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Theoretical article

A Model of Teaching Mathematics with the Effect of Developing the Probabilistic Style of Thinking in a Digital Educational Environment: Theoretical Justification and Empirical Verification

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Abstract. The article is focused on the problem of inducing students to develop the non-linear, probabilistic style of thinking in the context of the digitalization of modern education. The purpose of the study is to create effective scientific and methodological tools for organizing the process of teaching mathematics in schools and universities in order to achieve high pedagogical, socially demanded results and, on the basis of these tools, to create an improved didactic model with the effect of developing the probabilistic style of thinking, flexibility, creativity and criticality in students so as to enable them to actively participate in social life. The model of teaching mathematics is constructed in the unity of the target, theoretical and methodological, content, technological, diagnostic and effective components. The structure-forming factor is an information-intensive educational environment for teaching mathematics as a set of digital information and educational content that contributes to the effective development of probabilistic style of thinking. The content component of the model is implemented in the selection and structuring of educational material based on the fractal approach, in the methodological update of a complex of foundation spirals equipped with banks of applied and research tasks, taking into account the depth of the fractal representation of educational elements. To obtain guaranteed learning outcomes while solving technological problems of implementing the model, an adaptive learning system was used as a tool for developing probabilistic style of thinking in students and creating an objective means of management. The results of introducing the model into teaching practice with subsequent statistical verification based on the descriptive statistics methods and Student's *t*-test showed positive dynamics for all the structural components of the model with a confidence level of 95%. The prospect of the research is further intellectualization of the technological component of the model based on the hybridization of artificial intelligence methods to ensure the effective development of the probabilistic style of thinking with rapid changes in parameter values according to the set feedbacks.

Key words: digitalization of education, mathematical education, learning model, probabilistic style of thinking, innovative content, adaptive learning system

Introduction

The global transformations and crises in the modern period have drastically changed the organizational and functional frameworks of our society, destroying many of its most essential constructs. This complex process predetermines the qualitative changes in a person who has a new type of thinking adapted to the probabilistic nature of the processes taking place in the world. Therefore, an important aspect of the phenomenon of the manifestation of the educational effect is the formation of flexible and multifaceted thinking – probabilistic. Therefore, an important aspect of the manifestation of the educational effect is the development of a flexible and multifaceted style of thinking, i.e., probabilistic one.

Today it is impossible to introduce effective educational systems into practice without digital technologies, teaching materials and online services. Managing the formation of personal identity based on the ever-increasing use of constantly evolving digital and network technologies is a key task for any educational system. Modern digital technologies make it possible to form dynamic links between pedagogical control and management, to effectively organize the learning process both from the informational-prescriptive point of view and from the point of view of the effective development of the probabilistic style of thinking. Due to informatization of education, it becomes possible to create an infrastructure for the transfer of information, transitions of interdisciplinary interaction with the conjugation of intentional information flows, as well as to increase the efficiency of perception and processing of information dictated by the stochastic nature of the analyzed processes. Therefore, using digital tools, we are able to fully unlock the potential of an innovative educational strategy and develop the probabilistic style of thinking in the process of teaching mathematics.

The didactic aspect of the probabilistic style of thinking has been studied in the works of P. Nilsson, S. Prediger, S. Schnell, E. Sánchez, P. Rubén Landín, E. S. Mooney, C.W. Langrall, J.T. Hertel, C. Batanero and others (Nilsson, 2014; Prediger, Schnell, 2014; Sánchez, Landín, 2014; Batanero, Borovcnik, 2016). It is mathematical education that determines scientific knowledge from the concept of classical determinism to new concepts of neodeterminism, to the ideas of non-linearity and randomness that can provide the development of "high-order" skills and the probabilistic style of thinking, which is revealed in new competences and personal qualities:

1) the possession of basic mathematical knowledge and skills that help solve problems of real practice (the ability to analyze and compare information from various sources, presented in different forms; evaluate its reliability and usefulness; the ability to recognize and identify opportunities for applying mathematics, formulate situations mathematically; interpret, use and evaluate mathematical results, etc.);

2) the possession of mathematical knowledge and skills that are necessary to solve more complex problems in a situation of uncertainty and ambiguity, problems from other areas of knowledge with an unfamiliar context (critical and nonlinear thinking, creativity, ability to work in a team, etc.);

3) the ability to adapt to rapid changes in the environment (initiative, media literacy, ability to interact, work in a team and for results, multicultural competence, leadership qualities, emotional intelligence, etc.);

4) the proficiency in functional and digital literacy (ability to use mathematical knowledge and skills in solving a wide range of tasks outside of educational situations, in non-standard situations; readiness and ability to apply digital technologies in all areas of science, production and everyday tasks).

Two centuries of disputes and discussions about probabilistic theories and concepts have led to the need to develop the probabilistic style of thinking in students. In the era of big data and immersive interactive technologies, this problem needs to be rethought. Numerous advantages of the "digital heritage," the possibility of using virtual "soft control" of students' mental activity open up new opportunities for improving mathematical education in the context of knowledge integration, motivated and creative overcoming of "problem areas" of mathematical activity, improving the conceptual model of teaching mathematics with the effect of developing the probabilistic style of thinking. We believe that this style of thinking can be developed with the help of more advanced scientific and methodological tools for organizing the process of teaching mathematics at school and university.

Literature review

Historically, the probabilistic style of thinking has been studied from various perspectives.

The historical and philosophical view of probability and probabilistic thinking was revealed in the works of R. Carnap, J.M. Keynes, K. Popper, M. Borovcnik, R. Kapadia and others (Carnap, 1962; Keynes, 1921; Popper, 1957; Borovcnik, Kapadia, 2014). It should be noted that the debate about the philosophical basis of probability has not yet been fully resolved. Three main theories related to the classical, frequency, and subjectivist approaches are described. These philosophical ideas are key ones to the development of the content and methodology of training. Probabilistic concepts are closer to a consistent way of thinking about the world than deterministic ones.

The concept of "probabilistic style of thinking" is not generally accepted in psychology. The new term was introduced in 1945 by the psychologist B.M. Teplov to designate "a type of thinking, the structure of which includes judgments about the degree of probability of expected events" (Teplov, 1961). Investigating the specifics of this type of thinking, its structure, many researchers of cognitive psychology have identified the features of probabilistic thinking. One of these features is the role of the heuristic approach in formulating probabilistic judgments and conclusions (Gigerenzer, Gaissmaier, 2011). At the same time, it is advisable to recall the specific features of the probabilistic style of thinking, which drew the attention of D. Kahneman and A. Tversky (Kahneman, Tversky, 1973). The scientists focused on such its features as accessibility, representativeness, consolidation and correction.

W. McGuire and P.N. Johnson-Laird have explored the problem of probabilistic thinking based on mental models ("software based thinking"). They have proved that the extensional basis of probability theory is composed of deduction, induction and probabilistic thinking. The probabilistic style of thinking allows us to understand the world by constructing mental models (Collins, Michalski, 1989; Gigerenzer et al., 1991; Johnson-Laird, 1994), which are based on certain principles (Collins, Michalski, 1989). The research of G. Gigerenzer, U. Hoffrage, H. Kleinbölting (Gigerenzer et al., 1991) is focused on probabilistic mental models considered as the functional equivalent of a problem space in which a solution to a problem related to decision-making and risk assessment is being sought. In the study of A.V. Dobrin and A.M. Lopukhin, the probabilistic style of thinking is also determined through "an individually peculiar way of identifying and formulating a problem situation that contains an element of uncertainty" (Dobrin, Lopukhin, 2019). The ability to make probabilistic conclusions and justify uncertain judgments is determined by the ability to differentiate randomness and causality, the ability to predict based on a combination of induction, deduction and intuition methods.

Didacticians offer their own vision of the probabilistic style of thinking. For example, they believe that it is possible to develop "genuine" probabilistic thinking as a result of the analysis of the causes of errors and conflicts between the primary intuitive schemes for solving educational problems and specific types of reasoning specific to stochastic situations (Fischbein, Schnarch, 1997). According to Yu.I. Ponomarev and T.G. Shapovalenko, probabilistic thinking is understood as a type of thinking based on cause-effect relationship. The authors emphasize that this relationship is established not in single outcomes, but in an ensemble of random phenomena (Ponomarev, Shapovalenko, 2008). This approach is taken bay Rolf Biehler, "do not look for connections in an individual case" (Biehler, 1994). And then the scientist comes to an unexpected interpretation of probabilistic thinking: "Probabilistic thinking is the basis of enlightenment: randomness is not magic, but it can be calculated – at least to a certain extent, namely in relation to long-term behavior. The risks become calculable, and the uncertainty of decision-making can be rationalized" (Biehler, 1994).

An original understanding of probabilistic thinking through the activation of cognitive activity of students by means of a meaningful context containing uncertainty is proposed by D.I. Sari and D. Hermanto (Sari, Hermanto, 2017).

According to the long-term research conducted by S. Dvoryatkina (Dvoryatkina, 2013), and later by M. Borovcnik (Borovcnik, 2016), the probabilistic style of thinking allows us, using representations created on the basis of logic and intuition, to assess their probability, making it possible to quickly find the most correct decision in difficult situations of choice and uncertainty.

Summarizing the obtained data on the essence of the concept of "probabilistic style of thinking", we can define it as "an individual system of intellectual strategies, methods, techniques, principles, forms, ideas of probabilistic-statistical description and cognition of the laws of the surrounding world. This system ensures the application of scientific methods based on: a combination of perception modalities and primary assimilation of educational material; activation of the interaction of logical and intuitive types of thinking; integration of logical and probabilistic forms of thinking; qualitative enrichment of mental operations through the formation of systemic knowledge" (Dvoryatkina, 2013).

A model for teaching mathematics with the effect of developing the probabilistic style of thinking

The author's structural and functional model of teaching mathematics in schools and universities with the effect of developing the probabilistic style of thinking is presented as part of the components and their characteristics: theoretical and methodological, content, technological and diagnostic ones.

The goal-setting of the system for teaching mathematics with the effect of developing the probabilistic style of thinking is determined by:

– the social order (the need of society, production and economy for highly qualified specialists; the changed demand for the quality of general secondary and higher professional education);

– the changes in the educational and professional environment (the transition to federal educational standards of a new generation; the introduction of new professional standards, the digitalization of education); and

– the external prerequisites (the general trend in ensuring the global competitiveness of Russian education; the integration of Russian education into the European educational and scientific space).

An innovative component of the model is an information-intensive educational environment for teaching mathematics as a set of digital information and educational content that contributes to the effective development of the probabilistic style of thinking.

The theoretical and methodological basis for building the model is the concept of fractal formation and development of the probabilistic style of thinking through the example of teaching mathematics in schools and universities in the context of global informatization of education based on a combination of these approaches, which justify and characterize the leading ideas and interrelated provisions on the possibility of effective development of the probabilistic style of thinking in the process of teaching mathematics.

Let us now list the main provisions of the concept (Dvoryatkina, Scherbatykh, 2020):

– *the probabilistic style of thinking as a result of the development of professional competence of a modern specialist in all areas of training.* The main idea of this provision is that the independence and competence in solving problems in conditions of choice and uncertainty, the ability to overcome problems of social interaction, the accelerated decision-making and adaptive behavior in difficult situations are indicators that students have developed the probabilistic style of thinking;

– *the fractal structure of thinking.* The main idea here is that the use of fractal methods in modeling and analyzing complex, nonlinear thinking processes will provide a solution to the problem of effectively regulating information flows, filtering information and minimizing the relative delay in the controlled formation of integrative relationships;

– *the cyclic and systematic development of the probabilistic style of thinking*. The main idea is that the cyclic and systematic development of the probabilistic style of thinking is realized through the obligatory and sequential change of two phases (cumulative and bifurcational): a certain potential is accumulated, a bifurcation occurs and the process moves to a new fractal development level;

– *the foundation theory as a necessary methodological construct for future specialists to develop their probabilistic style of thinking, personal and professional qualities*. The main idea is that the management of the fractal development of the probabilistic style of thinking in the process of teaching mathematics is carried out through coordination with the foundation constructs of personal development: from updating the initial state of the student's individual experience to the implementation of solutions to specific applied problems through the use of digital technologies in a dynamically changing information environment at each level of the foundation spiral development;

– *the synergy of mathematical education as a system-forming factor in the design and organization of the educational process in schools and universities, focused on the development of the probabilistic style of thinking.* The main idea is that the synergy of mathematical education in schools and universities creates conditions for the effective development of the probabilistic style of thinking, providing opportunities for self-education, self-actualization and self-development, the fullest disclosure of communicative opportunities and actualization of creative independence in the educational process;

– *the dominant role of modern computing power and software in the controlled development of the probabilistic style of thinking.* The main idea is that the use of modern computing power and software for the controlled development of the probabilistic style of thinking will allow for rapid diagnostics, correction and forecast of the dynamics of this process in the direction of individualization and personalization of educational paths of students' development and self-development.

The methodological foundations of the concept are system-based, vector-contextual, competence-based, environmental, fractal and synergetic approaches. The synthesis of these approaches provides the disclosure of functional, operational and motivational components of the integrity of the perception of sign-symbolic activity in the direction of understanding and comprehending mathematical knowledge, developing creativity and critical thinking, activating the interaction of intuitive and logical types of thinking and, as a result, qualitative enrichment of mental activity.

An important methodological basis of the didactic system is the periodic system of principles using the ideas of synergetics. The authors introduced triads of principles into the theory and practice of teaching mathematics: the development of mathematical intuition – the logical structure of mathematical objects – the inclusion of probability in the structure and content of knowledge; fundamental – applied and professional orientation-Foundation; discreteness – continuity – fractality; openness – closeness – transdisciplinarity; expediency-causality – polymotivation; problems – clarity – flexibility and adaptability of the organizational structure; structural algorithmization – creativity – polyvariance; uncertainty – environmental conditionality – bifurcation; saturation of the educational environment – individualization and personalization of educational routes – intelligent management.

The content component of the model is implemented in the structural-functional, process-activity and motivational-applied modules based on a spiral scheme of deployment and modeling of basic educational elements in the direction of the development of thinking, personal and professional qualities of students in teaching mathematics.

The structural-functional module is implemented at the stage of analysis, selection and structuring of educational material. To select and structure the learning content, the fractal description method is used, which reflects the property of the self-similarity of the whole in any of its divisible parts due to the same scheme for constructing all the structural elements of the learning content. The fractal representation of educational elements with variable structuring of mathematical content intensifies the cognitive activity of students, ensuring the effective development of thinking mechanisms, managing the depth of establishing interdisciplinary connections and its operational control at the necessary levels of interpenetration.

The process-activity module includes: process stages of selecting and structuring the content of training:

1) the design of educational thesauri of mathematical disciplines on the basis of information-semantic and logical analysis with subsequent examination;

2) the definition of the structural interdisciplinary component of key concepts;

3) the deployment of the fractal structure according to the selected development vectors of the key concept;

4) the selection of parameters for limiting the growth of fractal conceptual structures (setting the number of iterations);

5) the content of training modules based on objectified thesauri;

6) the structuring of the bank of educational, cognitive and research tasks linked to the fractal structure of the conceptual apparatus (hierarchical complexes of multi-stage mathematical and informational tasks for schools and universities: professionally oriented, applied and research tasks classified by levels of complexity).

The main tasks of designing *motivational-applied module* include ensuring the motivation of educational-cognitive activity database spirals of foundation equipped with banks of applied and research tasks, taking into account the depth of the fractal representation of training elements. The motivational-applied component is implemented in a contextual and information-intensive educational environment and is updated at the stage of mastering the system of mathematical concepts, methods of action and operations with them. The fractality of structuring the content of teaching mathematics as it becomes more complex ensures its adaptability to specific learning conditions and the contingent of students.

The bank of problems for the basic school includes problems for analysis and synthesis, the ability to operate with a stock of knowledge, comparison, discrimination, abstraction, generalization, classification, systematization, induction and deduction, proof, mathematical problems on a chessboard, for the game Zhipto and Go, classified by levels difficulties; for higher education – these are professionally oriented and research tasks, multi-stage research tasks on computer modeling of nonlinear dynamic processes, etc.

The technological component is implemented in the form of a graph for coordinating the stages of developing the probabilistic style of thinking and a cluster of foundations of a generalized construct with the content of school or university mathematics, as well as means of intellectual support, i.e., adaptive learning system.

Description of approval stages (Figure 1):

1) creating a motivational field by means of visual modeling (presentations, video clips, simulation, business and didactic games) and motivational-applied situations that encourage integrating efforts to find a solution to the problem. Actualization of the motivational field is manifested in the expression of value and personal-adaptive characteristics of students' cognitive activities aimed at developing of standards of a generalized construct of mathematical knowledge and focus on finding research methods, as well as implementing interdisciplinary and interdisciplinary connections through the development of thinking mechanisms (induction, deduction, analogy, insight);

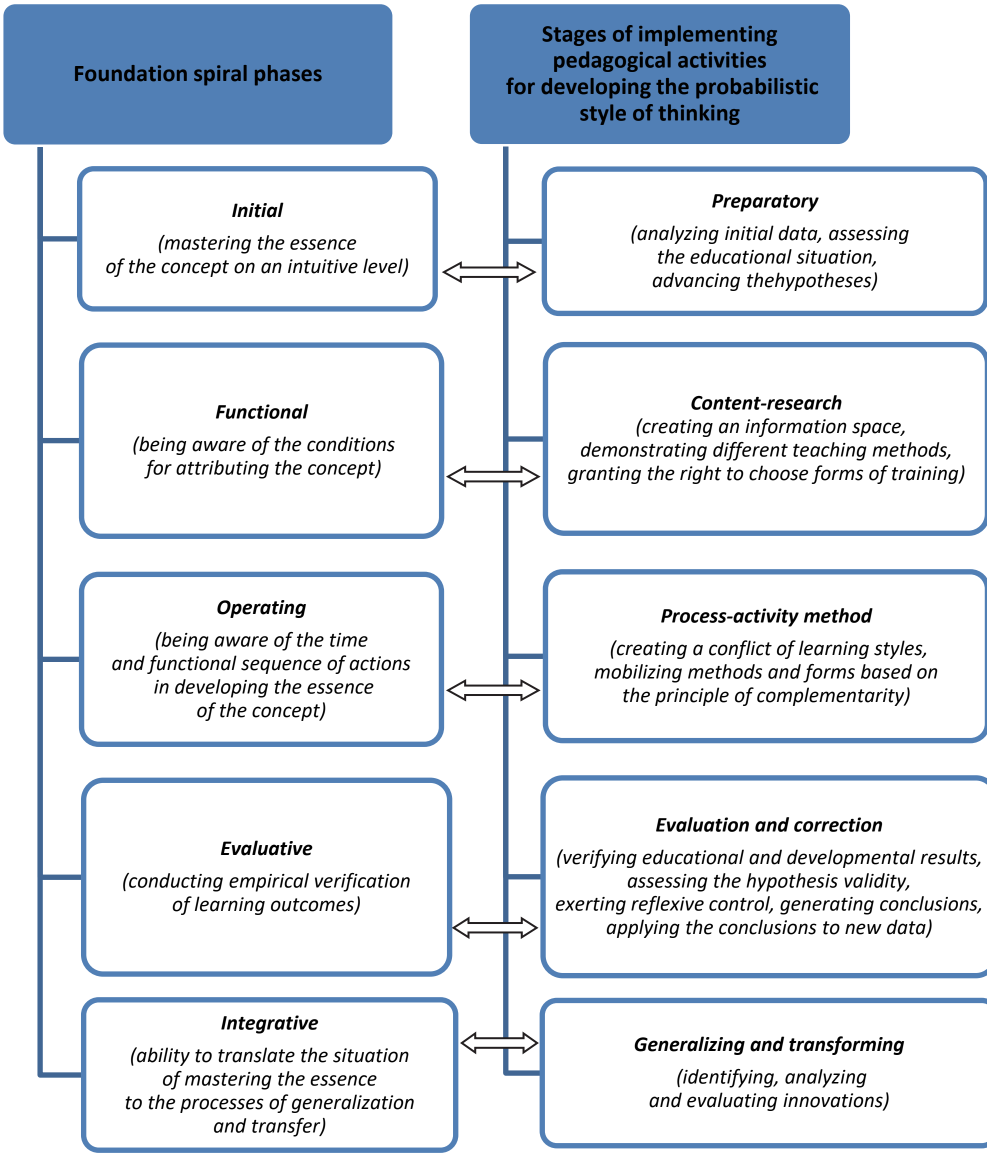

Figure 1. Coordination of the stages of developing the probabilistic style of thinking in the process of acquiring new knowledge (Dvoryatkina, Scherbatykh, 2020. P. 183)

2) creating a rich information-educational environment in implementing empirical tests and designing models of foundation procedures based on cognitive independence and actualization of thinking mechanisms (logical and critical analysis, forecasting, evaluation); designing situations of intellectual tension and self-organization for students, actualizing uncertainty, mechanisms of self-determination and self-actualization in problem situations when developing individual components of the essence of the generalized construct;

3) designing and organizing the process of learning research procedures and innovative manifestations of the essence of the generalized construct during the deployment of its foundation stages based on the actualized mechanisms of creative cognitive activity (analysis through synthesis, interaction of logical and intuitive principles, search for the unknown using heuristic techniques, combining).

The forms, methods and means of mastering the generalized construct are implemented through the integration of binary oppositions: traditional teaching methods and methods of teaching using web technologies, reproduction and search, interactive and traditional learning, active and passive perception, etc.

4) monitoring and developing diagnostic procedures for measuring the characteristics of students' personal qualities, the level of development of the probabilistic style of thinking in the course of their mastering the essence of the generalized construct. The leading thinking mechanisms at this stage are evaluating the truth of hypotheses, forecasting, evaluating the methods and procedures for finding results, and varying the conditions and data of the problem; and

5) organizing the content and transferring innovations to the mass practice of developing the school of mathematics; integrating the individual and the social in the design of an innovative synthesis of constructs; exchanging information, verifying innovations; identifying characteristics, parameters and indicators of the developed probabilistic thinking.

The fractality of mental activity determines the stages of manifestation and funding of self-actualization and self-development of the student's personality and the construction of the educational process according to the logic of personality-developing education.

To obtain guaranteed learning outcomes while solving technological problems of implementing the model, an adaptive learning system was used as a tool for developing probabilistic style of thinking in students and creating an objective means of management. The structure of the core of the adaptive system includes blocks with specified links between them: student access control; interactive information and instructional; accumulative bank of information about users; interface with the Internet; interface with administrators; graphical visualization of successful user's activities; educational and informational material; diagnostic material and artificial intelligence (Dvoryatkina et al., 2019). The use of the adaptive learning system contributes to the creation of an interactive intellectual environment, i.e., a subject-technological space that provides dynamic and effective intellectual development of the student's personality, including the probabilistic style of thinking.

The diagnostic component of the model includes the following components:

1) the characteristics of the levels of the developed probabilistic style of thinking in the process of teaching mathematics (reproductive, search, research, creative);

2) the criteria that reflect the effectiveness of activities to ensure the development of the probabilistic style of thinking in the process of teaching mathematics. This effectiveness is determined by logical, intuitive, predictive, reasoned-heuristic, critical and combinatorial criteria. A more detailed description is presented in (Dvoryatkina et el., 2019); and

3) the diagnostic methods and tools: to determine the effectiveness of the system of teaching mathematics with the effect of developing the probabilistic style of thinking, methods of mathematical statistics and the mathematical apparatus of the theory of fractals were used, as well as psychological testing was conducted using psychodiagnostic tools for diagnosing the probabilistic style of thinking (R. Amthauer's test of the structure of intelligence; S. Epstein's questionnaire of intuitive style; L.A. Regush's "predictive ability" method, P. Torrence's

creativity test; L.A. Regush's critical thinking test; L. Starkey's method for diagnosing the development of combinatorial thinking).

The software for visual assessment of the probabilistic style of thinking in the system of qualimetry of students' knowledge is based on the C++ language as a module of the diagnostic material block of the adaptive learning system. The use of the theory of fractals allowed us to correct the empirical distribution of system states at any time relative to the calculated one. The methods of correlation and factor analysis made it possible to determine the degree of influence of internal and external factors on the effectiveness of the educational process and its quality. We checked the reliable significance of individual indicators of personal development using statistical criteria.

The resulting component of the model is the final didactic product, to which all the other components of the system are oriented. As a result of the implementation of the model of the system of teaching mathematics, students are supposed to develop the probabilistic style of thinking of a high level, which provides their pre- and professional self-realization.

Empirical verification of the model

The effectiveness of the structural-functional model of teaching mathematics in schools and universities with the effect of developing the probabilistic style of thinking was based on diagnosing the level of the developed probabilistic style of thinking in schoolchildren and students using a comprehensive methodical complex, including diagnosing the established indicators of this style of thinking before and after experimental exposure. The experiment was conducted at schools of the Lipetsk and Tula regions as well as at Bunin Yelets State University. The experimental sample (n = 180) was made up of schoolchildren aged 13–17 years (n_1 = 51) and University students aged 17–22 years (n_2 = 66).

The formation (in school children) and development (in students) of the probabilistic style of thinking was diagnosed using elements of descriptive statistics of the sample mean $\bar{x}$ and standard deviation σ of comparing the indicators before and after experimental exposure in one sample) and indicators of the dynamics of development processes (growth rates), and also with the help of Student's *t*-test.

The results of statistical processing of empirical data for the entire sample showed significant differences in the output parameter "average indicator of the level of the developed probabilistic style of thinking" (Table 1, Figure 2).

Table 1

Differences in the statistical indicators of the developed probabilistic style of thinking in schoolchildren and students

Components to be diagnosed	Pupils (n_1 = 51)				Students (n_2 = 66)			
	Before ($\bar{x}$; σ)	After ($\bar{x}$; σ)	*t*	*p*	Before ($\bar{x}$; σ)	After ($\bar{x}$; σ)	*t*	*p*
Logical	(73;12)	(85;10.5)	5.37	0.000	(87; 18)	(98; 12)	4.13	0.000
Intuitive	(10;.5.2)	(12; 4.7)	2.04	0.023	(23; 4.9)	(27; 3.5)	5.40	0.000
Prognostic	(5; 3.1)	(8; 2.7)	5.31	0.000	(11; 4.3)	(14; 1.8)	5.23	0.000
Argumented-heuristic	(51; 3.2)	(58;1.9)	13.43	0.000	(79; 4.2)	(81; 3.2)	3.08	0.002
Critical	(3.8; 2.4)	(5,4; 1.9)	3.73	0.000	(19; 3.2)	(23.8; 2.5)	9.60	0.000
Combinatorial	(12; 5.8)	(17; 2.6)	5.62	0.000	(28; 8.5)	(43; 7.8)	10.56	0.000

The overall average was $\bar{X}_1 = 25.8$ and $\bar{X}_2 = 30.9$ for the sample of school children and $\bar{Y}_1 = 29.67$ and $\bar{Y}_2 = 47.67$ for the selection of students. There was a significant difference in the group of students and a slight difference in the group of schoolchildren.

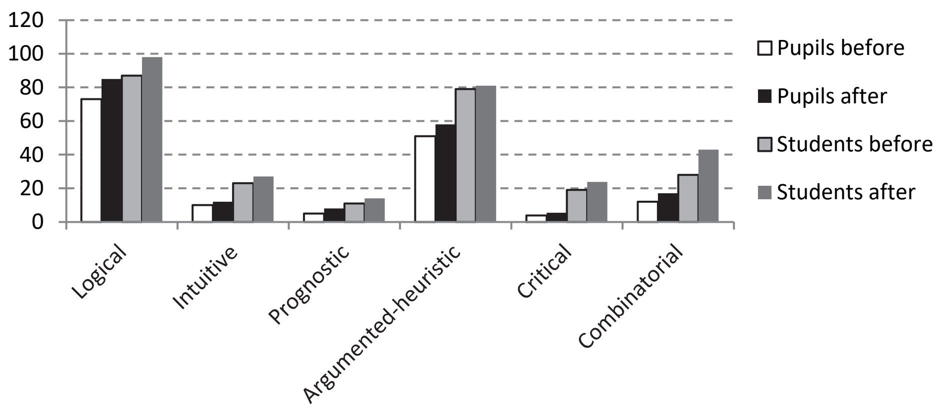

Figure 2. Graphical representation of differences in the statistical indicators of the developed probabilistic style of thinking

In general, we can note a slight positive change in the average indicators for all the studied components of the probabilistic style of thinking. The expected result is that the intuitive and combinatorial distributions between representatives at different stages of pre-professional and professional development are linearly approximated with positive coefficients, while the logical and reasoned-heuristic distributions are a normal histogram fitting function. The authors tend to interpret the results as confirming the effectiveness of the structural-functional model of teaching mathematics in schools and universities with the effect of developing the probabilistic style of thinking in schoolchildren and students.

The dynamics of changes in the process of developing the probabilistic style of thinking before and after experimental exposure are determined using Student's *t*-test. The empirical values for the group of schoolchildren are shown in Table 1, and the critical area is formed by the right-hand interval (2.01; +∞) for the significance level of 0.05. Since all the obtained values fall into the critical region, the null hypothesis about the equality of group averages (before and after experimental exposure) is rejected. The empirical values for the group of students are indicated in the last column of Table 1, the critical area is formed by the right-hand interval (1.9; +∞) for the significance level of 0.05. It can be seen that all the calculated values also exceed the critical one; therefore, there are no grounds for accepting the hypothesis of the equality of group averages. Thus, we have found a statistically significant influence of the implemented model on the positive change in the average indicators for all the studied components of the probabilistic style of thinking developed in schoolchildren and students.

The dynamics of changes in the development of the probabilistic style of thinking are calculated using the growth rate indicators (Table 2): $T_p = \frac{y_i}{y_0} 100\%$.

Table 2

Differences in the development rates of the probabilistic style of thinking

Components to be diagnosed	Pupils (n_1 = 51)		T_p, %	Students (n_2 = 66)		T_p, %
	Before	After		Before	After	
Logical	73	85	116.44	87	98	112.64
Intuitive	10	12	120.00	23	27	117.39
Prognostic	5	8	160.00	11	14	127.27
Reasoned-heuristic	51	58	113.73	79	81	102.53
Critical	3.8	5.4	142.11	19	23.8	125.26
Combinatorial	12	17	141.67	28	43	153.57

The positive dynamics of changes in the level of the developed probabilistic style of thinking characterizes the integral development of the conscious and intuitive spheres, i.e., the dominant components of the probabilistic style of thinking with resonant responses to the intensification of mental activity in the form of reasoned-heuristic, predictive, critical and combinatorial components that integrate and expand intuitive and abstract logical abilities. The dynamics of the predictive, critical, and combinatorial components is most pronounced, while the weakest dynamics is observed in the reasoned-heuristic component.

Conclusion

In conclusion, it should be noted that the relationship between the components of the projected conceptual model clearly demonstrates the prospects for modernizing the learning process, namely:

– increasing the role of educational potential (system of general scientific, professional, special and humanitarian knowledge);

– activating creative and intellectual potential (processes of self-actualization and self-realization);

– motivating students to achieve high levels of general and professional culture of students;

– developing meta-subject, universal and general professional competences for the effective use of personal development resources in the most promising areas of activity.

The prospect of the research is further intellectualization of the technological component of the model based on the hybridization of artificial intelligence methods to ensure the effective development of the probabilistic style of thinking with rapid changes in parameter values according to the set feedbacks.

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Теоретическая статья

Модель обучения математике с эффектом развития вероятностного стиля мышления в цифровой образовательной среде: теоретическое обоснование и эмпирическая проверка

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Аннотация. Рассматривается проблема развития нелинейного, вероятностного стиля мышления обучаемых в условиях цифровой трансформации современного образования. Целью является разработка эффективного научно-методологического инструментария организации процесса обучения математике в школе и вузе для достижения высоких педагогических, социально-востребованных результатов и создания на его основе усовершенствованной дидактической модели с эффектом развития вероятностного стиля мышления, гибкости, креативности и критичности личности для возможности ее активного участия в жизнедеятельности общества. Модель обучения математике с эффектом развития вероятностного стиля мышления разработана в единстве целевого, теоретико-методологического, содержательного, технологического, диагностического и результативного компонентов. Структурообразующим фактором выступает информационно-насыщенная образовательная среда обучения математике как совокупность цифрового информационного и образовательного контента, способствующая эффективному развитию вероятностного стиля мышления. Содержательный компонент модели реализуется в отборе и структурировании учебного материала на основе фрактального подхода, в методическом обновлении комплексом спиралей фундирования, оснащенного банками прикладных и исследовательских задач с учетом глубины фрактального представления учебных элементов. При решении технологических задач реализации модели, получения гарантируемых результатов обучения в качестве инструмента развития вероятностного стиля мышления обучаемых и объективного средства управления выступала адаптивная обучающая система. Результаты внедрения модели в практику обучения с последующей статистической проверкой на основе методов описательной статистики, *t*-критерия Стьюдента установили положительную динамику по всем структурным компонентам модели с доверительной вероятностью 95 %. Перспектива исследования состоит в дальнейшей интеллектуализации технологического компонента модели на основе гибридности методов искусственного интеллекта для обеспечения эффективного развития вероятностного стиля мышления с быстрой динамикой изменения значений параметров, согласно заданным обратным связям.

Ключевые слова: математическое образование, модель обучения, синтез подходов, вероятностный стиль мышления, система принципов, инновационное содержание, адаптивная обучающая система

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