becomes then:

- for $1{,}2 \leq K_p \leq 1{,}4$, $R_2\,(t) = (At + B).\frac{Q_{jmax}}{100}$. (12)

where $R_2\,(t)$ - Cumulative consumption at time t, m^3;

Q_{jmax} - maximum daily consumption, m^3 / day;

A and B - linear adjustment coefficients recorded in Table 1,

For $1.45 \leq Kp \leq 2.5$, $R_2\,(t) = (A't^3 + B't^2 + C't).\frac{Q_{jmax}}{100}$, (13)

where A ', B', C '- polynomial fit coefficients of degree 3, recorded in Table 2.

Table 1: Values of adjustment coefficients A and B based on the hourly peak coefficient

hourly peak coefficient	Values of adjustment coefficients	
	A	B
1,2	4,35	- 3,7
1,25	4,39	- 4,58
1,3	4,38	- 4,91
1,35	4,39	- 5,06
1,4	4,4	- 5,39

Table 2: Values of adjustment coefficients A ', B' and this based on the hourly peak coefficient

hourly peak coefficient	Values of adjustment coefficients		
	A'	B'	C'
1,45	- 0,00863	0,3425	0,931
1,5	- 0,0127	0,491	- 0,265
1,8	- 0,0135	0,533	- 0,816
1,9	- 0,0133	0,524	- 0,722
2	- 0,0128	0,512	- 0,671
2,5	- 0,0144	0,536	- 0,337

3 Determining the controlling reserves

Based on equations (10) and (11), it is observed that the cumulative water supply almost follow a straight trend line for a given time period. The cumulative consumption varies greatly during the day and depends on the variation of the hourly peak coefficient as indicated by equations (12) and (13). The controlling reserves which is a buffer between the cumulative supply during the day and the daily distribution, by the accumulation of excess water during periods of low consumption and its restitution during hours of high consumption. Its expression is therefore as follows:

$$X(t) = R_1\,(t) - R_2\,(t). \tag{14}$$

- ***Expression of X (t) in the case of a gravity water supply***

From equations (11), (12) and (14), we have:

$$X(t) = \frac{Q_{jmax}}{24}\,t - (At + B)\,.\frac{Q_{jmax}}{100}$$
$$= \left(\frac{Q_{jmax}}{24} - 0{,}01.Q_{jmax}.A\right)t - 0{,}01.B.Q_{jmax}\ . \tag{15}$$

From equations (11), (13) and (14), we have:

$$\boldsymbol{X(t) = \frac{Q_{jmax}}{24}\,t - (A't^3 + B't^2 + C't).\frac{Q_{jmax}}{100}}$$
$$= \mathbf{Q_{jmax}}\left[\mathbf{-0,01A't^3 - 0,01B't^2 + \left(\frac{1}{24} - 0,01C'\right)t}\right] \tag{16}$$

- ***Expression of X (t) in the case of pumped water supply***

Form equations (10), (12) and (14), we have:

$$X(t) = \sum_{n=1}^{+\infty}(Q_n - Q_{n-1})(t - a_{n-1})\,ua_{n-1}(t) - (At + B)\,.\frac{Q_{jmax}}{100}\,. \tag{17}$$

Form equations (10), (13) and (14), we have:

$$X(t) = \sum_{n=1}^{+\infty}(Q_n - Q_{n-1})(t - a_{n-1})\,ua_{n-1}(t) - (A't^3 + B't^2 + C't).\frac{Q_{jmax}}{100}\,. \tag{18}$$

The controlling reserves are obtained from the superposition of the curves of flow rates of cumulative water supply and the ones of the distribution. A parallel translation of the supply curve for covering the distribution curve allows the visualization of both maximum fluctuations. The sum of these two fluctuations indicates the minimum volume of the controlling reserves.

4 RESULTS AND DISCUSSIONS

The minimum volume of the controlling reserves, on the basis of equations (14) and (15), is the sum, in absolute values, of the largest value and smallest (negative) value of X (t).

4.1 Gravity water supply

From equation (15) and table 1, we have:

- $X(t) < 0 \Rightarrow \left(\frac{Q_{jmax}}{24} - 0{,}01.Q_{jmax}.A\right)t < 0{,}01.B.Q_{jmax}$,

whereas $\left(\frac{Q_{jmax}}{24} - 0{,}01.Q_{jmax}.A\right) < 0$, for all values of A,

hence $$t > \frac{0{,}01.B.Q_{jmax}}{\left(\frac{Q_{jmax}}{24} - 0{,}01.Q_{jmax}.A\right)} = t_\alpha.$$

With t_α - the limit time between filling up and the supplying of water to the network linked to the reservoir. $X(t) < 0$ for $t > t_\alpha$ $(t_\alpha \le t \le 24)$,then $R_2(t)$ is greater $R_1(t)$ and it occurs the double supply of water to the network by the reservoir and source. By framing $X(t)$ on this interval, we obtain its smallest negative value which is as follows:

$$V_2 = 24\left(\frac{Q_{jmax}}{24} - 0{,}01.Q_{jmax}.A\right) - 0{,}01B.Q_{jmax}, \quad (19)$$

$X(t) > 0$ for $0 < t < t_\alpha$, at that time of the day occurs the filling up of the reservoir. Similarly, framing $X(t)$ on this interval enables the finding of its maximum positive value which is as follows:

$$V_1 = -0{,}01.B.Q_{jmax}\,. \quad (20)$$

The minimum volume of controlling reserves becomes:

$$V_R = V_1 + |V_2| + \mu\,. \quad (21)$$

Equations (19) and (20) give:

$$V_R = Q_{jmax}(0{,}24.A - 1) + \mu, \quad (22)$$

with $$\mu = \frac{Q_{jmax}}{100}.\beta\,, \quad (23)$$

where μ - the volume correction factor which corrects the adjustment differences, β - the correction coefficient obtained by simulation with a MATLAB programme which adjusts the volume of the controlling reserves. It is based on the hourly peak coefficient in Figure 3.

In the conditions related to (16), $X(t)$ being a third degree polynomial, its maximum and minimum values are determined from the properties of its first and second derivatives. So:

$$X'(t) = Q_{jmax}\left[-0{,}03A't^2 - 0{,}02B't + \left(\frac{1}{24} - 0{,}01C'\right)\right], \quad (24)$$

$$X'(t) = 0 \Rightarrow \Delta = 0{,}0004B'^2 + 0{,}005A' - 0{,}0012A'C'.$$

Based on table 2, $\Delta > 0$ so there are two distinct solutions t_1 and t_2 of $X'(t) = 0$:

$$t_1 = \frac{-0{,}02B' - \sqrt{\Delta}}{0{,}06A'} \quad \text{and} \quad t_2 = \frac{-0{,}02B' + \sqrt{\Delta}}{0{,}06A'}.$$

Seeking the maximum and minimum of $X(t)$, let us integrate t_1 and t_2 in the second derivative of $X(t)$, we have:

$$X''(t_1) = Q_{jmax}\sqrt{\Delta} > 0,$$

then $X(t)$ has a minimum V_2 at point t_1 which represents the smallest negative value,

$$X''(t_2) = -Q_{jmax}\sqrt{\Delta} < 0,$$

then $X(t)$ has a maximum V_2 at point t_1, which represents the largest positive value.

From equation (21), the volume of the controlling reserves becomes:

$$V_R = X(t_2) + |X(t_1)| + \mu, \text{ then}$$

$$V_R = Q_{jmax}\left[0{,}01A'(t_1{}^3 - t_2{}^3) + 0{,}01B'(t_1{}^2 - t_2{}^2) - \left(\frac{1}{24} - 0{,}01C'\right)(t_1 - t_2)\right] + \mu. \quad (25)$$

Figure 3.a shows that the coefficient β is equal to 0 for Kp = 1.2 and reaches its maximum value equal to 1.8 at Kp = 1.32 and then decreases to 1 for Kp = 1.4. However, the volume of controlling reserves calculated with equation (21) is suitable for water supply conditions in cities with medium standing for μ_{max} equal to $0{,}018Q_{jmax}$.

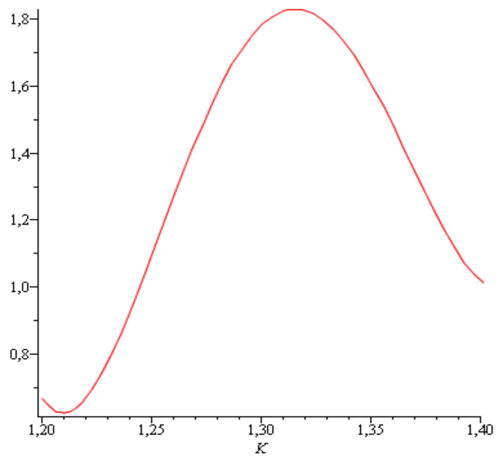


Figure3.a: Variation curve of according to the hourly peak coefficient K_p = 1,2-1,4

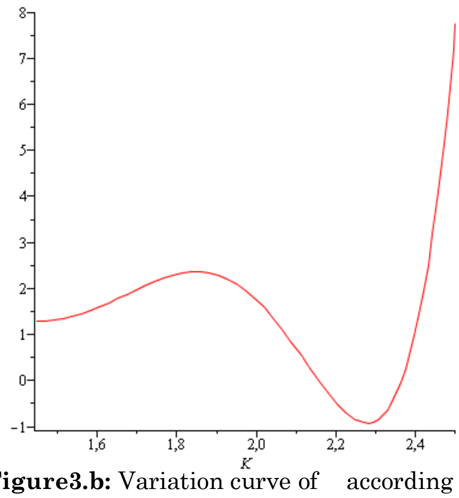


Figure3.b: Variation curve of according to the hourly peak coefficient K_p = 1,45-2,5

Figure 3.b shows that the coefficient increases from 1 for Kp = 1.45 to 2.4 for Kp = 1.9 by following a gradually varied progression where it reaches its maximum value at 2.4. It tends to infinity for Kp between 2.3 and 2.4, and vanishes at Kp = 2.16 ( ) then decreases down to -0.9. However, the sum of the differences expressed above decreases for cities with low population whose urbanization is spontaneous and for which Kp varies from 2.16 to 2.3. In fact, the expression (25) of the volume of controlling reserves is suitable for Kp between 1.45 to 2.3 with a maximum volume correction equal 0,024 .

4.1 Pumped water supply

In this case, two aspects are considered: the supplied flow rate is spread over 24 hours of the day and so it is constant and equal to or the supplied flow rate is variable and in fact enables to reduce the required volume of reservoir (which is important especially in the case of an elevated reservoir). The maximum hourly flow rate provided by the pump station thus depends on the selected operating mode; it is generally between (for uniform pumping) and . Therefore, the correction volume is set equal to 0.

The development of equation (10) by iterative method provides its simplified form:

$$R_1(t) = Q_n(t - a_{n-1}) + \sum_{i=1}^{n-1} Q_i . L_i, \quad for \;\; t \in [a_{n-1}, a_n[\;\; , \tag{26}$$

where - difference between the upper bound and the lower bound of the i[th] interval.

By incorporating equation (26) into equation (10), we proceed to the framing of (t) for t , and we have:

$$\sum_{i=1}^{n-1} Q_i . L_i \le R_1(t) \le \sum_{i=1}^{n} Q_i . L_i . \tag{27}$$

Here, the fact that the pumping rate is variable from one interval to another, equation (17) becomes then:

$$R_2(t) = [A(t - a_0) + B] . \frac{Q_{jmax}}{100} . \tag{28}$$

From equation (28), we proceed to the framing of (t) for t , and we have:

$$0.01\, Q_{jmax}.A.(a_{n-1} - a_0) + 0.01.Q_{jmax}.B \le R_2(t) \le 0.01\, Q_{jmax}.A.(a_n - a_0) + 0.01\, Q_{jmax}.B. \tag{29}$$

From equations (14) (27) (29) and 1,2≤ Kp ≤ 1.4, we proceed to the framing of on . This yields to:

$$\sum_{i=1}^{n-1} Q_i.L_i + 0.01.Q_{jmax}.A.a_0 - 0.01.Q_{jmax}(A.a_{n-1} + B) \le X(t) \le \sum_{i=1}^{n} Q_i.L_i + 0.01.Q_{jmax}.A.a_0 - 0.01.Q_{jmax}(A.a_n + B). \tag{30}$$

So : $I = \sum_{i=1}^{n-1} Q_i . L_i + 0{,}01. Q_{jmax}. A. a_0 - 0{,}01. Q_{jmax}(A. a_{n-1} + B)$, (31)

$$J = \sum_{i=1}^{n} Q_i . L_i + 0{,}01. Q_{jmax}. A. a_0 - 0{,}01. Q_{jmax}(A. a_n + B), \quad (32)$$

The values of I and J are respectively the maxima on each time interval considered.

Similarly to the considerations leading to the expression (28), equation (18) is for $t \in [a_{n-1},\ a_n[$ and 1,45≤ Kp ≤2,5:

$$R_2\,(t) = [A'(t - a_0)^3 + B'(t - a_0)^2 + C'(t - a_0)]. \frac{Q_{jmax}}{100}\ , \quad (33)$$

$$X(t) = -0{,}01. A'. Q_{jmax}(t - a_0)^3 - 0{,}01. B'. Q_{jmax}(t - a_0)^2 + (Q_n - 0{,}01. C'. Q_{jmax})t + 0{,}01. C'. Q_{jmax}. a_0 - Q_n a_{n-1} + \textstyle\sum_{i=1}^{n-1} Q_i . L_i\ . \quad (34),$$

So : $I = X(a_{n-1})$, and $J = X(a_n)$.

By setting: $u_n = a_{n-1} - a_0$ *and* $v_n = a_n - a_0$, we have :

$$I = -0{,}01. A'. Q_{jmax} u_n^{\ 3} - 0{,}01. B'. Q_{jmax} u_n^{\ 2} + (Q_n - 0{,}01. C'. Q_{jmax})(u_n + a_0) + 0{,}01. C'. Q_{jmax}. a_0 - Q_n a_{n-1} + \sum_{i=1}^{n-1} Q_i . L_i\ , \quad (35)$$

$$J = -0{,}01. A'. Q_{jmax} v_n^{\ 3} - 0{,}01. B'. Q_{jmax} v_n^{\ 2} + (Q_n - 0{,}01. C'. Q_{jmax})(v_n + a_0) + 0{,}01. C'. Q_{jmax}. a_0 - Q_n a_{n-1} + \sum_{i=1}^{n-1} Q_i . L_i\ . \quad (36)$$

If $I >\ 0$ and $J >\ 0$ then $X\,(t)$ admits only a positive maximum on $[a_{n-1},\ a_n[$,

$$V_{1n} = Max(I, J) \text{ and } V_{2n} = 0\ , \quad (37)$$

where V_{1n} - the largest positive difference between the totals of supply and consumption on the n[th] interval, V_{2n}- the smallest gap on the n[th] interval.

In this case, there is filling of the reservoir during this time slot.

If $I\ < 0$ and $J\ < 0$ then X (t) admits a negative maximum on $[a_{n-1},\ a_n[$,

$$V_{1n} = 0 \quad \text{and} \quad V_{2n} = |Max(I, J)|, \quad (38)$$

If $I\ < 0$ and $J > 0$ then $X\,(t)$ has a positive maximum and a negative maximum on $[a_{n-1},\ a_n[$,

$$V_{1n} = J, \text{ and } V_{2n} = |I|. \quad (39)$$

If $I >\ 0$ and $J\ < 0$ then $X\,(t)$ has a positive maximum and a negative maximum on $[a_{n-1},\ a_n[$.

$$V_{1n} = I \text{ and } V_{2n} = |J|\ . \quad (40)$$

The minimum volume of the controlling reserves in the reservoir becomes the sum of the maxima of differences V_{1n} and V_{2n} related to the conditions (37) (40) on the intervals $[a_{n-1},$ $a_n[$ spread over 24 hours of the day, and then the expression of $V_R$becomes:

$$\mathrm{V_R = Max(V_{1n}) + Max(V_{2n})}\ . \quad (41)$$

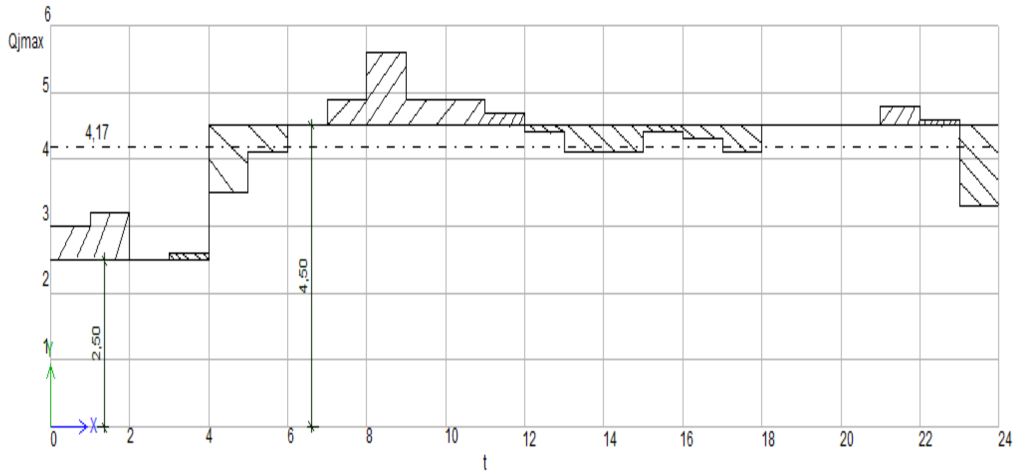
This approach shows that the capacity of the controlling reserves is defined using the integral curves whose ordinates give the quantity of the cumulative water consumed since the beginning of the day until the end of each specified time.

Applying this calculation model on drinking water systems in cities with spontaneous urbanization and medium standing in Benin (Figure 5) shows that the capacity of the controlling reserves in the reservoirs is 2.5 to 6 % of the daily peak consumption for a pumping system in steps and 15 to 30% when the pumping system is uniform throughout the day, which is close to the rate proposed by other researchers [2, 6, 9].

5 CONCLUSION

The general expressions of curves of the cumulative gravity water supply and pumped water supply as well as the distribution over time formulated using the Laplace transform, have enabled the determination of the volume of the controlling reserves from the daily peak consumption, the pumping system and the hourly peak coefficient. The cumulative consumption scales linearly for a peak coefficient Kp between 1.2 and 1.4, and follows a polynomial curve of degree 3 for Kp ranging from 1.45 to 2.5. It is established the relationship between the totals respectively gravity supply systems and delivery and the cumulative consumption, which superimposed have enabled the finding of the mathematical model for determining the control volume ranging from 3 to 30% of the daily peak consumption.

a)



b)

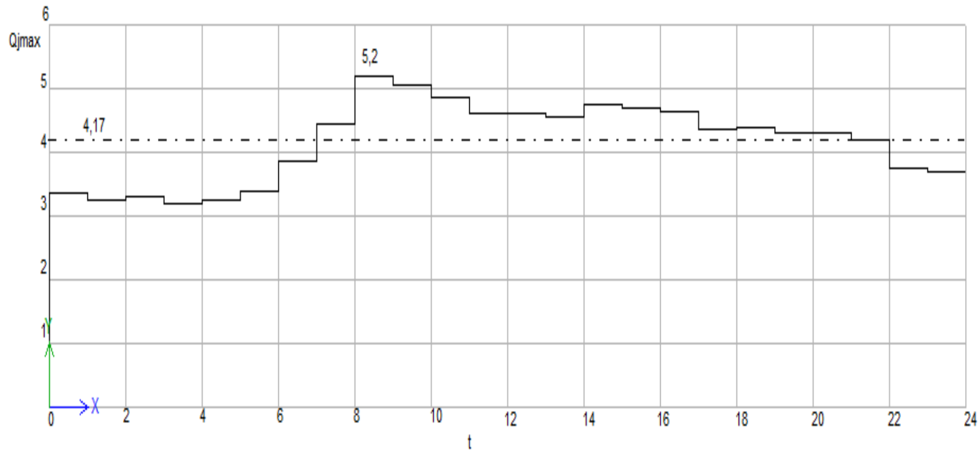


Figure 4: Hourly consumption of the peak day for hourly peak coefficients
a) $K_P = 1.35$ and b) $K_P = 1.25$

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