NUMERICAL MODELING OF MASSÈ SHRINKING CLAYEY SOILS IN THE DEPRESSION OF LAMA IN THE SOUTH OF BENIN

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This article presents the results of an experimental study about four soil samples coming from Massè, a region crossed by the East depression of LAMA in the south of Benin, where important damages have been recorded (rising of the foundations, deformation and cracking of the infrastructures, tearing of the surface coat of pavements...). In the first stage, a physicomechanical characterization of the soil samples is achieved by laboratory tests. The second stage of this study was reserved to the numerical modeling of the shrinking by basing on the analysis of the behaviour of the curves obtained at the end of the laboratory odometer tests. This approach allowed us to describe the variation of the indication of the voids ratio according to the loading pressure, to elaborate a mathematical model permitting to simulate the odometer curve of a loading-discharge cycle and to determine the parameters of compressibility. The results show a good quality of the adjustment. A negligible mistake of the order of 0,3% has been recorded.

KEYWORDS: physico-mechanical characterization, evaluation, shrinking, modeling, simulation.

1. Introduction

The new architectural challenges call by force to designers for a real taking in charge of the different aspects bound to the dimensioning and the safety of the works. The study of the stability of those works constitutes one of important problems to which the engineer is often confronted. Among the causes of instability of works the more encountered in general in the world and in particular in Benin, the phenomenon of shrinking-swelling of the supporting soil is probably the most dangerous. This instability, that concerns about 2,5% of the world surface of the grounds[1] and essentially a precise zone of Benin, named "median depression or depression of Lama" causes enormous damages (deformations, cracking, tearing, the wear and tear the surface coat and the destruction of the foundations) on civil engineering works. Therefore, the designer must take in account the effects induced by this phenomenon.

In the reach of the goals fixed at conferences of the lasting quality and development Forum [2], the phenomenon of shrinking-swelling has caused several investigations and varied research topics in order to minimize the risks connected to this one. Within this context, different models have then been proposed to describe the swelling pressure and the deformation of swelling soils [3]. Thus, A. Djedid, A. Bekkouche and Mamoune [4], show in their study "Identification and estimates of the swelling of a few soils of the region of Tlemcen (Algeria)", that the models of estimate are sensitive and are only applicable to the soils that served to their establishment. In the same order of ideas A. Medjnoun [5], after an analysis,

characterization, estimate and modeling of the behavior of swelling clays concluded in his works that the empiric models are inapplicable for the studied sites and that the mistake is very important. Sohe concluded that the evaluation of the swelling potential using the methods of classifications has for purpose to orientate the campaign of exploration toward the tests of swelling. It is deduced therefore that the solution would be to search for some models clean to every soil. The processing of a model adapted to the clayey soils of the East depression of LAMA in Benin is revealed main to warn the precocious defacements recorded on level of the infrastructures erected in this zone where several works of research have been done on the phenomenon of shrinking-swelling of these soils [6]. The conclusion of this works shows that the development of a model to predict the mechanical parameters of shrinking-swelling of these soils is not landed.

The works, subject of this study, consist to the processing of a numerical model of prediction of the compaction of the clayey soils of Massè district, located in the aforesaid depression. Indeed, the results of the identification tests presented in this article allow characterizing these clays on the one hand and on the other hand to give the means to the designer for a real hold in account of this phenomenon thanks to the elaborated model. The obtained results allow to predict the pressure from identification tests. Besides, the systematic resort to the tests of laboratory, generally lead, is appreciably reduced thanks to the established numerical model.

2. Methodology

2.1. Environment Of Study

The site, object of the present study, is located in Massè, township of Adja-Ouèrè situated in Issaba depression in the East of Lama depression. According to IGUE MOUINOU, WELLER ULRICH (2000)-Geology and geomorphology of South Benin quoted[7], this depression of Lama is formed by a directed band WSW –ENE of maximal width 25km. It covers an area that extends from the East to the West with an area of more than 3000km².

The depression of Lama forms a wide drill of 130km long and a variable width from 5km (Tchi) to 25km (Issaba). It is divided into three zones to know the depression of Issaba in the East, depression of Khô in the center and depression of Tchi in the West quoted [7].



Figure 1: Geographical localization of the taking site

2.2. Sampling

The samples are taken in Massè in the township of Adja-Ouèrè. The choice of the site has been justified by the visual inspection of precocious defacements caused by the phenomenon of shrinking-swelling of soil in place.

For the sampling, a well has been drilled from which we have extracted intact samples and remolded to different depths (0,00m to 0,50m; 0,50m to 1,00m; 1,00m to 2,00m and 2,00m to 3,00m).

The National Center of the Tests and Research in Public Works (NCTRPW) of Benin and the LAB-TP laboratory of Lomé in Togo served as surroundings for the achievement of the different and mechanical identification tests on samples. The results of these tests were the object of analysis and discussions.

3. Results and discussions

3.1. Physicalcharacteristics

3.1.1. The tests of identification

The sizing of the four samples of Massè site shows a variation of the particles lower to 80 μ m of (67 à 75%) and even better the majority of these particles is lower to 2 μ m (53 to 63%) which represent the clayey particles. (See graph 1). As for the results of the other tests of identification, they are presented in the board 1.





Identification of clays of Mas	Depth Z(m)				
Paramètres	0,00-0,50	0,50-1,00	1,00-2,00	2,00-3,00	
Particles content< $2\mu m$ (%)C2		46,352	51,235	41,353	52,235
Water content(%) W		26,60	31,80	34,41	31,03%
Dry density γ		1.54	1.40	1,389	1,41
Liquidity Limits(%) W _L		67,80	69,70	86,00	95,00
Plasticity Limits(%) W _P		46,30	41,35	52,80	60,90
Index of plasticity	IP	21,50	28,35	33,20	34,10
Specific weight (g/cm ³) γs		2,52	2,50	2,53	2,54
Activity of clay A		0,464	0,553	0,803	0,653

Organic matter content (%)		Смо	4,31	3,76	2,99	5,87
	Bond of soil in (kPa)	С	34,00	24,00	38,50	39,00
	Angle of internal friction (°)	φ	3,00	5,00	5,00	7,50

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The values of found specific weights are of the order of 2,50 to 2,54 and allow us to conclude according to Agbelele K. Judicaël et al., (2016) [7] that the tested samples are of clayey nature.

3.2. Mechanicalcharacteristics

3.2.1. Odometer test

In geotechnics, the odometer test is generally used like direct method to determine the pressure and the amplitude of swelling (Ozer et al., 2011) [8]. At the end of this test on the intact samples, the recapitulative of the results and the odometer curves are presented as follows:

Parameters		Depths (m)				
		0,00-0,50	0,50-1,00	1,00-2,00	2,00-3,00	
Voids ratio ei		0,526	0,650	0,651	0,725	
Voids ratio of the soil in place		0,516	0,644	0,648	0,720	
Pre consolidation stress (KPa) σp		73,250	129,02	155	162	
Swelling pressure (KPa)		5,180	3,440	56,63	57.8	
Compression ratio (0,304	0,202	0,206	0.218	
Swelling ratio Cs		0,052	0,082	0,015	0,010	

Board 2: Recapitulative board of the results of the odometer tests



Graph 4: Odometer curve sample Massè depth 2 m Graph 5: Odometer curve sample Massè depth 3 m

4. Numerical modeling of the compaction of the studied soils

In the worry to write a mathematical model according to the voids ratio, we present the odometer curves above to the depths (1,00-2,00m) and (2,00-3,00m) by considering voids ratio in abscissa and the loading pressures corresponding in ordinates. This choice of depth is justified by the typology of works encountered in the study zone whose foundations are essentially superficial and their depth of anchorage varies from 2,00 m to 3,00 m.

4.1. Geometrical interpretation of the curve loading pressure - voids ratio

From the observation of the curves pressures of loading-indication of the voids ratio, we note that each of them presents at least three points of inflection and three extrema. According to Rolle's theorem, the polynomial function expressing the pace of this curve would be at least of degree 6 [9].

4.2 Approach of the curve by the polynomial form of order 6at the depth 2,00 m $P_g = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + gx^6$ Let's apply the method of the least squares to this equation to determine the constants

a, b, c, d, e, f and g. The sum to minimize is

$$\sum_{i=1}^{22} [P_i - (ai + bxi + cxi^2 + dxi^3 + exi^4 + fxi^5 + gxi^6)]^2 = 0$$

Let's nullify the first derivatives in relation to a, b, c, d, e and f: $\partial()/\partial a = 0 \Leftrightarrow$

$$\sum_{i=1}^{22} a + b \sum_{i=1}^{22} xi + c \sum_{i=1}^{22} xi^{2} + d \sum_{i=1}^{22} xi^{3} + e \sum_{i=1}^{22} xi^{4} + f \sum_{i=1}^{22} xi^{5} + g \sum_{i=1}^{22} xi^{6} = \sum_{i=1}^{22} Pi$$

$$a \sum_{i=1}^{22} xi + b \sum_{i=1}^{22} xi^{2} + c \sum_{i=1}^{22} xi^{3} + d \sum_{i=1}^{22} xi^{4} + e \sum_{i=1}^{22} xi^{5} + f \sum_{i=1}^{22} xi^{6} + g \sum_{i=1}^{22} xi^{7} = \sum_{i=1}^{22} Pixi$$

$$\frac{\partial()}{\partial c} = 0 \Leftrightarrow$$

$$a \sum_{i=1}^{22} xi^{2} + b \sum_{i=1}^{22} xi^{3} + c \sum_{i=1}^{22} xi^{4} + d \sum_{i=1}^{22} xi^{5} + e \sum_{i=1}^{22} xi^{6} + f \sum_{i=1}^{22} xi^{7} + g \sum_{i=1}^{22} xi^{8} = \sum_{i=1}^{2} Pixi^{2}$$

$$\frac{\partial()}{\partial d} = 0 \Leftrightarrow$$

$$\sum_{i=1}^{22} xi^{3} + b \sum_{i=1}^{22} xi^{4} + c \sum_{i=1}^{22} xi^{5} + d \sum_{i=1}^{22} xi^{6} + e \sum_{i=1}^{22} xi^{7} + f \sum_{i=1}^{22} xi^{8} + g \sum_{i=1}^{22} xi^{9} = \sum_{i=1}^{2} Pixi^{3}$$

$$\frac{\partial()}{\partial e} = 0 \Leftrightarrow$$

$$a \sum_{i=1}^{22} xi^{4} + b \sum_{i=1}^{22} xi^{5} + c \sum_{i=1}^{22} xi^{6} + d \sum_{i=1}^{22} xi^{7} + e \sum_{i=1}^{22} xi^{9} + g \sum_{i=1}^{22} xi^{9} = \sum_{i=1}^{2} Pixi^{4}$$

$$\frac{\partial(\)}{\partial f} = 0 \Leftrightarrow$$

$$a \sum_{i=1}^{22} xi^5 + b \sum_{i=1}^{22} xi^6 + c \sum_{i=1}^{22} xi^7 + d \sum_{i=1}^{22} xi^8 + e \sum_{i=1}^{22} xi^9 + f \sum_{i=1}^{22} xi^{10} + g \sum_{i=1}^{22} xi^{11} =$$

$$= \sum_{i=1}^{2^2} Pixi^5$$

$$\frac{\partial(\)}{\partial g} = 0 \Leftrightarrow$$

$$a \sum_{i=1}^{2^2} xi^6 + b \sum_{i=1}^{2^2} xi^7 + c \sum_{i=1}^{2^2} xi^8 + d \sum_{i=1}^{2^2} xi^9 + e \sum_{i=1}^{2^2} xi^{10} + f \sum_{i=1}^{2^2} xi^{11} + g \sum_{i=1}^{2^2} xi^{12} =$$

$$= \sum_{i=1}^{2^2} Pixi^6$$

The CRAMER method of resolution of the equations system formed by the first derivatives allowed us to determine respectively the different values of the coefficients a, b, c, d, e, f and g [9].

Coefficients	Values	Coefficients	Values
a	-5,91474 E-07	d	5870787,642
b	185238,6942	е	-10376421,92
С	-1652879,471	f	9127859,635
-	-	σ	-3197723 079

Board 3: Value of the coefficients of the polynomial form of degree 6 at the depth 2,00 m

By rewriting the equation with the values of the coefficients, we obtain: $P_q =$

 $-5,91474.10^{-7}185238,6942x - 1652879,471x^{2} +$ 5870787,642 x^{3} -10376421,92 x^{4} + 9127859,635 x^{5} -3197723,079 x^{6} .

By this method the different pressures obtained for every voids ratio allowed us to draw the curve loading pressure-voids ratio and to compare it with the one of the test:



Graph 6: Curve loading pressure-voids ratio (polynomial form of degree 6) at the depth 2,00 m

We notice that the two curves are confounded in a first time and then we observe a gap. We can conclude that the model under the polynomial form of order 6 presents some results with a margin of mistake. The gap of mistake is very big for some ranges. We will try to solve the problem by a mathematical model defined by intervals.

4.3. Approach of the curve loading pressure - voids ratio by a defined function by intervals at the depth 2,00m

It will be about describing the pace of the curve by a mathematical function defined on several intervals. So the curve answers to a function defined by four intervals.

• First interval [0,447365255; 0,468333412[:

For this interval of voids ratio we have kept for the loading pressure a function of the type: $P_g = a + bx + cx^2$.

By the method of the least squares we have determined the coefficients *a*, *b*, and *c*. So the pressure can be written: $P_g = 1640.9 - 6738.8x + 6913x^2$.

• Second interval [0,468333412;0,501657804[:

The expression of the loading pressure on this interval is in the form: $P_g = a + bx$. The expression with the values of the coefficients *a* and *b* gives us:

 $P_q = 16,732 - 33,249x$

Third interval [0,501657804 ; 0,508397569[:

The expression of the loading pressure on this interval is also in the form: $P_a = a + bx$.

The expression with the values of the coefficients *a* and *b* gives us: $P_a = -488,08 + 973,03x$.

• Forth interval [0,508397569;0,656485176]:

On this last interval, the expression of the loading pressure is:

$$P_q = a + bx + cx^2 + dx^3.$$

By the method of the least squares we determine the coefficients and the expression becomes: $P_q = 20,576 + 68,01x - 313,2x^2 + 246,52x^3$.

We can rewrite our function therefore in the form: $P_a =$









4.4. Approach of the loading curveloading pressure –voids ratio by a defined function by inter valsat the depth 3,00m

The approach is identical to the previous and allows us at the end of works to find again the expression of the pressure in the form: $P_a =$

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	$\forall x \in [0.515777501; 0.53983929]$; $P_q = 7250,51 - 27079x + 25282x^2$
= {	$\forall x \in [0,53983929; 0,553613806]$	$P_q = -244,03 + 452,14x$
	$\forall x \in [0,553613806; 0,72675412]$; $P_a = 299,8 - 1263,6x + 1816,2x^2 - 887,58x^3$

4.5. Comparison between the test to the laboratory and the one simulated by the model

Through the figure below we superpose the odometer curve of the test and the one obtained thanks to the mathematical model defined by intervals. We notice the similitude of behaviour of two curves. They nearly have the same paces allowing us to determine the parameters of compressibility.



Graph 9: Odometer curve of the test in laboratory and simulation of the model at the depth 2,00m



Graph10: Odometer curve of the test in laboratory and simulation of the model at the depth 3,00m

 $\sigma_r^2 = \frac{1}{n} \sum_i \left(P_{gi} - \frac{1}{n} \sum P_i \right)^2.$

- 4.6. Quality of the adjustment
 - Salvage or conditional variance: $\sigma_c^2 = \frac{1}{n} \sum_i (P_i P_{gi})^2$.
 - Variance due to the regression:
 - Variance of the observed values: $\sigma_p^2 = \frac{1}{n} \sum_i \left(P_i \frac{1}{n} \sum P_i \right)^2$. With: *Pi :the pressure of calculation of the odometer test, Pgi :the pressure estimated by the numerical model.*

For the analysis and the interpretation of the results, the relations: $\frac{\sigma_c^2}{\sigma_r^2}$; $\frac{\sigma_c^2}{\sigma_p^2}$ have been calculated: $\frac{\sigma_c^2}{\sigma_r^2} = 0.003700659$, $\frac{\sigma_c^2}{\sigma_p^2} = 0.00370217$.

0,37% of the Pi variance are due to the gaps of the Pi to their conditional average. We deduct from these results, a good quality of the adjustment. The recorded mistake is of the order of 0,03%.

Quality of the adjustment								
Quality of the adjustment								
Voids ratio	Loading Pressure (Pi)	Pressure calculated by the model (Pgi)	Equations	$(P_i - P_{gi})^2$	$\left(P_{gi}-\frac{1}{n}\sum P_i\right)^{\frac{1}{2}}$	$\left(P_i-\frac{1}{n}\sum P_i\right)$		
0,447365255	9,727	9,732915864	Pg = 1640,9- 6738,8x+6913x^2	3,49975E-05	70,249329	70,150196		
0,468333412	1,16	1,160382384	Pg=16,732-33,249x	1,46218E-07	0,036498	0,036644		
0,501657804	0,052	0,052379675	Pg=16,732-33,249x	1,44153E-07	1,687528	1,688514		
0,508397569	6,61	6,626086564	Pg=-488,08+973,03x	0,000258778	27,822016	27,652573		
0,563064549	3,5	3,580065378	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	0,006410465	4,966822	4,616359		
0,602379	1,935	1,780088	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	0,023997489	0,183749566	0,340555612		
0,618480	1,16	1,155887	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	1,69145E-05	0,0382364	0,036644898		
0,639074118	0,766	0,467297	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	0,089223225	0,781687874	0,342726612		
0,639074	0,766	0,467297431	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	0,089223225	0,781687874	0,342726612		
0,639822	0,377	0,444727809	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	0,004587056	0,822106273	0,949511041		
0,640946	0,247	0,411209961	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	0,026964911	0,884011035	1,219762469		
0,641882	0,377	0,383588382	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	4,34068E-05	0,936714632	0,949511041		
0,642069	0,182	0,378097981	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	0,038454418	0,947372438	1,367563184		
0,642818	0,117	0,356249	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	0,057240489	0,990380693	1,523813898		
0,642818	0,182	0,356249	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	0,030363009	0,990380693	1,367563184		
0,642818	0,247	0,356249	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	0,011935529	0,990380693	1,219762469		
0,64843402	0,377	0,19823034	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	0,031958558	1,329865943	0,949511041		
0,649183765	0,247	0,177948436	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	0,004768118	1,377055628	1,219762469		
0,65068149	0,182	0,137946702	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	0,001940693	1,472538247	1,367563184		
0,65068149	0	0,137946702	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	0,019029293	1,472538247	1,826359184		
0,655923529	0,117	0,0039093	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	0,012789507	1,815808188	1,523813898		

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0,656485176	0,052	-0,009895298	Pg = 20,576+68,01x- 313,2x^2+246,52x^3	0,003831028	1,853202679	1,688514612
SUM	28,38	28,35486072	0	0,4530714	122,429912	122,3799543
				Salvage or conditional variance	variance due to the regression	variance of the observed values
$1/n \sum Pi$	1,351428 571		Interpretations	0,021574829	5,829995809	5,827616873
				0,003700659		
				0,00370217		

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5. Conclusion

This work permitted us to write a model capable to predict the compaction of the clayey soils of Massè from four samples taken from different depths where the tests of identifications and mechanical have been achieved in a first time in the laboratory. Then, a geometric interpretation of the Odometer curves gotten from the tests has been achieved. It emerges that the polynomial function expressing better the pace of these curves would be at least of degree 6 because each of the curves loading pressures-voids ratio presents at least three points of inflection and three extrema. Thanks to the obtained results we noticed that the gap of mistake is very big with the model in the polynomial form of degree 6 for some ranges. So we tried to solve the problem by intervals with a defined mathematical model. The results reveal a good quality of the adjustment. The observed mistake is negligible of the order of 0,3%.

Bibliographical references

[1] ASSIOU. F.Z. et NECHNECH. A. (2011), *Retrait et gonflement des marnes argileuses d'Alger « étude d'un risque naturel »*, INVA CO2 : Séminaire International INNOVATION et VALORISATION en Génie – Civil et Matériaux de Construction 5P-050 Université Houari Boumediene Algérie .7 p

[2] Emmanuelle PLAT (2009), Sols argileux et risque sécheresse : nouveaux outils et retour d'expérience, BATIMAT – Conférences du Forum Qualité & Développement Durable, Agence Qualité Construction, 39 p.

[3] Zohra DERRICHE et Mustapha KEBAILI (2000), Prévision du gonflement des argiles d'In-Aménas, BULLETIN DES LABORATOIRES DES PONTS ET CHAUSSÉES 218 NOVEMBRE-DÉCEMBRE 1998-RÉF.3865-PP15-23

[4] BEKKOUCHE A., DJEDID A., AISSA MAMOUNE S.M. (2001), *An experimental investigation on the assessment of the swelling parameters*, IV. International Congress, Eastern Mediterranean University, Gazimagusa, North Cyprus, Vol. 2, pp. 627-633).

[5] A. MEDJNOUN (2014), Analyse, caractérisation, prévision et modélisation du comportement des argiles gonflantes, Thèse de Doctorat, Université de Mouloud Mammeri de Tizi-Ouzou, 209 Pages.

[6]. V. GBAGUIDI et al (2010), Etude du phénomène de gonflement des sols argileux du bassin sédimentaire côtier au Bénin, Journal de la recherche scientifique de l'Université de Lomé, vol. 12, N°2, pp. 19-29

[7] K.J. AGBELELE et al (2016), Caractérisation physico-mécanique des sols argileux de la dépression d'Issaba au Sud-Est du Bénin, Afrique sciences, vol. 12, N°2, pp. 206-221.

[8] Ozer *et al.* (2011), cité par Kelly GUERRIER, Dominique BOISSON *et al* (2014), *Etude du gonflement d'échantillons de marne formation géologique "morne DELMAS" en Haïti*, Journées Nationales de Géotechnique et de Géologie de l'Ingénieur JNGG2014 – Beauvais 8-10 juillet 2014,10 p.

[9] Joël M. ZINSALO (2012), Cours de Mathématiques pour ingénieurs de l'Ecole Polytechnique d'Abomey Calavi (EPAC), 433 Pages.

[10] J. LERAU (2006), Cours de Géotechnique 1, Chapitres 4, Tome 2, 25 p.