THE FINITE ELEMENTS OF A QUADRILATED SHAPE FOR ANALYSIS OF SHELLS TAKING INTO CONSIDERATION A DISPLACEMENT OF A BODY WITH RIGID BODY MODES

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On the basis of a curvilinear quadrilated finite element the new mode of approximation of fields of displacements is realized, which essence consists that at a stage of approximation of interior magnitudes through nodal unknown of a finite element accept not separate components of a vector of displacement and their derivative, and immediately vector of displacement of nodal points of a finite element and its derivative.

On an example of account of an equalizer loaded by interior pressure with presence of constructional displacement as rigid body is shown. That use of the developed algorithm solves a well-known problem FEM – account of displacement of finite element as rigid body.

Key words: finite element, approximation, vector displacement, displacement as rigid body.

Introduction

The method of approximation of displacement fields for the method of the finite elements is proposed, allowing to take automatically into account a displacement of the finite element with rigid body modes. Its essence consists of a choice of a column of nodal unknown quantities, containing the self vectors of the nodal points and their derivatives. Through the chooses column of nodal unknown quantities is approximated the displacement vector of the internal point of the finite element, that allows to express components of the displacement vector and its derivatives through all displacements and derivatives of the displacement of the nodal points of the finite element.

Numeral examples are shown. This approach, natural to approximation of the displacement, allows solving a problem of the displacement of the finite element with rigid body modes.

1. Basic geometrical relations of any nonshallow shell. The middle surface of any nonshallow shell in the Cartesian system of coordinates can be described by a position vector

$$\vec{R}^0 = x^1(\theta^1,\theta^2)\vec{i}_1 + x^2(\theta^1,\theta^2)\vec{i}_2 + x^3(\theta^1,\theta^2)\vec{i}_3 = x^k(\theta^a)\vec{i}_k, \quad (1)$$

where x^k, $\vec{i}_k$ are coordinates and unit vectors of the Cartesian system of coordinates, θ^a are the curvilinear coordinates of the shell middle surface. Here and below, the Latin indexes accept the meanings 1, 2, 3 and the Greek ones meaning 1, 2.

The tangent vectors of the local basis of the point of the middle surface in the initial condition can be received by the differential (1) on curvilinear coordinates θ^α [1]

$$\vec{a}^0_\alpha = \vec{R}^0_{,\alpha} = x^k_{,a}(\theta^\gamma)\vec{i}_k. \quad (2)$$

The unit normal vector of the local basis is determined by the vector product

$$\vec{a}^0 = \frac{\vec{a}^0_1 \times \vec{a}^0_2}{\left|\vec{a}^0_1 \times \vec{a}^0_2\right|} = \frac{x^k_{,1}\vec{i}_k \times x^m_{,2}\vec{i}_m}{\left|x^k_{,1}\vec{i}_k \times x^m_{,2}\vec{i}_m\right|}. \quad (3)$$

The relations (2) and (3) can be expressed in the matrix form

$$\{\vec{a}^0\} = [m]\{\vec{i}\}, \quad (4)$$

where $\{\vec{a}^0\}^T = \{\vec{a}^0_1, \vec{a}^0_2, \vec{a}^0_3\}$, $\{\vec{i}\} = \{\vec{i}_1, \vec{i}_2, \vec{i}_3\}$. According to [4], the covariant components $a^0_{\alpha\beta}$ of the metric tensor and its determinant a^0, the covariant $b_{\alpha\beta}$ and mixed

b_α^β components of the tensor of curvature, the covariant vectors of the basis $\vec{a}^{0\alpha}$ and the Christoffel symbols of the first $\Gamma_{\alpha\beta}$ and second Γ_β^α ranks are determined,

$$a_{\alpha\beta}^0 = \vec{a}_\alpha^0 \cdot \vec{a}_\beta^0; \; a^0 = \sqrt{a_{11}^0 a_{22}^0 - a_{12}^0 a_{21}^0}; b_{\alpha\beta} = \vec{R}_{,\alpha\beta}^0 \cdot \vec{a}^0;$$

$$\vec{a}^{0\alpha} = a^{0\alpha\rho} \vec{a}_\rho^0; \; \Gamma_{\alpha\beta\gamma} = \frac{1}{2}(a_{\alpha\beta,\gamma}^0 + a_{\alpha\gamma,\beta}^0 - a_{\beta\gamma,\alpha}^0); \; \Gamma_{\beta\gamma}^\alpha = a^{0\alpha\rho} \Gamma_{\rho\beta\gamma}. \tag{5}$$

The displacement vector of any point of the middle surface can be submitted in the initial basis by the expression $\vec{W} = W^\alpha \vec{a}_\alpha^0 + W\vec{a}^0$. (6)

Using the relation of the derivatives of the vectors of the basis [4] one can write

$$\vec{a}_{\alpha,\beta}^0 = \Gamma_{\alpha\beta}^\rho \vec{a}_\rho^0 + b_{\alpha\beta}\vec{a}^0; \; \vec{a}_{,\alpha}^0 = -b_\alpha^\rho \vec{a}_\rho^0. \tag{7}$$

It is possible to determine the derivatives of the displacement vector on the curvilinear coordinates with the help of expressions

$$\vec{W}_{,1} = \vec{a}_1^0 (W_{,1}^1 + W^1 \Gamma_{11}^1 + W^2 \Gamma_{21}^1 - W b_1^1) + a_0^2 (W_{,1}^2 + W^1 \Gamma_{11}^2 + W^2 \Gamma_{21}^2 - W b_1^2) +$$
$$\vec{a}^0 (W_{,1} + W^1 b_{12} + W) = m_1^1 \vec{a}_1^0 + m_1^2 \vec{a}_2^0 + m_1 \vec{a}^0,$$
$$\vec{W}_{,2} = \vec{a}_1^0 (W_{,2}^1 + W^1 \Gamma_{12}^1 + W^2 \Gamma_{22}^1 - W b_2^1) + (W_{,2}^2 + W^1 \Gamma_{12}^2 + W^2 \Gamma_{22}^2 - W b_2^2) +$$
$$\vec{a}^0 (W_{,2} + W^1 b_{12} + W) = m_2^1 \vec{a}_1^0 + m_2^2 \vec{a}_2^0 + m_2 \vec{a}^0,$$
$$\vec{W}_{,11} = \vec{a}_1^0 (\Gamma_{11}^1 m_1^1 + \Gamma_{21}^1 m_1^2 + W_{,11}^1 + W_{,1}^1 \Gamma_{11}^1 + W^1 \Gamma_{11,1}^1 + W_{,2}^1 \Gamma_{21}^1 + W^2 \Gamma_{21,1}^1 -$$
$$- W_{,1} b_1^1 - W b_{1,1}^1 - b_1^1 m_1) + \vec{a}_2^0 (\Gamma_{11}^2 m_1^1 + \Gamma_{21}^2 m_1^2 + W_{,11}^2 + W_{,1}^1 \Gamma_{11}^2 + W^1 \Gamma_{11,1}^2 +$$
$$+ W_{,2}^1 \Gamma_{21}^2 + W^2 \Gamma_{21,1}^2 - W_{,1} b_1^2 - W b_{1,1}^2 - b_1^2 m_1) + \vec{a}^0 (b_{11} m_1^1 + b_{21} m_1^2 + W_{,11} +$$
$$W_{,1}^1 b_{11} + W b_{11,1} + W_{,1}^2 b_{21} + W^2 b_{21,1}) = t_{11}^1 \vec{a}_1^0 + t_{11}^2 \vec{a}_2^0 + t_{11} \vec{a}^0, \tag{8}$$
$$\vec{W}_{,12} = \vec{a}_1^0 (\Gamma_{12}^1 m_1^1 + \Gamma_{22}^1 m_1^2 + W_{,12}^1 + W_{,2}^1 \Gamma_{11}^1 + W^1 \Gamma_{11,2}^1 + W_{,2}^2 \Gamma_{21}^1 + W^2 \Gamma_{21,2}^1 -$$
$$- W_{,2} b_1^1 - W b_{1,2}^1 - b_2^1 m_1) + \vec{a}_2^0 (\Gamma_{12}^2 m_1^1 + \Gamma_{22}^2 m_1^2 + W_{,12}^2 + W_{,2}^1 \Gamma_{11}^2 + W^1 \Gamma_{11,2}^2 +$$
$$+ W_{,12}^2 \Gamma_{21}^2 + W^2 \Gamma_{21,2}^2 - W_{,2} b_1^2 - W b_{1,2}^2 - b_2^2 m_1) + \vec{a}^0 (b_{12} m_1^1 + b_{22} m_1^2 + W_{,12} +$$
$$W_{,2}^1 b_{11} + W^1 b_{11,2} + W_{,2}^2 b_{21} + W^2 b_{21,2}) = t_{12}^1 \vec{a}_1^0 + t_{12}^2 \vec{a}_2^0 + t_{12} \vec{a}^0,$$
$$\vec{W}_{,22} = \vec{a}_1^0 (\Gamma_{12}^1 m_2^1 + \Gamma_{22}^1 m_2^2 + W_{22}^1 + W_{,2}^1 \Gamma_{12}^1 + W^1 \Gamma_{12,2}^1 + W_{,2}^2 \Gamma_{22}^1 + W^2 \Gamma_{12,2}^1 -$$
$$- W_{,2} b_2^1 - W b_{2,2}^1 - b_2^1 m_2) + \vec{a}_2^0 (\Gamma_{12}^2 m_2^1 + \Gamma_{22}^2 m_2^2 + W_{,22}^2 + W_{,2}^2 \Gamma_{12}^2 + W^1 \Gamma_{12,2}^2 +$$
$$+ W_{,2}^2 \Gamma_{22}^2 + W^2 \Gamma_{22,2}^2 - W_{,2} b_2^2 - W b_{2,2}^2 - b_2^2 m_2) + \vec{a}^0 (b_{12} m_2^1 + b_{22} m_2^2 + W_{,22} +$$
$$W_{,2}^1 b_{12} + W^1 b_{12,2} + W_{,2}^2 b_{22} + W^2 b_{22,2}) = t_{22}^1 \vec{a}_1^0 + t_{22}^2 \vec{a}_2^0 + t_{22} \vec{a}^0.$$

The location of any point of the shell's middle surface in the deformed condition will be determined by the radius-vector

$$\vec{R} = \vec{R}^0 + \vec{W}. \tag{9}$$

By the differentiation (9) the tangent vectors of the local basis in the deformed condition can be received as

$$\vec{a}_1 = \vec{R}_{,1} = \vec{a}_1^0 (1 + m_1^1) + \vec{a}_2^0 m_1^2 + \vec{a}^0 m_1; \; \vec{a}_2 = \vec{R}_{,2} = \vec{a}_1^0 m_2^1 + \vec{a}_2^0 (1 + m_2^2) + \vec{a}^0 m_2. \tag{10}$$

The unit normal vector of the deformed middle surface can be determined by the vector product

$$\vec{a}=\frac{\vec{a}_1\times\vec{a}_2}{|\vec{a}_1\times\vec{a}_2|}\approx\frac{1}{\sqrt{a^0}}(\sqrt{a^0}\,\vec{a}^0+\vec{a}_1^0\times\vec{W}_{,2}+\vec{W}_{,1}\times\vec{a}_2^0). \quad (11)$$

2. Deformations and strains in a shell. The location of any shell point, being distant from the middle surface at the distance of ζ in the initial and deformed conditions is described by the suitable position vectors

$$\vec{R}^{0\zeta}=\vec{R}^0+\zeta\vec{a}^0,\ \vec{R}^{\zeta}=\vec{R}^{0\zeta}+\vec{V}. \quad (12)$$

Included in the (12) the displacement vector of the point, being distant from the middle surface at the distance of ζ, using the hypothesis of the straight unit vectors is determined by the relation

$$\vec{V}=\vec{W}+\zeta(\vec{a}-\vec{a}^0)=\vec{W}+\zeta\vec{W}^n. \quad (13)$$

For the determination of deformations of an any shell's layer it is possible to take advantage of the relations of the mechanics of a solid medium [4]

$$\varepsilon_{\alpha\beta}^{\zeta}=\frac{1}{2}(g_{\alpha\beta}-g_{\alpha\beta}^0), \quad (14)$$

where $g_{\alpha\beta}, g_{\alpha\beta}^0$ are the components of the metric tensors of the initial and deformed conditions, which are determined be the differentiation (12) on the global coordinates

$$g_{\alpha\beta}^0=\vec{g}_\alpha^0\cdot\vec{g}_\beta^0;\ g_{\alpha\beta}=\vec{g}_\alpha\cdot\vec{g}_\beta. \quad (15)$$

Included in (14) the tangent vectors of the basis are determined by the differentiation (12) on the global coordinates

$$\vec{g}_\alpha^0=\vec{a}_\alpha^0-\zeta\, b_\alpha^\rho\vec{a}_\rho^0;\ \vec{g}_\alpha=\vec{g}_\alpha+\vec{W}_{,\alpha}+\zeta(\vec{W}_{,\alpha}^n-\vec{a}_{,\beta}^0). \quad (16)$$

As a result of the consecutive substitution (16) and (15) in (14) it is possible to describe the deformations $\varepsilon_{\alpha\beta}^{\zeta}$ of an any shell's layer by the deformations $\varepsilon_{\alpha\beta}$ and the curatives $\chi_{\alpha\beta}^{\zeta}$ of the shell's middle surface in the final form

$$\varepsilon_{\alpha\beta}^{\zeta}=\varepsilon_{\alpha\beta}+\zeta k_{\alpha\beta}, \quad (17)$$

where $\varepsilon_{\alpha\beta}=\frac{1}{2}(\vec{a}_\alpha^0\cdot\vec{W}_{,\beta}+\vec{a}_\beta^0\cdot\vec{W}_{,\alpha})$; $\chi_{\alpha\beta}=\frac{1}{2}\left[\vec{a}_{,\alpha}^0\cdot\vec{W}_{,\beta}+\vec{a}_{,\beta}^0\cdot\vec{W}_{,\alpha}+\vec{a}_\alpha^0\vec{W}_{,\beta}^n+\vec{a}_\beta^0\vec{W}_{,\alpha}^n)\right]$, (18)

When obtaining (18) are omitted the items $\vec{W}_{,\alpha}\cdot(\vec{a}_{,\beta}-\vec{a}_{,\beta}^0)$ as quantities of the trifle's second rank. Taking into account the relations (16), (8) and (7), the expressions (18) can be represented as

$$\varepsilon_{11}=a_{11}^0 m_1^1+a_{12}^0 m_2^2;\qquad \varepsilon_{22}=a_{12}^0 m_2^1+a_{22}^0 m_2^2;$$
$$\varepsilon_{12}=\frac{1}{2}(a_{11}^0 m_1^1+a_{12}^0 m_1^2+a_{12}^0 m_2^1+a_{22}^0 m_2^2); \quad (19)$$
$$\chi_{11}=-m_1^1 b_{11}+m_1\Gamma_{11}^1+m_2\Gamma_{11}^2-m_2^{21}b_{11}-t_{11};$$
$$\chi_{22}=-m_2^2 b_{12}+m_2\Gamma_{22}^2-t_{22}-m_1^1 b_{22}+m_1\Gamma_{22}^1;$$
$$\chi_{12}=-m_2^2 b_{12}-m_1^1 b_{12}+m_1\Gamma_{12}^1+m_2\Gamma_{12}^2-t_{12}.$$

When obtaining (19) was used the known relation

$$\frac{1}{\sqrt{a^0}}\frac{\partial\sqrt{a^0}}{\partial\theta^\alpha}=\Gamma_{\rho\alpha}^\alpha. \quad (20)$$

The describing of the components of the strain tensor $\tau^{\alpha\beta}$ through the components of the deformation tensor of an any shell's layer $\varepsilon^{\zeta}_{\rho\gamma}$ can be represented as

$$\tau^{\alpha\beta} = E^{\alpha\beta\rho\gamma}\varepsilon^{\zeta}_{\rho\gamma}, \text{ where } E^{\alpha\beta\rho\gamma} = \frac{E}{1-\nu^2}\left[(1-\nu)g^{0\alpha\gamma}g^{0\beta\rho} + \nu g^{0\alpha\beta}g^{0\gamma\rho}\right]; \tag{21}$$

E is the module of elasticity of the shell's material; ν is the factor of Poisson (Poisson's factor).

3. The finite elements and the interpolation of the displacement.

As the finite element is accepted the fragment of an any shell in the form of the curvilinear quadrangle on its middle surface with nodes *i, j, k, I,* which is reflect on the square with the local system of coordinates —1 ≤ ξ^{α} ≤ 1. The connection between the global coordinates θ^{α} and the local coordinates ξ^{α} is represented by the bilinear dependence

$$\theta^{\alpha} = \frac{(1-\xi^1)}{2}\frac{(1-\xi^2)}{2}\theta^{\alpha i} + \frac{(1+\xi^1)}{2}\frac{(1-\xi^2)}{2}\theta^{\alpha j} + \\ + \frac{(1+\xi^1)}{2}\frac{(1+\xi^2)}{2}\theta^{\alpha k} + \frac{(1-\xi^1)}{2}\frac{(1+\xi^2)}{2}\theta^{\alpha i}, \tag{22}$$

where θ^{α} is the global coordinate of the internal point of the finite element; $\theta^{\alpha i}$ and $\theta^{\alpha l}$ are the global coordinates of the finite element's nodes.

By the differentiation of the relations (22) the derivatives of the global coordinates on the local ones $\frac{\partial\theta^{\alpha}}{\partial\xi^{\beta}}, \frac{\partial^2\theta^{\alpha}}{\partial\xi^1\partial\xi^2}$ and the derivatives of the local coordinates on the global ones $\frac{\partial\xi^{\alpha}}{\partial\theta^{\beta}}, \frac{\partial^2\xi^{\alpha}}{\partial\theta^1\partial\theta^2}$ can be received

3.1. The finite element with the matrix of rigidity by the size 36x36. The vectors of the nodal unknowns containing the displacement vectors of the finite element's nodes and their first derivatives in the local and global systems of coordinates look like

$$\left\{\vec{W}^{\text{л}}_{y}\right\}^T = \left\{\vec{W}^i\vec{W}^j\vec{W}^k\vec{W}^l\vec{W}^i_{,\xi^1}\vec{W}^j_{,\xi^1}\vec{W}^k_{,\xi^1}\vec{W}^l_{,\xi^1}\vec{W}^i_{,\xi^2}\vec{W}^j_{,\xi^2}\vec{W}^k_{,\xi^2}\vec{W}^l_{,\xi^2}\right\}; \\ \left\{\vec{W}^{\Gamma}_{y}\right\}^T = \left\{\vec{W}^i\vec{W}^j\vec{W}^k\vec{W}^l\vec{W}^i_{,1}\vec{W}^j_{,1}\vec{W}^k_{,1}\vec{W}^l_{,1}\vec{W}^i_{,2}\vec{W}^j_{,2}\vec{W}^k_{,2}\vec{W}^l_{,2}\right\}. \tag{23}$$

The displacement vector of the internal point of the finite element is approximated through the nodal vectors (23) by the expression

$$\vec{W} = \underset{1\times12}{\{\varphi\}^T}\underset{12\times1}{\left\{\vec{W}^{\text{л}}_{y}\right\}} = \underset{1\times12}{\{\varphi\}^T}\underset{12\times12}{[L]}\underset{12\times1}{\left\{\vec{W}^{\Gamma}_{y}\right\}}, \tag{24}$$

where $\{\varphi\}^T = \{G_{11}G_{21}G_{22}G_{12}G_{31}G_{41}G_{42}G_{32}G_{13}G_{23}G_{24}G_{14}\}$;

$$G_{km} = h_k(\xi^1)h_m(\xi^2); \quad h_1(\lambda) = \frac{1}{4}(\lambda^3 - 3\lambda + 2);$$

$$h_2(\lambda) = -\frac{1}{4}(\lambda^3 - 3\lambda - 2); \; h_3(\lambda) = \frac{1}{4}(\lambda^3 - \lambda^2 - \lambda + 1); \; h_4(\lambda) = \frac{1}{4}(\lambda^3 + \lambda^2 - \lambda - 1);$$

λ is the coordinate ξ^1 от ξ^2; *[L]* is the matrix of the transfer of the nodal displacement vector from the local system of coordinates into the global one. By the differentiation (24), the first and second derivatives of the displacement vector of the internal point of the finite element's in the global system of coordinates can be received

$$\vec{W}_{,\alpha}=\left(\left\{\varphi_{,\xi^1}\right\}^T\frac{\partial\xi^1}{\partial\theta^\alpha}+\left\{\varphi_{,\xi^2}\right\}^T\frac{\partial\xi^2}{\partial\theta^\alpha}\right)\left\{\vec{W}_y^{л}\right\};$$

$$\vec{W}_{,\alpha\beta}=\left(\left\{\varphi_{,\xi^1\xi^1}\right\}^T\frac{\partial\xi^1}{\partial\theta^\alpha}\frac{\partial\xi^1}{\partial\theta^\beta}+\left\{\varphi_{,\xi^1\xi^2}\right\}^T\left(\frac{\partial\xi^1}{\partial\theta^\alpha}\frac{\partial\xi^2}{\partial\theta^\beta}+\right.\right.$$
$$\left.+\frac{\partial\xi^1}{\partial\theta^\beta}\frac{\partial\xi^2}{\partial\theta^\alpha}\right)+\left\{\varphi_{,\xi^2\xi^2}\right\}^T\frac{\partial\xi^2}{\partial\theta^\alpha}\frac{\partial\xi^2}{\partial\theta^\beta}+\left\{\varphi_{,\xi^1}\right\}^T\frac{\partial^2\xi^1}{\partial\theta^\alpha\partial\theta^\beta}+ \tag{25}$$
$$\left.+\left\{\varphi_{,\xi^2}\right\}^T\frac{\partial^2\xi^2}{\partial\theta^\alpha\partial\theta^\beta}\right)\left\{W_y^{л}\right\}=\left[Z_{\alpha\beta}\right]^T\left\{W_y^{л}\right\}.$$

The column of the nodal unknowns of the vector in the global system of coordinates taking into account (6) and (8) can be represented by the matrix relation.

$$\underset{12\times1}{\left\{\vec{W}_y^{Г}\right\}}=\underset{12\times36}{\left[\vec{A}\right]}\underset{36\times1}{\left\{u_y^{Г}\right\}} \tag{26}$$

where $\left\{u_y^{Г}\right\}^T=\{W^{1i},\ldots,W^{1l}W^{2i},\ldots,W^{2l}W^{i},\ldots,W^{l}m_1^{1i},\ldots,m_1^{1l}m_1^{2i},\ldots m_1^{2l}m_1^{i},\ldots m_1^{l}\times$
$\times m_2^{1i},\ldots m_2^{1l}m_2^{2i},\ldots m_2^{2l}m_2^{i},\ldots m_2^{l}\};$

[A] is the matrix, which elements are the vectors of the local basis of the nodal points of the element $\vec{a}_1^{0n}$, $\vec{a}_2^{0n}$, $\vec{a}^{0n}$, $n=i,\ j,\ ,k,\ l.$

As the result of the substitution (26) into (24) the last relation will accept the form $\vec{W}=\{\varphi\}^T[L][\vec{A}]\{u_y^{Г}\}.$ (27)

It is possible to choose the matrix $[G]$ so, that the equality will be carried out

$$[L][\vec{A}]=[\vec{A}][G]. \tag{28}$$

When using (4) and expressing the vectors of the local bases of the nodal points through the vectors of the basis of the internal point of the finite element, the matrix $[A]$ can be represented by the sum $[\vec{A}]=\vec{a}_1^0[L_1]+\vec{a}_2^0[L_2]+\vec{a}^0[L_3].$ (29)

Taking into account (28), (29) and (6), the relation (27) can be given the form

$$W^1\vec{a}_1^0+W^2\vec{a}_2^0W\vec{a}^0=\{\varphi\}^T(\vec{a}_1^0[L_1]+\vec{a}_2^0[L_2]+\vec{a}^0[L_3][G]\{u_y^{Г}\}. \tag{30}$$

From that one are received the expressions of the approximation for the component of the displacement vector of the internal point of the finite element

$$W^1=\underset{1\times12}{\{\varphi\}^T}\underset{12\times12}{[L_1]}\underset{12\times36}{[G]}\underset{36\times1}{\{u_y^{Г}\}}=\underset{1\times36}{\{\mu_1\}^T}\underset{36\times1}{\{u_y^{Г}\}};\quad W^2=\{\varphi\}^T[L_2][G]\{u_y^{Г}\}=\{\mu_2\}^T\{u_y^{Г}\};$$
$$W=\{\varphi\}^T[L_3][G]\{u_y^{Г}\}=\{\mu_3\}^T\{u_y^{Г}\}. \tag{31}$$

The derivatives (25), taking into account (24), (26), (28) and (29), can be represented in the form

$$\vec{W}_{,\alpha}=\left(\left\{\varphi_{,\xi^1}\right\}^T\frac{\partial\xi^1}{\partial\theta^\alpha}+\left\{\varphi_{,\xi^2}\right\}^T\frac{\partial\xi^2}{\partial\theta^\alpha}\right)\times\left(\vec{a}_1^0[L_1]+\vec{a}_2^0[L_2]+\vec{a}^0[L_3]\right)[G]\{u_y^{Г}\};$$
$$\vec{W}_{,\alpha\beta}=\left\{Z_{\alpha\beta}\right\}^T\left(\vec{a}_1^0[L_1]+\vec{a}_2^0[L_2]+\vec{a}^0[L_3]\right)[G]\{u_y^{Г}\}. \tag{32}$$

Taking into account the relations (8), it is possible to receive from (32) the expressions of the components of the derivatives of the displacement vector of the point of the finite element through all nodal quantities

$$m_\alpha^1 = \left(\{\varphi_{,\xi^1}\}^T \frac{\partial \xi^1}{\partial \theta^\alpha} + \{\varphi_{,\xi^2}\}^T \frac{\partial \xi^2}{\partial \theta^\alpha} \right) [L_1][G]\{u_y^\Gamma\} = \{\mu_\alpha^1\}\{u_y^\Gamma\};$$
$$t_{\alpha\beta}^1 = \{Z_{\alpha\beta}\}^T [L_1][G]\{u_y^\Gamma\} = \{\mu_{\alpha\beta}^1\}\{u_y^\Gamma\};$$
$$m_\alpha^2 = \left(\{\varphi_{,\xi^1}\}^T \frac{\partial \xi^1}{\partial \theta^\alpha} + \{\varphi_{,\xi^2}\}^T \frac{\partial \xi^2}{\partial \theta^\alpha} \right) [L_2][G]\{u_y^\Gamma\} = \{\mu_\alpha^2\}\{u_y^\Gamma\};$$
$$t_{\alpha\beta}^2 = \{Z_{\alpha\beta}\}^T [L_2][G]\{u_y^\Gamma\} = \{\mu_{\alpha\beta}^2\}\{u_y^\Gamma\}; \quad (33)$$
$$m_\alpha = \left(\{\varphi_{,\xi^1}\}^T \frac{\partial \xi^1}{\partial \theta^\alpha} + \{\varphi_{,\xi^2}\}^T \frac{\partial \xi^2}{\partial \theta^\alpha} \right) [L_3][G]\{u_y^\Gamma\} = \{\mu_\alpha^3\}\{u_y^\Gamma\};$$
$$t_{\alpha\beta} = \{Z_{\alpha\beta}\}^T [L_3][G]\{u_y^\Gamma\} = \{\mu_{\alpha\beta}^3\}\{u_y^\Gamma\};$$

As it is obvious from (31) and (33), the proposed method of the interpolation differ by that, that every component of the displacement vector of the internal point and its derivatives depends on the nodal meanings of all the three components, whereas at the traditional approach [2, 3] a separate component is expressed only through the nodal meanings of this component and its derivatives.

For the formation of the stiffness matrix of the finite element are introduced the matrix designations

$$\{\varepsilon^\zeta\}^T = \{\varepsilon_{11}^\zeta \varepsilon_{22}^\zeta 2\varepsilon_{12}^\zeta\}; \quad \{\varepsilon\}^T = \{\varepsilon_{11} \varepsilon_{22} 2\varepsilon_{12} k_{11} k_{22} 2k_{12}\}; \quad \{\tau\}^T = \{\tau^{11} \tau^{22} \tau^{12}\} \quad . \quad (34)$$

and on the basis of (17), (21) and (19), the matrix relations are formed

$$\{\varepsilon^\zeta\} = [\Gamma]\{\varepsilon\}; \quad \{\tau\} = [c]\{\varepsilon^\zeta\}; \quad \{\varepsilon\} = [B]\{u_y^\Gamma\}. \quad (35)$$

From the equality of works of the external and internal forces one may write

$$\int_v \tau^{\rho\gamma} \varepsilon_{\rho\gamma}^\zeta dv = \int_s \vec{W} \cdot \vec{P} ds, \quad (36)$$

where $\vec{P} = P^1 \vec{a}_1^0 + P^2 \vec{a}_2^0 + P\vec{a}$ is the vector of the external forces, taking into account (34) the relation

$$\underset{36\times36}{[k]} \underset{36\times1}{\{u_y^\Gamma\}} = \underset{36\times1}{\{f\}}. \quad (37)$$

can be received . Here $[k] = \int_v [B]^T [\Gamma]^T [c][\Gamma][B] dV$ *is* the stiffness matrix of the finite element; $\{f\} = \int \left[\{\mu_1\}^T (a_{11}^0 P^1 + a_{12}^0 P^2) + \{\mu_2\}^T (a_{12}^0 P^1 + a_{22}^0 P^2) + \{\mu_3\}^T P \right] \times \{\varphi\} ds$ is the vector of loads.

3.2. The finite element with the stiffness matrix by the size 72 x 72. The column of the regard unknowns, containing the nodal displacement vectors, their first and second derivatives in the local and global systems of coordinates have the form

$$\{\vec{W}_y^л\} = \{\vec{W}^i \vec{W}^j \vec{W}^k \vec{W}^l \vec{W}^i_{,\xi^1} \ldots \vec{W}^l_{,\xi^1} \vec{W}^i_{,\xi^2} \ldots \vec{W}^l_{,\xi^2} \vec{W}^i_{,\xi^1\xi^1} \ldots \vec{W}^l_{,\xi^1\xi^1} \vec{W}^i_{,\xi^2\xi^2} \ldots \vec{W}^l_{,\xi^2\xi^2} \vec{W}^i_{,\xi^1\xi^2} \ldots \vec{W}^l_{,\xi^1\xi^2}\};$$
$$\{\vec{W}_y^\Gamma\} = \{\vec{W}^i \vec{W}^j \vec{W}^k \vec{W}^l \vec{W}^i_{,1} \ldots \vec{W}^l_{,1} \vec{W}^i_{,2} \ldots \vec{W}^l_{,2} \vec{W}^i_{,11} \ldots \vec{W}^l_{,11} \vec{W}^i_{,22} \ldots \vec{W}^l_{,22} \vec{W}^i_{,12} \ldots \vec{W}^l_{,12}\}. \quad (38)$$

The displacement vector of the internal point of the finite element is approximated through the nodal vectors (38), using the polynomials of the fifth power.

$$\vec{W} = \underset{1\times 24}{\{\psi\}^T} \underset{24\times 1}{\{\vec{W}_y^{л}\}} = \underset{1\times 24}{\{\psi\}^T} \underset{24\times 24}{[N]} \underset{24\times 1}{\{\vec{W}_y^{\Gamma}\}}, \tag{39}$$

where $[N]$ is the matrix of the transformation of the column of the unknowns $\{\vec{W}_y^{л}\}$ in the local system of the coordinates through the column of the unknowns $\{\vec{W}_y^{\Gamma}\}$ in the global one.

The line, containing functions of the form looks like

$$\{\psi\}^T = \{H_{11}H_{21}H_{22}H_{12}H_{31}H_{41}H_{42}H_{32}H_{13}H_{23}H_{24}H_{14}H_{51}H_{61} \\ H_{62}H_{52}H_{15}H_{25}H_{26}H_{16}H_{33}H_{43}H_{44}H_{34}\}, \tag{40}$$

where $H_{km} = g_k(\xi^1)g_m(\xi^2)$ is the multinominal, which represents the product of the polynomials of the fifth power on the coordinates of the local system

$$g_1(\lambda) = \frac{1}{16}(-3\lambda^5 + 10\lambda^3 - 15\lambda + 8);\ g_2(\lambda) = \frac{1}{16}(3\lambda^5 - 10\lambda^3 + 15\lambda + 8);$$

$$g_3(\lambda) = \frac{1}{16}(-3\lambda^5 + \lambda^4 + 10\lambda^3 - 6\lambda^2 - 7\lambda + 5);$$

$$g_4(\lambda) = \frac{1}{16}(-3\lambda^5 - \lambda^4 + 10\lambda^3 + 6\lambda^2 - 7\lambda - 5); \tag{41}$$

$$g_5(\lambda) = \frac{1}{16}(-\lambda^5 + \lambda^4 + 2\lambda^3 - 2\lambda^2 - \lambda - 1);\ g_6(\lambda) = \frac{1}{16}(\lambda^5 + \lambda^4 - 2\lambda^3 - 2\lambda^2 + \lambda + 1);$$

λ is the symbol, designating the coordinates of the local system ξ^1 or ξ^2.

By the differentiation (39) on the global coordinates, the derivatives of the displacement vector can be presented in the form

$$\vec{W}_{,\alpha} = \left(\{\psi_{,\xi^1}\}^T \frac{\partial\xi^1}{\theta^\alpha} + \{\psi_{,\xi^2}\}^T \frac{\partial\xi^2}{\theta^\alpha}\right)[N]\{\vec{W}_y^{\Gamma}\} = \{\eta_\alpha\}[N]\{\vec{W}_y^{\Gamma}\};$$

$$\vec{W}_{,\alpha\beta} = \left(\{\psi_{,\xi^1\xi^1}\}^T \frac{\partial\xi^1}{\partial\theta^\alpha}\frac{\partial\xi^1}{\partial\theta^\beta} + \{\psi_{,\xi^1\xi^2}\}^T \left(\frac{\partial\xi^1}{\partial\theta^\alpha}\frac{\partial\xi^2}{\partial\theta^\beta} + \frac{\partial\xi^2}{\partial\theta^\alpha}\frac{\partial\xi^1}{\partial\theta^\beta}\right) + \right.$$

$$\left. + \{\psi_{,\xi^2\xi^2}\}^T \frac{\partial\xi^2}{\theta^\alpha}\frac{\partial\xi^2}{\theta^\beta} + \{\psi_{,\xi^1}\}^T \frac{\partial^2\xi^1}{\partial\theta^\alpha\partial\theta^\beta} + \{\psi_{,\xi^2}\}^T \frac{\partial^2\xi^2}{\partial\theta^\alpha\partial\theta^\beta}\right) \times [N]\{\vec{W}_y^{\Gamma}\} =$$

$$= \{\eta_{\alpha\beta}\}^T[N]\{\vec{W}_y^{\Gamma}\}. \tag{42}$$

The column of the nodal vectors $\{W_y^{\Gamma}\}$ can be represented by the product of the matrices

$$\underset{24\times 1}{\{\vec{W}_y^{\Gamma}\}} = \underset{24\times 72}{[\vec{A}]} \underset{72\times 1}{\{u_y^{\Gamma}\}}, \tag{43}$$

where the column of the regard nodal unknowns has the following form

$$\{u_y^{\Gamma}\} = \{W^{1i}\ldots W^{1l}W^{2i}\ldots W^{2l}W^{i}\ldots W^{l}m_1^{1i}\ldots m_1^{1l}m_1^{2i}\ldots m_1^{2l}m_1^{i}\ldots m_1^{l} \\ m_2^{1i}\ldots m_2^{1l}m_2^{2i}\ldots m_2^{2l}m_2^{i}\ldots m_2^{l}t_{11}^{1i}\ldots t_{11}^{1l}t_{11}^{2i}\ldots t_{11}^{2l}t_{11}^{i}\ldots t_{11}^{l}t_{22}^{1i}\ldots t_{22}^{1l}t_{22}^{2i}\ldots t_{22}^{2l} \\ t_{22}^{i}\ldots t_{22}^{l}t_{12}^{1i}\ldots t_{12}^{1l}t_{12}^{i}\ldots t_{12}^{l}t_{22}^{1i}\ldots t_{22}^{1l}t_{22}^{2i}\ldots t_{22}^{2l}\}; \tag{44}$$

$\left[\vec{A}\right]$ is the matrix, which elements contain only the basis vectors; $\vec{a}_1^0$, a_2^0 and $\vec{a}^0$ of the nodal points of the finite element (n=i, j, k, l).

Taking into account (43) the displacement vector of the internal point of the finite element and its derivatives will accept the form

$$\vec{W} = \underset{1\times24}{\{\psi^T\}}\underset{24\times24}{[N]}\underset{24\times72}{\left[\vec{A}\right]}\underset{72\times1}{\{u_y^\Gamma\}};\ \vec{W}_{,\alpha} = \{\eta_\alpha\}^T[N]\left[\vec{A}\right]\{u_y^\Gamma\};\ \vec{W}_{,\alpha\beta} = \{\eta_{\alpha\beta}\}^T[N]\left[\vec{A}\right]\{u_y^\Gamma\}. \quad (45)$$

The product of the matrices $[N]\left[\vec{A}\right]$ can be replaced through the other product

$$[N]\left[\vec{A}\right]=\left[\vec{A}\right][H]. \quad (46)$$

When expressing the basis vectors of the nodal points through the vectors of the internal point of the finite element $\vec{a}_1^{0n}$, a_2^{0n}, $\vec{a}^{0n}$ the product (46) will have the form

$$[N]\left[\vec{A}\right]=\left(\vec{a}_1^0[N_1]+\vec{a}_2^0[N_2]+\vec{a}^0[N_3]\right)[H]\{u_y^\Gamma\}. \quad (47)$$

When substituting (47) into (45) and comparing the received relations with (8) it is possible to express the components of the displacement vector and its derivatives through the column of the nodal unknowns of the finite element

$$W^1 = \underset{1\times24}{\{\psi\}^T}\underset{24\times24}{[N_1]}\underset{24\times72}{[H]}\underset{72\times1}{\{u_y^\Gamma\}} = \underset{1\times72}{\{v_1\}}\underset{72\times1}{\{u_y^\Gamma\}};\ W^2 = \{\psi\}^T[N_2][H]\{u_y^\Gamma\} = \{v_2\}^T\{u_y^\Gamma\};$$

$$W = \{\psi\}^T[N_3][H]\{u_y^\Gamma\} = \{v_3\}^T\{u_y^\Gamma\};\ m_\alpha^1 = \{\eta_\alpha\}^T[N_1][H]\{u_y^\Gamma\} = \{v_\alpha^1\}^T\{u_y^\Gamma\};$$

$$m_\alpha^2 = \{\eta_\alpha\}^T[N_2][H]\{u_y^\Gamma\} = \{v_\alpha^2\}^T\{u_y^\Gamma\};\ m_\alpha = \{\eta_\alpha\}^T[N_3][H]\{u_y^\Gamma\} = \{v_\alpha^3\}^T\{u_y^\Gamma\};$$

$$t_{\alpha\beta}^1 = \{\eta_{\alpha\beta}\}^T[N_1][H]\{u_y^\Gamma\} = \{v_{\alpha\beta}^1\}^T\{u_y^\Gamma\};$$

$$t_{\alpha\beta}^1 = \{\eta_{\alpha\beta}\}^T[N_2][H]\{u_y^\Gamma\} = \{v_{\alpha\beta}^2\}^T\{u_y^\Gamma\};\ t_{\alpha\beta} = \{\eta_{\alpha\beta}\}^T[N_3][H]\{u_y^\Gamma\} = \{v_{\alpha\beta}\}^T\{u_y^\Gamma\}. \quad (48)$$

The algorithm of the formation of the stiffness matrix by the size 72 x 72 is similar to the above-stated in item (3.1). It is necessary to note, that the numerical integration is carried out as it is accepted in [2, 3].

The advantages of the supposed method of the approximation of the displacement of the points of the finite element of a quadrilated shape are shown on the numerical examples.

4. The calculation examples.

Example 1. As the example, the sum about the determination of the tension-deformed condition of the compensator, being under the influence of the inner pressure of the intensively q (fig) was solved. The radius of the rotation was given by the functional dependence of the kind

$$r=A+B\cos(x/c).$$

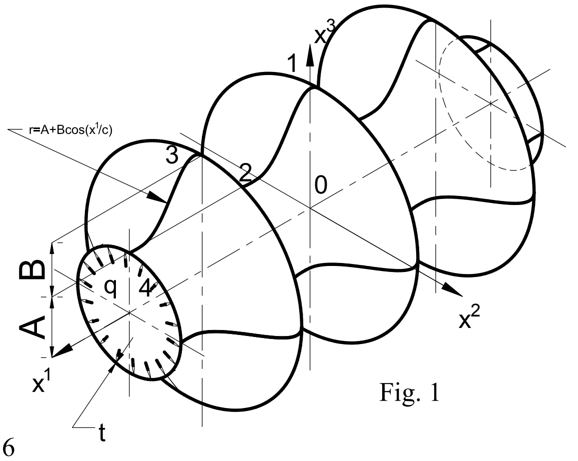

Fig. 1

Owing to the presence of symmetry planes was examined 1/8 of the shell. The following initial data was accepted: q = 0,2 MPa, A=1,3 m, B=0,4 m, C=12 m, t = 0,01 m, E = 2 ·10^5 MPa, ν=0, 3. The coordinate x varied in the limits $0 < x < 36\pi m$.

The calculations were carried out in the two variants. In the first variant when constructing the matrix of rigidity of the finite element of a quadrilated shape by the size 36 x 36 was used the traditional independent interpolation procedure [2, 3]. In the second variant was used the proposed way of the vector interpolation of the vectors displacements (see the ratios (23)...(33)). The calculation results are shown in the table 1, in which are listed the values of the ring (for the points 1,2,3 of the calculated construction) and meridian (for the point 4) tensions in the inner fibers of the shell in depending on the meridian direction. Along the ring the compensator was parted in the 2 or 3 finite elements.

The analysis of the calculation results listed in the table 1, shows, that in the first calculation variant the convergent of the calculation process is practically absent, in spite of the sufficiently small network of the discrete elements. In the second calculation variant one can see the fast gathering of the calculation process by the comparatively small number of the finite elements.

The chosen calculation scheme, by which the left edge of the shell (the point 4 on fig. 1) remains free, allows us to compare the received numerical value of the meridian tension with the precise one, which must be equal to zero.

From the table 1 one can see, that in the first calculation variant even by the dividing of the compensator in 120 elements in the meridian direction the controlling tension (the last line in the table 1) remains very far of the meaning of zero $(a_m = -634{,}7 Mpa)$. In the second calculation variant the meridian tension on the right edge of the shell is going up zero by the increasing of the number of the discrete element. For the determination of the tensions in the nodal points of the finite element it is necessary to calculate the second derivatives of the normal displacement w_{ap} (in the first calculation variant) and the polynomials t_{ap} (for the second variant), which supposes the necessity to use the full set of the nodal variable parameters of the discrete element 36 x 36. By the correct calculation the tensions in the nodal point calculated when using of the nodal unknowns of the adjacent finite elements, must be quite near in the values.

In the table 2, the numerical values of the ring tension in the inner fibers in the point 2 and 3 of the compensator are given, which are calculated when using the nodal unknowns of the finite elements, which side left and right with the examined know.

The analysis of the numerical tension values, listed in the table 2, shows, that in the first calculation variant along with the unsatisfactory gathering of the calculation process one can see the sharp difference in the tension values, which were calculated when using of the adjacent finite elements. Even by the dividing of the compensator in 120 discrete elements the controlling tensions differ not only in the value, but also in the sign. In the second calculation variant by any number of the discrete elements one can see the full coincidence of the numerical tension values, calculated when using of the finite elements, which side to the examined knot, and the good gathering of the calculation process by the comparatively small number of the finite elements.

Based on the analysis of the above mentioned table's material one can draw the conclusion, that the use of the quadrilated finite element by the size of the matrix of stiffness 36 x 36 for the calculation of the shells with the considerable gradients of the curvature of the middle surface demands the realization of the supposed way of the interpolation of the fields of the shift vectors (the ratio (23)...(33)). The use of the quadrilated discrete elements 36 x 36, in which by the formation of the matrix of stiffness was realized the traditional interpolation procedure (2,3), does not allow by the calculations of the above mentioned constructions to get the satisfactory results even by the small network of the finite elements.

Table 1

Variant of the way of interpolation of the shifts	I						II			
Number of FE along the meridian and coordinates of points	48	60	72	90	108	120	49	60	72	90
p.1, $x=0$	32.5	17.3	-4.9	-39.5	-65.2	-77.5	-136.1	-133.6	-132.9	-132.8
p.2, $x=12\pi$	-139.0	-127.4	-108.0	-63.7	-27.0	-9.2	49.6	49.4	49.2	49.0
p.3, $x=24\pi$	203.7	269.9	281.9	205.2	111.9	61.3	-136.1	-133.6	-132.9	-132.8
p.4, $x=36\pi$	-1193	-1356	-1356	-1099	-800.1	-634.7	-3.6	-1.9	-1.2	-0.6

Here and below FE is Finite element

Table 2

Variant of the way of interpolation of the shifts		I						II			
Number of FE along the meridian and coordinates		49	60	72	90	108	120	49	60	72	90
p. 2, $x=12\pi$	Left	-139.0	-127.4	-108.0	-63.7	-27.0	-9.2	49.6	49.4	49.2	49.0
	Right	-89.3	-17.2	39.5	72.7	73.6	70.5	49.6	49.4	49.2	49.0
p. 3, $x=24\pi$	Left	203.7	269.9	281.9	205.2	111.8	61.3	-136.1	-133.6	-132.9	-132.8
	Right	-141.8	-249.4	-311.1	-300.0	-251.9	-224.6	-136.1	-133.6	-132.9	-132.8

Example 2. The quadrilated finite element by the size of the matrix of stiffness 72 X 72 was used for the calculation of the compensator, described in the previous example. The calculations also were carried out in the 2 variants. In the first one in the algorithm of the formation of the matrix of stiffness of the finite element 72 x 72 was realized the traditional interpolation procedure [2,3]. In the second one — the suppose vector procedure. The calculation results are listed in the table 3, which structure coincides with the structure of the table 1. The analysis of the calculation shows, that in the first calculation variant one can watch the slow convergent of the calculation process by the considerable number of the discrete elements.

Table 3

Variant of the way of interpolation of the shifts	I						II			
Number of FE along the meridian and coordinates of points	12	18	30	48	72	90	12	15	18	24
p.1, $x=0$	41.9	50.1	-32.8	-115.5	-130.4	-131.7	-139.3	-136.9	-136.9	-138.0
p.2, $x=12\pi$	-68.6	-70.6	-7.2	-37.6	45.2	46.1	42.7	45.5	46.6	47.4
P.3, x = 24π	-109.2	-106.6	-35.1	-116.1	-132.4	-133.3	-140.6	-138.7	-138.5	-138.7
p.4, $x=36\pi$	-495.1	-1006.8	-230.7	-392.8	141.1	59.0	-12.2	-8.0	-4.2	-0.9

The meridian tension on the right edge of the shell remains very far from the zero value $(\sigma_m = 59Mpa)$ even by the dividing of the compensator in 90 finite elements in

Table 4

Variant of the way of interpolation of the shifts	I						II			
Number of FE along the meridian and coordinates of points	15	36	60	72	96	120	15	24	36	48
p.1, 1 = 0	29.1	351.0	93.0	17.1	-78.5	-108.2	-136.2	-123.5	-123.8	-122.5
p.2, $x = 12\pi$	79.7	-590.9	-283.4	-178.2	-34.9	10.8	35.2	45.0	48.8	49.5
p.3, $x = 24\pi$	-103.4	456.8	119.6	0.63	-93.8	-116.8	-134.6	-125.6	123.4	121.4
p.4, $x — 36\pi$	-343.5	-843.9	1815.5	2024.8	1467.7	748.6	-43.3	-7.4	0.7	-0.8

the meridian direction. In the second variant one can see the rapid gathering on the calculation process by the considerable smaller (in comparison with the first variant) number of the finite elements. The finite tension on the right edge of the shell becomes near enough to zero ($\sigma_m = 0{,}9 Mpa$) by the comparatively small number of the discrete elements (equal to 24). By the increasing of the compensator's wave frequency in 1,5 time $c = 8{,}0\ m \le x \le 24\pi\, m$ in the first calculation example (see table 4) the convergent of the calculation process is practically absent and the meridian tension on the right edge of the compensator becomes equal to 768,6 M Pa (instead of $\sigma_m = 0$) even by the dividing of the shell in 120 finite elements. In the second calculation variant one can see the fast gathering of the calculation process and σ_m in the point 4 is very near to zero already by the number of the discrete elements equal to 36.

Based on the analysis of the above given table's material one can draw the conclusion, that the use of the traditional independent interpolation procedure in the algorithms of the construction of the matrix of rigidity of the quadrilated finite elements [2,3] by the calculation of the shells with high gradients of the curvature of the middle surface becomes very un effective, in spite of the increasing of the number of the nodal changeable parameters of the discrete element. For the correct determination of the tension-deformed condition of such the constructions it is necessary to use the interpolation of the fields of the shifts (correlations (23)...(33)).

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КОНЕЧНЫЙ ЭЛЕМЕНТ ЧЕТЫРЕХУГОЛЬНОЙ ФОРМЫ ДЛЯ РАСЧЕТА ОБОЛОЧЕК С УЧЕТОМ СМЕЩЕНИЯ КАК ЖЕСТКОГО ЦЕЛОГО

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На базе криволинейного четырехугольного конечного элемента реализован новый способ аппроксимации полей перемещений, суть которого заключается в том, что на этапе аппроксимации внутренних величин через узловые неизвестные за узловые неизвестные конечного элемента выбираются не отдельные компоненты вектора перемещения и их производные, а непосредственно сам вектор перемещений узловых точек конечного элемента и его производные. На примере расчета компенсатора нагруженного внутренним давлением с наличием конструкционного смещения как жесткого целого показано, что использование разработанного алгоритма решает общеизвестную проблему МКЭ – учета смещения конечного элемента как жесткого целого.

Ключевые слова: метод конечного элемента, учет смещения как жесткого целого.