Buckling analysis

BUCKLING ANALYSIS OF FUNCTIONALLY GRADED EPITROCHOIDAL SHELLS STRUCTURES

GIL-OULBÉ MATHIEU, *associate professor,*
ISMAEL TAHA FARHAN, *second year of master student,*
DAU TYEKOLO, *lecturer,*
Peoples' Friendship University of Russia
115419, Moscow, Ordjonikidze, 3

In the present article, stability of functionally graded epitrochoidal shells under pressure and thermal environment is examined. Material properties are taken as temperature dependent. Finite element solutions are obtained through commercially available tool ANSYS. The linear eigenvalue buckling problem is solved using Block Lanczos method. The effect of different geometry and material parameters on the critical buckling temperature of functionally graded Epitrochoidal Shells under pressure and thermal environment is demonstrated. Finally, the change of the stresses, displacements, rotations and stains were investigated and presented.

KEY WORDS: Functionally Graded Materials, Cyclic Shells, Epitrochoidal Shells, mechanical and thermal material properties, conservative, nonconservative, Stability, Instability, buckling analysis.

Concepts of Stability and Instability:

Instability is a universal phenomenon, which may occur in various material bodies. The fundamental concepts of stability and instability are clarified through the following definitions:

The state of a system is *the collection of values of the system parameters at any instant of time.* For example, the positions of material points in a structure and the temperature field at various points constitute the state of that system. The state of a system depends on system parameters and environmental conditions. For example, in a shell structure, the system parameters are geometrical and material properties. And the environmental conditions are the applied forces and thermal conditions.

Stability - The state of a system, at any instant of time, is called stable if the relatively small changes in system parameter and / or environmental conditions would bring about relatively small changes in the existing state of the system.

Instability - The state of a system at any instant of time is called unstable if relatively small. Changes in system parameter and / or environmental conditions would cause major changes in the existing state.

Stability and Instability of Equilibrium - The equilibrium state of a system is called *stable* if small perturbations in that state, caused by load changes or other effects would be confined to a vicinity of the existing equilibrium state. The equilibrium state of a system is called *unstable* if slight changes in conditions related to that state would force the system away from that equilibrium state; an unstable system would find other equilibrium state(s); the new equilibrium state(s) may be in the vicinity of the initial state or may be far away from the initial equilibrium configuration. The concepts of stable and unstable equilibrium are illustrated in Fig. 1 [1]. This figure

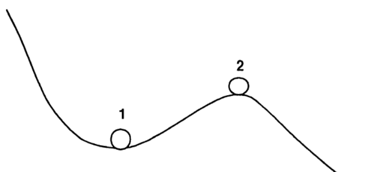

Fig. 1. Concept of stability and instability of equilibrium

shows a small ball lying on a smooth surface. According to the forgoing definitions, the equilibrium state 1 is stable while state 2 is unstable [2]. The relativity of the foregoing definitions is clearly demonstrated in this figure; the state 1 may be stable in a certain limited region, but may be unstable in a larger domain.

Buckling is a special mode of instability of equilibrium which may occur in deformable bodies subjected mostly to compressive loadings. So far as the structural problems are concerned, an existing state of equilibrium or trend of behavior of the structure subjected to applied loadings and / or temperature variations could be altered and the structure could acquire a new equilibrium state or a new trend of behavior. This phenomenon is termed the *buckling* of that particular structure. A well-known example of elastic buckling instability is the flexural buckling of an axially compressed slender elastic column subjected to a concentric compressive force. The type of applied loading affects the modes of elastic instability. Loading systems are classified as ***conservative*** or ***nonconservative***. Dead loadings, such as the weight of structures, are conservative forces; time dependent loadings, and the forces which depend on the state of the system are generally nonconservative. Conservative loadings are derivable from a potential function whereas nonconservative forces have no generating potential. From this viewpoint, frictional forces are nonconservative. Elastic bodies subjected to conservative forces may loose their current equilibrium state and find other equilibrated configurations; this mode of elastic instability is normally of the *buckling* type. The equilibrium of the same elastic bodies subjected to nonconservative forces may become *dynamically* unstable; the system could undergo oscillations with increasing amplitude. This mode of elastic instability is *called flutter.* Thin panels or shells in contact with flowing fluids could develop a flutter mode of elastic instability.

An Overview of Shell Buckling:

The equilibrium of thin elastic shells subjected to certain force fields may become unstable and the shell may undergo *prebuckling, buckling,* and *postbuckling* deformation. The occurrence of buckling in thin shells is quite probable due to the fact that the thickness to span ratio of shells is usually much lower than other structural elements. The response of thin shells to compressive forces is essentially very different from the behavior of other structural elements such as struts, columns, and plates; some types of thin shells are extremely sensitive to geometrical and loading imperfections. Geometrical imperfections include all deviations in the shape of the structural member from an ideally assumed geometrical configuration. Thus, a slightly crooked column, in comparison with a perfectly straight bar is considered imperfect. In the case of shells, the geometrical imperfection is marked by deviation of middle surface geometry from a conceived ideal shape.

Loading imperfections are probable deviations of magnitudes and / or directions of applied Forces from assumed values and / or directions. As an example, an eccentrically applied axial force to a straight column can be considered an imperfect loading. Loading Imperfections, may be quantified by the so-called "imperfection parameters"; in the column problem, the axial force eccentricity could be chosen as an imperfections parameter.

Experiments performed on column and plates, under in-plane compressive conservative forces, have shown that such elements are relatively insensitive to slight geometric and loading imperfections. This is not the case in shell problems.

Buckling experiments carried out on shells have shown that some shells are very sensitive to geometrical and loading imperfections. Thus the buckling load of laboratory shell samples is normally smaller than the critical loads that a perfect system could sustain. This is, on one hand due the fact that the actual shells are, by production, never geometri-

cally perfect and also that an ideally perfect conceived loading can never be produced and, on the other hand due to imperfection sensitivity of real shells.
The imperfection sensitivity of shells has important analysis and design implications; to obtain a realistic estimate of buckling strength of shells, geometrical and loading imperfections must be taken into account.

Finite element modelling

An epitrochoidal shell structure [3] with fixed supports is depicted in Fig 2. It has been analyzed under pressure and thermal loading. Thickness of the shell (*h=1.0 cm*) including two layers (, shown in Fig.3.The mechanical and thermal material properties used in the present study have been listed in the Table 1.

Table 1. The mechanical and thermal material properties:

Material	Ceramic (	Metallic (Steel)
Thermal expansion coefficient		
Poisons' ratio (	0.25	0.25
Young's' modulus,	390	210
Density (kg/m^3)	3890	7850
Conductivity(	25	40

In this study, finite element modelling of functionally graded cyclic shell (Epitrochoidal shells) structures with uniform thickness *h* is considered as shown in Figure 2. Here, FG epitrochoidal shell is modeled and analyzed in ANSYS through ANSYS parametric design language (APDL) code. A shell element (SHELL181), defined in the ANSYS library, is utilized to discretize the FG Epitrochoidal shell. This shell element has total six degrees of freedom per node i.e., translations and rotations in the *x*, *y* and *z* directions.

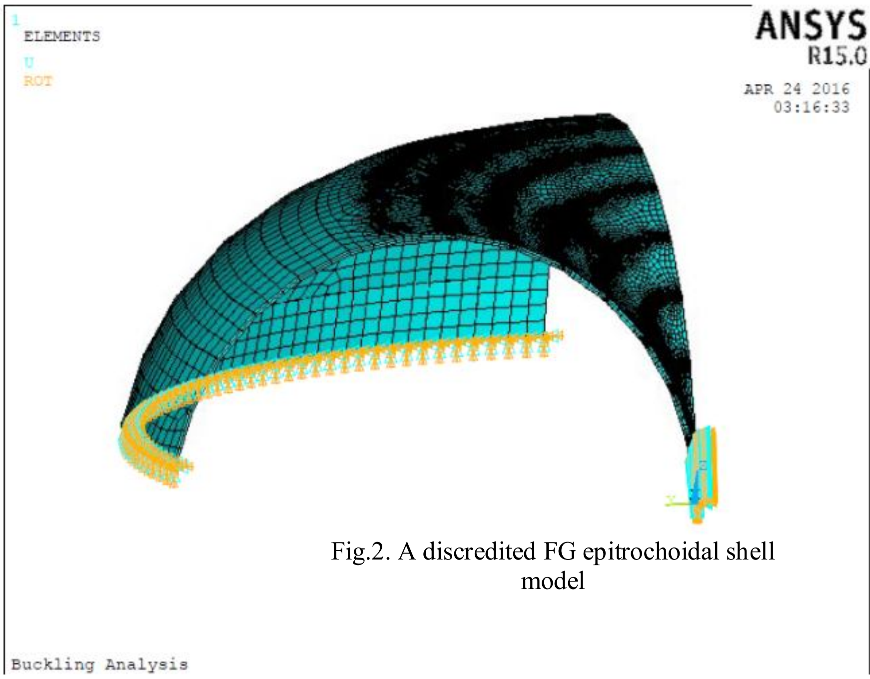

Fig.2. A discredited FG epitrochoidal shell model

4. Results and Discussions

In this section, the stability behavior of FG epitrochoidal shell is performed under pressure and uniform temperature field (*T* =700ºK). The FG epitrochoidal shell is discretized and solved using finite element steps in ANSYS APDL platform [4]. Block Lanczos method is used to obtain the eigenvalue bucking responses.

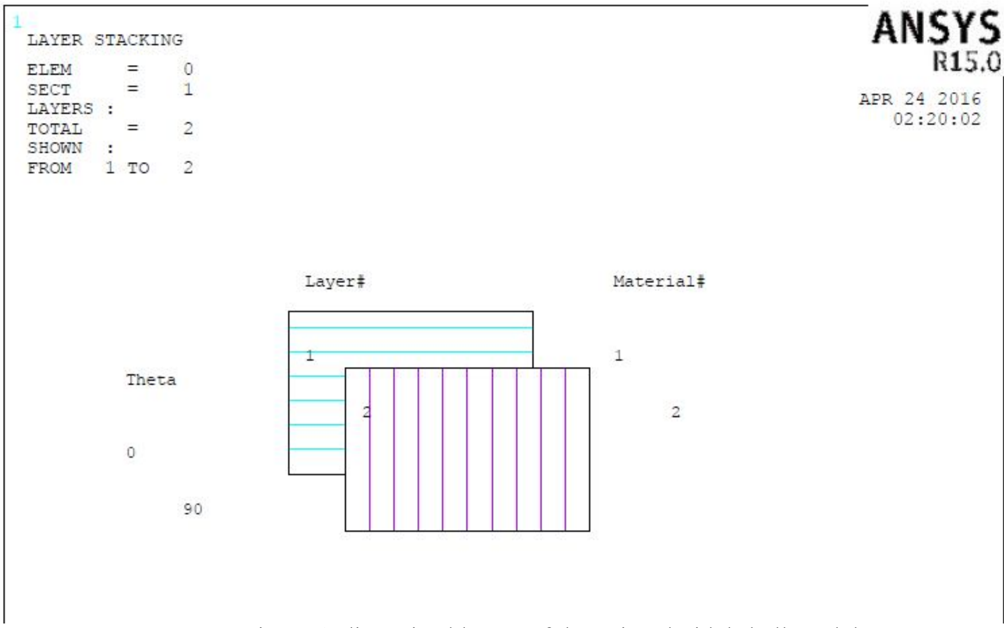

Fig. 3. A discretized layers of the epitrochoidal shell model

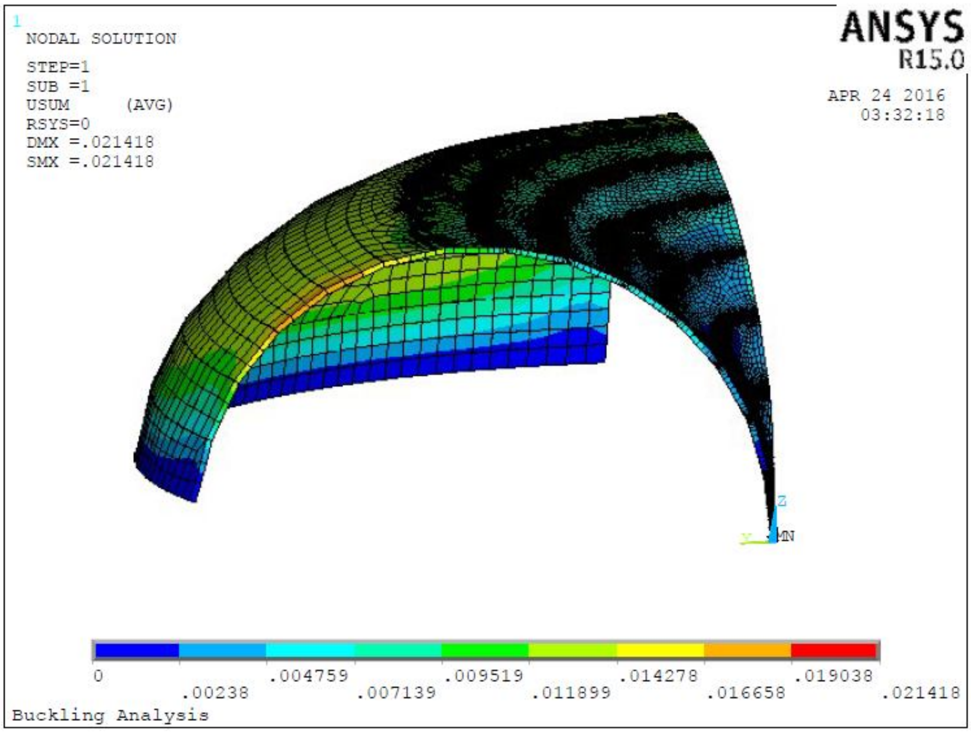

Fig.4. Displacement variation for FG epitrochoidal shell

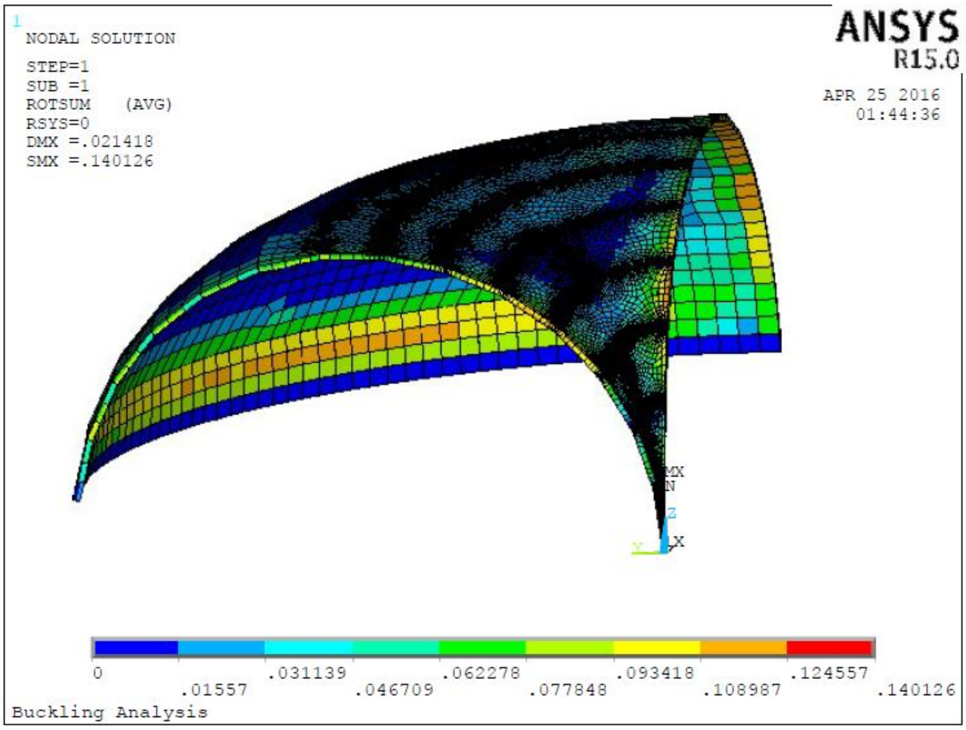

Fig.5. Rotation variation for FG epitrochoidal shell

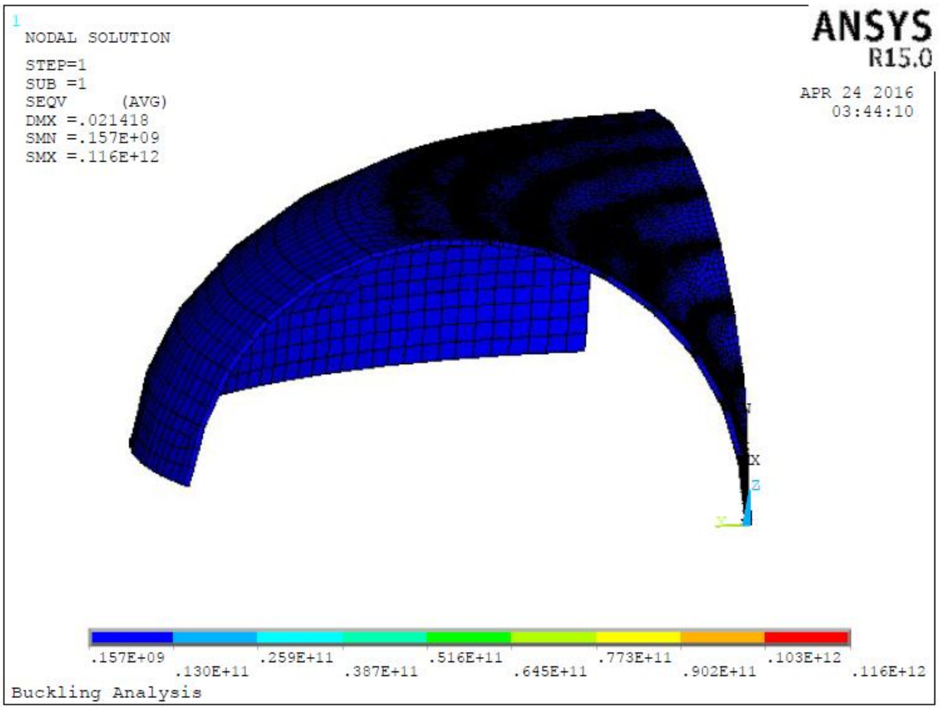

Fig. 6. Von Mises Stress variation for FG epitrochoidal shell

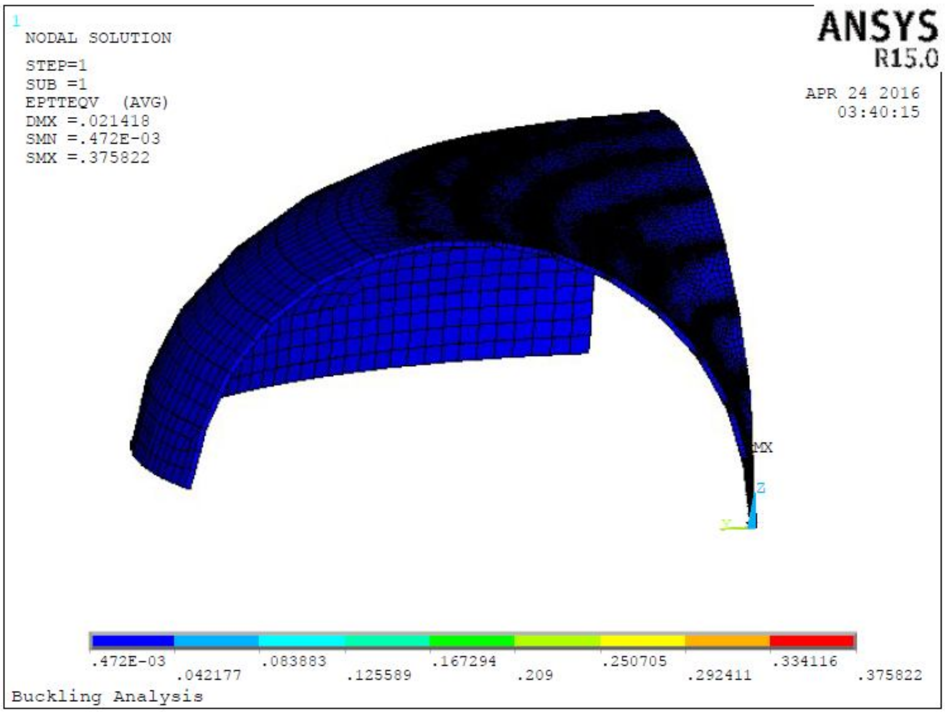

Fig.7. Von Mises of total mechanical and thermal strain variation for FG epitrochoidal shell

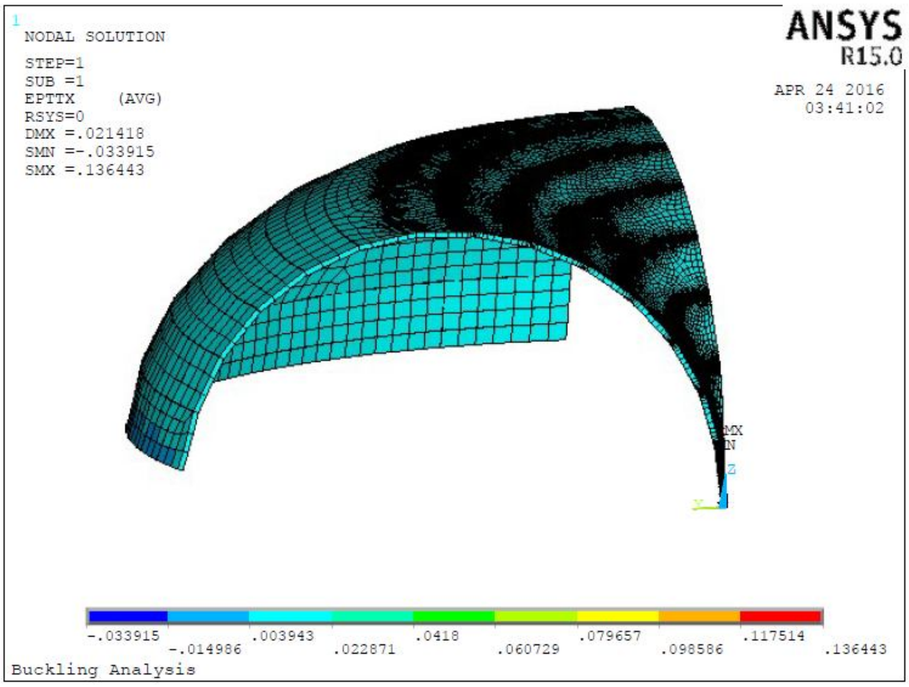

Fig. 8. X-component total mechanical and thermal strain variation for FG epitrochoidal shell

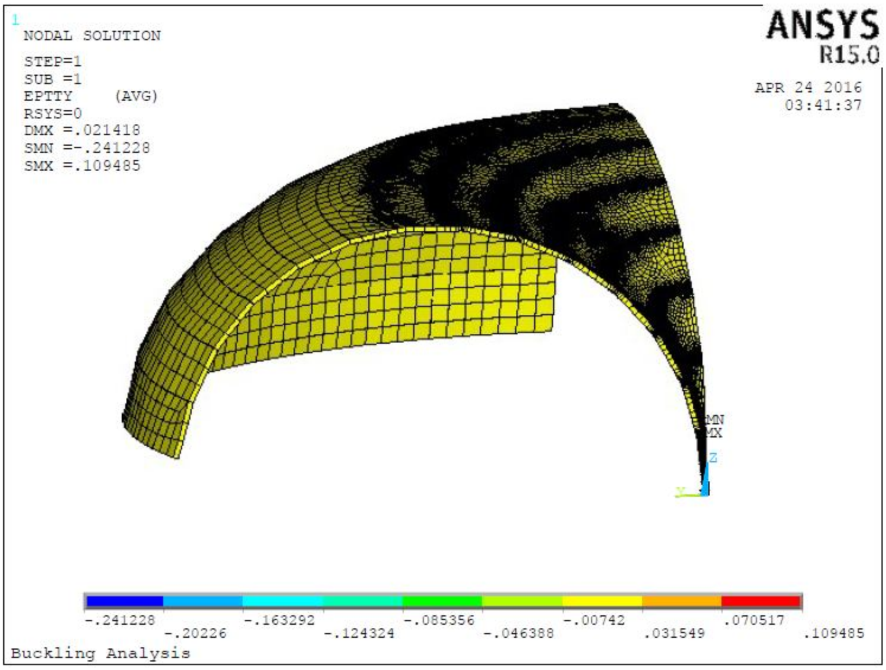

Fig. 9. Y-component total mechanical and thermal strain variation for FG epitrochoidal shell

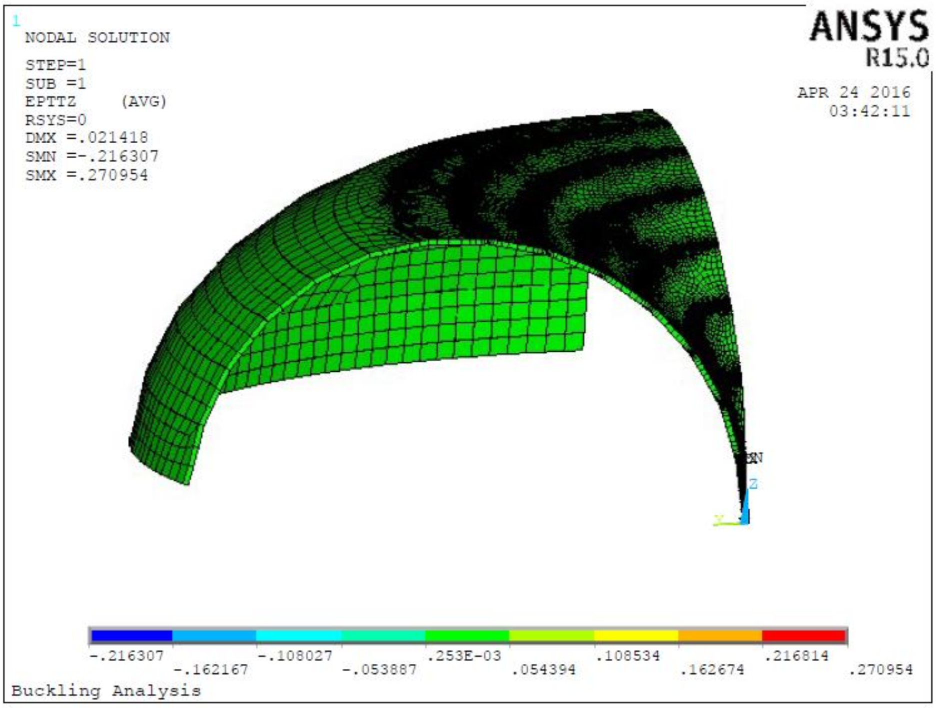

Fig.10. Z-component total mechanical and thermal strain variation for FG epitrochoidal shell

Fig. 4 shows Displacement variation for FG epitrochoidal shell under pressure and thermal loading. The overall displacement varies from $0.00\ m$ to $0.021418\ m$.

Fig. 5 shows Rotation variation for FG epitrochoidal shell under pressure and thermal loading. The overall Rotation varies from $0.00\ rad$ to $0.140126\ rad$.
Fig. 6 shows the Von Mises Stress variation for FG epitrochoidal shell under pressure and thermal loading. The stress varies from $0.157 \cdot 10^{9} Pa$ to $0.116 \cdot 10^{12} Pa$.
Fig. 7 shows Von Mises of total mechanical and thermal Strain variation for FG epitrochoidal shell under pressure and thermal loading. The strain varies from

$$-0.472 \cdot 10^{-3} \quad \text{to} \quad 0.375822.$$

Fig. 8 shows X- component of total mechanical and thermal variation for FG epitrochoidal shell under pressure and thermal loading. The strain varies from

$$-0.033915 \quad \text{to} \quad 0.136443.$$

Fig. 9 shows Y- component of total mechanical and thermal Strain variation for FG epitrochoidal shell under pressure and thermal loading. The strain varies from

$$-0.241228 \quad \text{to} \quad 0.109485.$$

Fig. 10 shows Z- component of total mechanical and thermal variation for FG epitrochoidal shell under pressure and thermal loading. The strain varies from

$$-0.216307 \quad \text{to} \quad 0.270954.$$

5. Conclusions

In this study, the thermal bucking behavior of FG epitrochoidal shell under pressure and uniform temperature field are investigated. In addition, temperature dependent material properties of FGM constituents are considered. Finite element solution for the buckling behavior of present FG model is proposed using Block Lanczos method. The influences of different material and geometrical parameters on the thermal buckling of FG epitrochoidal shell are illustrated. Finally, the change of the stresses, displacements, rotations and stains were investigated and presented.

R e f e r e n c e s

1. *M. Farshad Empa. Design and Analysis of Shell Structures,* Switzerland. Springer Netherlands, 423p.
2. *Hui-Shen Shen* (2009). *Functionally Graded Materials: Nonlinear Analysis of Plates and Shells*, LAP Lambert Academic Publishing, 200p.
3. *Krivoshapko, S.N., Ivanov, V.N.* (2015). *Encyclopedia of Analytical Surfaces*, Springer International Publishing Switzerland, 752 p.
4. *Mathieu, Gil-oulbé* (1997). *The Stress-Strain Analysis of Epitrochoidal Shells: PhD Thesis*, Moscow: RUDN, 16 p.

РАСЧЕТ НА УСТОЙЧИВОСТЬ ФУНКЦИОНАЛЬНО-ГРАДИЕНТНЫХ ЭПИТРОХОИДАЛЬНЫХ ОБОЛОЧЕК

Матье Жиль-улбе, И.Т. Фархан, Д. Теколо

В настоящей статье рассматривается устойчивость функционально-градиентных эпитрохоидальных оболочек под давлением и тепловой среды. Свойства материала принимаются как зависящие от температуры. Конечно-элементные решения получены с помощью программного комплекса ANSYS. Линейные задачи на собственные значения устойчивости решается с помощью блочного метода Ланцоша. Влияние различной геометрии и параметров материала на критическую температур функционально-градиентных эпитрохоидальных оболочек под давлением и тепловой средой наглядно показано. В конце, изменение напряжений, перемещений, вращений и деформаций изучены и представлены.

КЛЮЧЕВЫЕ СЛОВА: функционально-градиентные материалы, циклические оболочки, эпитрохоидальные оболочки, механические и термические свойства материала, консервативный, неконсервативный, стабильность, нестабильность, анализ потери устойчивости.