МАТЕМАТИЧЕСКАЯ ЛОГИКА

COMBINED PARACONSISTENT LOGICS AND THEIR (CO)EXPONENTIALS

Vladimir L. Vasyukov

Institute of Philosophy, Russian Academy of Science
Volkhonka Str., 14, Moscow, Russia, 119991

Combined logics of sentences and events consist of two parts: the external logic depending on epistemological assumptions and the internal logic depending on ontological ones. They were introduced by V.A. Smirnov following some G. Frege's and N. Vasiliev's ideas. An analysis of the structure of combined logics shows that, in fact, they employs the links between two logical systems postulating in the role of ontological part algebras which serve as the models of respective logics. It prompts us to consider systems which describe the direct interplay of two logics on syntactic level where we have an access to these logics without mediation of their models. In the role of those systems might be used the so-called coexponential and exponential ones which were introduced in [15]. In the paper the case of two paraconsistent combined logics (having paraconsistent algebras as their ontological part) is considered.

Key words: combined lopgics, Jaskowski's discursive logic, da Costa paraconsistent logics, translation, (un)constrained possible translating

1. Introduction

The combined logic introduced by V.A. Smirnov essentially exploits some G. Frege's and N.A. Vasiliev's ideas. Since Vasiliev distinguished two levels in logic then combined logic consists of two parts: the abstract (external) logic and the empirical (internal) logic. The former depends on epistemological assumptions while ontological ones determine the latter. Such an approach becomes more transparent if we patently discern acts of assertion (the relation of mental content with the way things are) and acts of predication (the synthesis of a property with the object). Following this course we, in effect, maintain Frege's differentiation of mental process (*Gedanke*) and assertion statement (*Urteil*). In order to emphasize it Frege even introduces the special sign: "...we need a special sign to assert that something or other is true. For this purpose I write the sign '$\vdash$' before the name of the truth-value, so that in '$\vdash 2^2 = 4$' it is asserted that the square of 2 is 4. I make a distinction between *judgment* and *thought*, and understand by *judgment* the acknowledgement of the truth of a *thought*" [3. P. 156].

Being inspired by these ideas V.A. Smirnov introduces several combined calculi of sentences and events (cf. [9; 10; 11; 12]) when both external and internal logic are subjected to change. The language of those calculi includes two sorts of variables: event variables (terms) and propositional ones. If a and b are terms then $a \cup b$, $a \cap b$, $\sim a$ will be also terms (complex events) while θa, θb are the formulas along with the formulas $\theta a \vee \theta b$, $\theta a \wedge \theta b$, $\neg a$. Clearly, postulating some equivalencies like $\theta(a \cup b) \equiv \theta a \vee \theta b$, $\theta(a \cap b) \equiv \theta a \wedge \theta b$ etc. we arrive at different combination of algebras of events and propositional calculi in the framework of one logic.

Meanwhile, if we bear in mind that N.A. Vasiliev assumed inconsistency on ontological level, but denied it on logical one, then it would be desirable to pursue this program for combined logic too. One of proposals in this case consists in approaching algebra of events as the discursive system which notion goes back to S. Jaśkowski. In his seminal paper "*Propositional Calculus for Contradictory Deductive Systems*" S. Jaśkowski [4] offers a system of discursive logic by adding to *S*5 modal system a conditional $\rightarrow$ (often written as $\supset d$ and called discursive implication) and defining $\alpha \rightarrow \beta$ as $\Diamond\alpha \rightarrow \beta$. The logical truths of the pure $\rightarrow$ fragment of discursive logic are the same as those of the pure $\supset$ fragment of classical logic but unlike of the latter $\models \alpha \rightarrow (\neg\alpha \rightarrow \beta)$ fails, since $\models_{S5} \Diamond(\Diamond\alpha \supset (\Diamond\neg\alpha \supset \beta))$ fails too.

Approaching algebra of events as *S*5-modal algebra we are in position to cope with contradictory character of ontological level by introducing counterpart of Jaśkowski's type conditional in algebra of events and then θ-translating it into sentential calculus. The only question arising now is the nature of the possible event. What does it mean intuitively? Are there any mechanisms allowing to separate real events from possible? Or there are some criteria for dividing events into possible and real one?

The proposal consisting in exploitation of the notion of ontological modality would be the remedy we search for. Its basic idea can be expressed by means of the "*making possible*" modal operator $\boldsymbol{MP}(x,y) \leftrightarrow y \in \sigma(x)$ (x makes possible y iff y is synthetizable from x) [7]. Hence, we may treat the possible event ontologically by (i) purporting possibility as the case when a relation between some event and possible event take place and (ii) identifying with this relation the relation "*making possible*" (usually such relations are called "makers"). Thus, in a sense, one can consider possible events as "ontologically generated" by some other events.

But there is also another, more popular in logical semantics, opportunity of event treatment. In this case we assign to every event the non-empty set of possible worlds in which this event comes about. In fact, such an approach purports the exploitation of the usual technique of modal semantics and as consequence we arrive at the possible world semantic frame where the accessibility relation must be taken into account. Again we can treat accessibility relation as "*making possible*" relation: since some possible worlds are accessible from another ones then the collection of the later could be observed as event and thus determines the former as event too. Indeed, under such treatment the former would be acknowledged as the "possible" event.

Finally, there is another way to fulfill Vasiliev's program. Instead of *S5*-algebra in a role of event-ontology in this case more elaborated paraconsistent theories should be adopted describing different cases and models of paraconsistency. Following this course of consideration one can accept da Costa algebra [2] reflected the most of logical properties of da Costa systems C_n as internal logic in assumed combined system. In this case the resulting system of combined logic also would be inconsistent (paraconsistent) on ontological level but consistent on logical.

An analysis of the structure of combined logics shows that, in fact, they employs the links between two logical systems postulating in the role of ontological part algebras which serve as the models of respective logics. In case of Jaśkowski—Vasiliev combined discursive system in this role we have S5-algera which is the algebraic model of S5-modal system, in case of da Costa combined logic da Costa algebra is employed which is an algebraic model of paraconsistent da Costa logic. It prompts us to consider systems which describe the interplay of two logics on syntactic level where we have a direct access to these logics without mediation of their models.

In the role of those systems might be used the so-called coexponential and exponential ones which were introduced in [15]. In a nutshell they would be described in the following way.

Let L_1 and L_2 be logical systems. Then a *coexponential* (or the unconstraint possible translating) of L_2 into L_1 is a system $L_{1\Leftarrow 2}$ in which we have $\Gamma\vdash_{1\Leftarrow 2}\phi$ iff $g[\Gamma]\vdash_2 g(\phi)$ for all translations g:$\mathrm{L}_1\to\mathrm{L}_2$. An *exponential* (or the constraint possible translating) of L_2 into L_1 is a system $L_{2\Rightarrow 1}$ in which $\Gamma\vdash_{2\Rightarrow 1}\phi$ iff there exist translations h:$L_2\to L_1$ and g:$L_1\to L_2$ such that $h(g[\Gamma]))\vdash_1 h(g(\phi))$.

2. A System JVCD of Jaśkowski—Vasiliev Combined Discursive Logics

The language of the JVCD-system of Jaśkowski—Vasiliev Combined Discursive Logics can be described as follows. Let *p, q*, ... be event's variables and we assume (as in [9]) that event's variables make terms. If *a* and *b* are terms, then $a\cap b$, $a\cup b$, $\sim a$ are the terms as well. If *a* is a term, then θa is a formula; if α and β are formulas, then $\alpha\vee\beta$, $\alpha\wedge\beta$, $\alpha\supset\beta$, $\neg\alpha$ are formulas too. It is forbidden to mix terms and formulas. Hence, for example, expressions of the form $\theta p\supset q$, $a\cap\beta$, $\theta a\cap b$ would be neither term, nor formula: it is simply non-well-formed expression. Let event variables be defined as above but we also have that $\Diamond a$ will be the term too. In his paper from 1948 S. Jaśkowski defined a system D_2 of discursive logic as follows: "The system D_2 *of the two-valued discursive sentential calculus* is the set of formulae *T*, termed the theses of the system D_2 and marked by the following properties:

1) *T* includes sentential variables and at the moment the following functions: $\to$, $\leftrightarrow$, $\vee$, $\wedge$, $\neg$.

2) Preceding *T* with the symbol $\Diamond$ yields a theorem in the two-valued sentential calculus of modal sentences M_2" [5. P. 150—151].

He proved also the following methodological theorems:

Methodological theorem 1 [5. P. 151]. Every thesis T in the two-valued sentential calculi L_2, which does not include constant symbol other then $\supset$, $\equiv$, $\vee$, becomes a thesis Td in the discursive sentential calculi D_2 when in T the implication symbols $\supset$ are replaced by $\to$, and the equivalence symbols $\equiv$ are replaced by $\leftrightarrow$.

Methodological theorem 2 [5. P. 152]. If T is a thesis in the two-valued sentential calculus L_2 and includes variables and at the most functors $\vee$, $\wedge$, $\neg$, then

1) T
2) $\neg T \to q$;

are theses in D_2.

Methodological theorem 3 [5. P. 153]. If in a thesis that belongs to the discursive sentential calculus D_2 $\to$ is replaced by $\supset$, and $\leftrightarrow$ by $\equiv$, a thesis belonging to the sentential calculus L_2 is obtained.

In order to explicate those theorems in the system of combined logic we add to the axiom schemata of classical sentential logic and the rule *modus ponens* the following schemes:

A1. $\theta a \vee \theta b \equiv \theta(a \cup b)$
A2. $\neg\theta a \equiv \theta(\sim a)$
B1. $\theta(\Diamond(a \cup b)) \equiv \theta(\Diamond a) \vee \theta(\Diamond b)$
B2. $\theta a \supset \theta(\Diamond a)$
B3. $\theta(\Diamond\Diamond a) \supset \theta a$
B4. $\theta(\Diamond a) \supset \theta(\sim\Diamond\sim\Diamond a)$

Let us hereafter $a \to b$ means $\sim\Diamond a \cup b$, $a \leftrightarrow b$ means $(\sim\Diamond a \cup b) \cap (\sim\Diamond a \cup \Diamond b)$. It easily can be seen that the axioms A1—A2 provide us with a Boolean algebra structure of the set of events and the following theses will take place:

A3. $\theta a \wedge \theta b \equiv \theta(a \cap b)$
B5. $\theta(a \to b) \supset (\theta a \supset \theta b)$
B6. $\theta(a \leftrightarrow b) \supset (\theta a \equiv \theta b)$

Let us denote $\theta(\Diamond a)$ as $\delta(a)$. As it may be easily checked, $\Diamond(\Diamond\alpha \supset \beta)$ is *S5*-logically equivalent to $(\Diamond\alpha \supset \Diamond\beta)$ which leads to $\Diamond(\sim\Diamond a \vee b) = \sim\Diamond a \vee \Diamond b$ as it algebraic counterpart. Since "$\to$" and "$\leftrightarrow$" are, in effect, algebraic counterparts of the discussive implication and discussive equivalence respectively, then we are entitled to introduce an algebraic counterpart "$\cap_d$" of discussive conjunction $\Diamond\alpha \wedge \beta$ ($a \cap_d b$ means $\Diamond a \cap b$) and an algebraic counterpart "∇" of discussive negation $\neg\Diamond\alpha$ (∇a means $\sim\Diamond a$). All this operators would be together characterised with the help of the following theses:

D1. $\delta(a \cup b) \equiv \delta(a) \vee \delta(b)$
D2. $\delta(a \cap_d b) \equiv \delta(a) \wedge \delta(b)$
D3. $\delta(\nabla a) \equiv \neg\delta a$
D4. $\delta(a \to b) \supset (\delta a \supset \delta b)$
D5. $\delta(a \leftrightarrow b) \equiv (\delta a \equiv \delta b)$

It is easy to check by direct computation that $\to$, $\cap_d$, $\cup$, $\leftrightarrow$, ∇ possess all the properties of Boolean algebra operations $\supset$, $\wedge$, $\vee$, $\neg$ respectively. Obviously, the set of all δ-formulas will be closed under the rule of discussive *modus ponens*:

$$(\delta\text{MP})\ \frac{\delta a \quad \delta(a \to b)}{\delta b}.$$

For the time being it seems that everything is going well. But detailed analysis quickly shows that point 1) of Jaśkowski's methodological theorem 2 was left beyond the scope of our considerations. We know how to obtain a discussive formula from the event and meanwhile we still have no idea of the correlation between usual formula and respective event. But this is precisely the condition we ought to satisfy in our interpretation according to the sense of Jaśkowski's methodological theorem 2.

In order to bypass those difficulties arisen we borrow some notion from Smirnov's General Logic of Sentences and Events in [11]. There is an operator [–] in the language of this calculus such that if α is a formula then $[\alpha]$ is a sentential term. Loosely speaking, we relate with an every formula the respective event (e.g. as her referent). By means of such an operator we enrich our system with the axioms:

B7. $\theta[\alpha] \equiv \alpha$

B8. $\theta[\alpha \vee \beta] \equiv \theta([\alpha] \cup [\beta])$

B9. $\theta[\neg\alpha] \equiv \theta({\sim}[\alpha])$

B10. $\alpha \supset \theta({\sim}[\alpha] \to b)$,

where b is an arbitrary event in the event algebra. Those give us an explication of Jaśkowski's methodological theorem 3 we search for. It easy to see that we obtain

$\alpha \supset \delta[\alpha]$

$\alpha \supset (\delta({\sim}[\alpha]) \supset \delta b)$

as the corollaries of the newly introduced axioms.

It seems that an idea of the [–]-operator could be trace back to the J. Słupecki's idea from [8]. He proposed, namely, to enrich the language of modal logic with the expressions $p{*}x$ which might be read as follows:

(1) saying that p, we state (the event) x;

(2) sentence p states the event x.

Following this course we can read the expressions $[\alpha]$ as "saying that α we state (the event) $[\alpha]$" or "sentence α state the event $[\alpha]$".

If we would consider formula θa as, in a sense, a description of the event a and δa as a discussive description of the event a then following N. C. A. da Costa we can define a discussive theory $\boldsymbol{T}$ of event interpretation in case when the following conditions are satisfied:

(1) If a is an event, then $\delta a \in \boldsymbol{T}$;

(2) $\boldsymbol{T}$ is closed under discussive detachment: if $\delta a \in \boldsymbol{T}$ and $\delta(a \to b) \in \boldsymbol{T}$, then $\delta(b) \in \boldsymbol{T}$.

Discussive theories seem to be reflecting the characteristic marks of Jaśkowski's discursive system while describing or treating some set of events. According to Jaśkowski "a system which cannot be said to include theses that express opinion in agreement with one another, be termed a *discursive system*... if a thesis A is recorded in a discursive system, its intuitive sense ought to be interpreted so as if it were preceded by the symbol *Pos*, that is, the sense 'it is possible that A'. This is how an impartial arbiter might understand the theses of the various participants in the discussion" [4. P. 149].

We must take now into account that the cases of θ- and δ-operators are of a different nature relatively to the notion of inconsistency. In fact, we need to distinguish external and internal inconsistency where the former notion is the usual one due to the classical character of our external logic (which is would not be the rule). There are any peculiarities concerning θa and $\theta(\sim a)$ formulas because of the A2 axiom we have $\neg\theta a$ instead of $\theta(\sim a)$ and everything is going on as usually. But in case of δa and $\delta(\sim a)$ situation is rather different.

Let Γ be a set of formulas. (Γ) denotes the least theory, containing all elements of Γ. Then the following proposition will be true:

Proposition 1. There exist internally-inconsistent discussive theories of event interpretation which are not over-complete (i.e. if **T** is such a theory then it is not always the case that **T** = F where F is the set of all formulas).

Proof. If $\Gamma = \{\delta a, \delta(\sim a)\}$, then Γ is internally-inconsistent but not over-complete, since $\delta(a \cap_d \sim a \rightarrow b)$ is not a thesis of $\boldsymbol{T}$, where b is any event distinct from a. ■

Hereafter ■ means the end of the proof.

In algebraic way semantics of *JVCD*-logic might be obtained approaching propositions and events as two different kind of entities. Then an algebraic *JVCD*-bundle will be a 4-tuple $\langle \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{f}, \boldsymbol{g} \rangle$ where $\boldsymbol{A} = \langle A, +, - \rangle$ is a Boolean algebra (A contains two elements at least), $\boldsymbol{B} = \langle B, \oplus, ', \bullet \rangle$ is an *S5*-algebra, $\boldsymbol{f}: \boldsymbol{B} \rightarrow \boldsymbol{A}$, $\boldsymbol{g}: \boldsymbol{A} \rightarrow \boldsymbol{B}$ are embedding functions. Let 0, 1, $\circ$ and $\leq$ be in both algebras defined as usual. For $\boldsymbol{f}$ and $\boldsymbol{g}$ the following conditions are fulfilled:

$\boldsymbol{f}(k \oplus l) = \boldsymbol{f}(k) + \boldsymbol{f}(l)$,

$\boldsymbol{f}(k') = -\boldsymbol{f}(k)$,

$\boldsymbol{f}(\boldsymbol{g}(x)) = x$,

$\boldsymbol{g}(x + y) = \boldsymbol{g}(x) \oplus \boldsymbol{g}(y)$,

$\boldsymbol{g}(-x) = \boldsymbol{g}(x)'$,

$x \leq \boldsymbol{f}((\bullet \boldsymbol{g}(x)')' \oplus y)$,

where $x, y \in A$ and $k, l \in B$.

If F is a set of well-formed formulas and E is a set of events then a valuation $\boldsymbol{v}$ is defined by:

$\boldsymbol{v}: F \bigcup E \rightarrow A \bigcup B$,

$\boldsymbol{v}(\alpha \vee \beta) = \boldsymbol{v}(\alpha) + \boldsymbol{v}(\beta)$,

$\boldsymbol{v}(\neg\alpha) = -\boldsymbol{v}(\alpha)$

(where α, β are wff and $\boldsymbol{v}(\alpha), \boldsymbol{v}(\beta) \in A$),

$\boldsymbol{v}(a \cup b) = \boldsymbol{v}(a) \oplus \boldsymbol{v}(b)$,

$\boldsymbol{v}(\sim a) = \boldsymbol{v}(a)'$,

$\boldsymbol{v}(\Diamond a) = \bullet \boldsymbol{v}(a)$,

$\boldsymbol{v}(\theta a) = \boldsymbol{f}(\boldsymbol{v}(a))$,

$\boldsymbol{v}([\alpha]) = \boldsymbol{g}(\boldsymbol{v}(\alpha))$

(where a, b are events and $\boldsymbol{v}(a), \boldsymbol{v}(b) \in B$).

Theorem 2. Axioms PC + (A1 – A2, B1 – B4, B7 – B10) are valid in any ⟨**A**, **B**, **f**, **g**⟩-bundle.

Proof is straightforward ■.

The sense of exploiting the notion of a JVCD-bundle becomes more transparent in case when in role of $\boldsymbol{B}$ we take a modal algebra $\boldsymbol{B}^+ = \langle M, \cup, \cap, —, \Diamond \rangle$ where

(i) M = $\boldsymbol{P}(W)$ (a set of all subsets of W);

(ii) $\cup, \cap,$— are set-theoretical join, meet and complementation in M;

(iii) for $A \in M$ $\Diamond A = \{x: \exists y(y \in A \text{ and } x \leq y)\}$

if we use the standard Lemmon's method of obtaining modal algebra for a frame (cf. [Lemmon 1966]). On the one hand this construction of JVCD-bundle $\langle \boldsymbol{A}, \boldsymbol{B}^{+}, \boldsymbol{f}, \boldsymbol{g}\rangle$ preserves our treatment of event as a set of possible worlds and from the other hand it shows the "mechanism" of fibre bundles in combined logics semantics.

3. Coexponentials and Exponentials of JVCD

In a standard way we can also introduce a valuation $\boldsymbol{v}_{S5}$: $F_{S5} \to \boldsymbol{B}$ for *S5*-logic by means of the following definitions:

$\boldsymbol{v}_{S5}(\alpha\vee\beta) = \boldsymbol{v}_{S5}(a) \oplus \boldsymbol{v}_{S5}(b)$,

$\boldsymbol{v}_{S5}(\alpha\wedge\beta) = \boldsymbol{v}_{S5}(a) \otimes \boldsymbol{v}_{S5}(b)$,

$\boldsymbol{v}_{S5}(\neg\alpha) = (\boldsymbol{v}_{S5}(a))'$,

$\boldsymbol{v}_{S5}(\Diamond\alpha) = \bullet\ \boldsymbol{v}_{S5}(\alpha)$,

where α, β are wff of *S5*-logic and $\boldsymbol{v}_{S5}(\alpha)$, $\boldsymbol{v}_{S5}(\beta) \in \boldsymbol{B}$.

If we split *JVCD*-valuation $\boldsymbol{v}$ into a valuation $\boldsymbol{v}^{f}_{JVCD}$ which is defined on formulas and a valuation $\boldsymbol{v}^{e}_{JVCD}$ which is defined on events then this gives us the following diagram of valuations and embeddings:

$$\begin{tikzcd}
S5 \arrow[r, shift left, "\boldsymbol{h}"] \arrow[d, "\boldsymbol{v}_{S5}"'] & JVCD \arrow[l, shift left, "\boldsymbol{i}"] \arrow[dl, "\boldsymbol{v}^{e}_{JVCD}"'] \arrow[d, "\boldsymbol{v}^{f}_{JVCD}"] \\
\boldsymbol{B} \arrow[r, shift left, "\boldsymbol{g}"] & \boldsymbol{A} \arrow[l, shift left, "\boldsymbol{f}"]
\end{tikzcd}$$

where $\boldsymbol{h}$ is a translation $\boldsymbol{h}$: *JVCD* $\to$ *S5* defined by the conditions

$\boldsymbol{h}(a) = p_a$,

$\boldsymbol{h}(p_i) = p_i$,

$\boldsymbol{h}(\alpha\vee\beta) = \boldsymbol{h}(\alpha) \vee \boldsymbol{h}(\beta)$,

$\boldsymbol{h}(\neg\alpha) = \neg\ \boldsymbol{h}(\alpha)$

$\boldsymbol{h}(a\cup b) = \boldsymbol{h}(a) \vee \boldsymbol{h}(b)$,

$\boldsymbol{h}(\sim a) = \neg\ \boldsymbol{h}(a)$,

$\boldsymbol{h}(\Diamond a) = \Diamond_{S5}\ \boldsymbol{h}(a)$,

$\boldsymbol{h}(\theta a) = \boldsymbol{h}(a)$,

$\boldsymbol{h}([\alpha]) = \boldsymbol{h}(\alpha)$

and $\boldsymbol{i}$ is a translation $\boldsymbol{i}$: *S5* $\to$ *JVCD* defined by the condition $\boldsymbol{i}(\alpha) = \theta(\boldsymbol{v}^{*}_{S5}(\alpha))$ where $\boldsymbol{v}^{*}_{S5}(\alpha)$ means substitution for *S5*-operators respective events-operators of *JVCD* i.e. we replace $\oplus$, $'$, $\bullet$ with $\cup$, $\sim$, $\Diamond$ respectively. Now it is easy to define $\Gamma\vDash_{JVCD}\alpha$ and $\Gamma\vDash_{S5}\alpha$ iff for any $\beta \in \Gamma$ and an every valuation $\boldsymbol{v}_{JVCD}$ $(= \boldsymbol{v}^{f}_{JVCD} \cup \boldsymbol{v}^{e}_{JVCD})$ and $\boldsymbol{v}_{S5}$ we have $\boldsymbol{v}_{JVCD}(\beta) \le \boldsymbol{v}_{JVCD}(\alpha)$ and $\boldsymbol{v}_{S5}(\beta) \le \boldsymbol{v}_{S5}(\alpha)$ respectively.

Such a construction allows us develop a possible translating semantics (PST) of *JVCD* according to the general scheme of PST elaborated by W. Carnielli in [1]. In this case we define a local forcing relation $\vDash^{\boldsymbol{i}}_{JVCD}$ for *JVCD* (in respect to $\boldsymbol{h}$) as

$$\Gamma \vDash^{\boldsymbol{h}}_{JVCD} \alpha \text{ iff } \boldsymbol{h}[\Gamma] \vDash_{S5} \boldsymbol{h}(\alpha)$$

for every set $\Gamma \cup \{\alpha\}$ of formulas of *JVCD*. Generalizing we get a definition:

$\Gamma \vDash_{JVCD} \alpha$ iff for any translation $\boldsymbol{h}$ it is the case that $\Gamma \vDash^{\boldsymbol{h}}_{JVCD} \alpha$.

Since $\boldsymbol{i}(\alpha) = \theta(\boldsymbol{v}^*_{S5}(\alpha))$ then substituting we analogously obtain $\Gamma \vDash^{vS5}{}_{S5}\alpha$ iff $\theta(\boldsymbol{v}^*_{S5}[\Gamma]) \vDash_{JVCD} \theta(\boldsymbol{v}^*_{S5}(\alpha))$ and $\Gamma \vDash_{S5}\alpha$ iff for any valuation $\boldsymbol{v}_{S5}$ we have $\theta(\boldsymbol{v}^*_{S5}[\Gamma]) \vDash_{JVCD} \theta(\boldsymbol{v}^*_{S5}(\alpha))$. In essence, the role of θ-operator would be depicted on the diagram above with the help of the function $\boldsymbol{j}\colon \boldsymbol{B} \to JVCD$, i.e. as

$$\begin{tikzcd}
S5 \arrow[r, shift left, "\boldsymbol{h}"] \arrow[d, "\boldsymbol{v}_{S5}"'] & JVCD \arrow[l, shift left, "\boldsymbol{i}"] \arrow[d, "\boldsymbol{v}^f_{JVCD}"] \arrow[dl, shift left, "\boldsymbol{v}^e_{JVCD}"] \\
\boldsymbol{B} \arrow[r, shift left, "\boldsymbol{g}"] \arrow[ur, shift left, "\boldsymbol{J}"] & \boldsymbol{A} \arrow[l, shift left, "\boldsymbol{f}"]
\end{tikzcd}$$

and we obtain $\boldsymbol{i}(\alpha) = \boldsymbol{j}(\boldsymbol{v}_{S5}(\alpha))$ where

$\boldsymbol{j}(a) = \theta a$
$\boldsymbol{j}(a \oplus b) = \boldsymbol{j}(a) \vee \boldsymbol{j}(b)$
$\boldsymbol{j}(a') = \neg \boldsymbol{j}(a)$
$\boldsymbol{j}(\bullet a) = \boldsymbol{j}(\Diamond a)$

and respectively modified condition:

$\Gamma \vDash_{S5} \alpha$ iff for any valuation $\boldsymbol{v}_{S5}$ we have $\boldsymbol{j}(\boldsymbol{v}_{S5}[\Gamma]) \vDash_{JVCD} \boldsymbol{j}(\boldsymbol{v}_{S5}(\alpha))$.

What will take place if we replace $\vDash$ with $\vdash$? In this case we can define local consequence relation $\vdash^{\boldsymbol{h}}{}_{JVCD}$ for *JVCD* (relative to $\boldsymbol{h}$) as

$$\Gamma \vdash^{\boldsymbol{h}}_{JVCD} \alpha \text{ iff } \boldsymbol{h}[\Gamma] \vdash_{S5} \boldsymbol{h}(\alpha),$$

and we define consequence relation on *S5* via JVCD as

$\Gamma \vdash_{JVCD} \alpha$ iff for any translation $\boldsymbol{h}$ it is the case that $\Gamma \vdash^{\boldsymbol{h}}_{JVCD} \alpha$.

In fact, our definition of $\Gamma \vdash_{JVCD} \alpha$ gives us the construction of coexponential of *JVCD* to *S5* which would be denoted $JVCD \Leftarrow S5$ since the last is the system in which the consequence relation is determinate by the consequence relation of *S5* depending of all translations from *JVCD* to *S5*.

If as above $\boldsymbol{i}(\alpha) = \boldsymbol{j}(\boldsymbol{v}_{S5}(\alpha))$ then every translation of $\boldsymbol{i}$-type will be determined by the respective $\boldsymbol{v}_{S5}$, i.e. $\Gamma \vdash^{\boldsymbol{v}}_{S5} S5\alpha$ iff $\boldsymbol{j}(\boldsymbol{v}_{S5}[\Gamma]) \vdash_{JVCD} \boldsymbol{j}(\boldsymbol{v}_{S5}(\alpha))$ and $\Gamma \vdash_{S5} \alpha$ iff for any valuation $\boldsymbol{v}_{S5}$ we have $\boldsymbol{j}(\boldsymbol{v}_{S5}[\Gamma]) \vdash_{JVCD} \boldsymbol{j}(\boldsymbol{v}_{S5}(\alpha))$. In this case we deal with the coexponential $S5 \Leftarrow JVCD$ of *S5* to *JVCD* where translations $\boldsymbol{i}\colon S5 \to JVCD$ are defined via $\boldsymbol{v}_{S5}$.

But if we replace *JVCD* with *PC* then the situation immediately changes: $\boldsymbol{i}$ should not now depends on θ-operator and a valuation $\boldsymbol{v}_{S5}$ and in this case instead of $\Gamma \vdash_{S5} \alpha$ we deal with $\Gamma \vdash_{S5 \Leftarrow PC} \alpha$ where $S5 \Leftarrow PC$ (*coexponential* of *S5* to *PC*) is a system in which $\Gamma \vdash_{S5 \Leftarrow PC} \alpha$ iff for any translation $\boldsymbol{i}$: $S5 \to PC$ it is the case that $\Gamma \vdash^{\boldsymbol{i}}_{S5 \Leftarrow PC} \alpha$ i.e. iff for any $\boldsymbol{i}$: $S5 \to PC$ we have $\boldsymbol{i}[\Gamma] \vdash_{PC} \boldsymbol{i}(\alpha)$.

The only problem is that since we did not impose constraints on different $\boldsymbol{i}$ then it might be the case that $\boldsymbol{i}_1(\alpha) = \boldsymbol{i}_2(\beta)$ i.e. $\Gamma \vdash_{S5 \Leftarrow PC} \alpha$ and $\Gamma \vdash_{S5 \Leftarrow PC} \beta$ would be determined by one and the same *PC*-formula. If we would like to have more precise definition of the consequence relation by means of translation then it seems promising to employ analogous to $\boldsymbol{g}$ translation $\boldsymbol{k}$: $PC \to S5$ and characterize a system $PC \Rightarrow S5$ (of *constraint possible translating* of *PC* to *S5* or *exponential* of *PC* to *S5*) with the help of condition $\Gamma \vdash PC \Rightarrow S5 \alpha$ iff there are translations $\boldsymbol{i}$: $S5 \to PC$, $\boldsymbol{k}$: $PC \to S5$ such that $\Gamma \vdash^{\boldsymbol{ki}}_{S5} \alpha$ i.e. $\boldsymbol{k}(\boldsymbol{i}([\Gamma])) \vdash_{S5} \boldsymbol{k}(\boldsymbol{i}(\alpha))$.

Finally, if we take into account that Jaśkowski's methodological theorems, in fact, specify us the translation $\boldsymbol{m}$: $PC \to S5$ (methodological theorems 1 and 2) and translation $\boldsymbol{n}$: $S5 \to PC$ (methodological theorem 3) then we can speak of Jaśkowski's discursive logic D_2 as the exponential $S5 \Rightarrow PC$ of *PC* to *S5* where $\Gamma \vdash_{S5 \Rightarrow PC} \alpha$ iff there is the case that $\Gamma \vdash^{\boldsymbol{nm}}_{PC} \alpha$ i.e. $\boldsymbol{n}(\boldsymbol{m}([\Gamma])) \vdash_{PC} \boldsymbol{n}(\boldsymbol{m}(\alpha))$.

4. Coexponentials and Exponentials of da Costa Combined Logic

We can try to adopt more elaborated paraconsistent theories describing different cases and models of paraconsistency while fulfilling Vasiliev's program. Following this course of consideration we propose to accept da Costa algebra [2] reflected the most of logical properties of da Costa systems C_n as internal logic in our assumed combined system. In this case the resulting system of combined logic also would be inconsistent (paraconsistent) on ontological level but consistent on logical. Since our further theoretic constructions are essentially based on da Costa algebra then for the further proceedings we adduce the complete definition.

Definition [2. P. 81]. By a *da Costa algebra* we mean a structure

$$A = \langle S, 0, 1, \leq, \cap, \cup, \to, \sim \rangle$$

such that for every a, b, c, x in S the following conditions hold:

1) $\leq$ is a quasi-order;
2) $a \cap b \leq a$, $a \cap b \leq b$;
3) if $c \leq a$ and $c \leq b$ then $c \leq a \cap b$;
4) $a \cap a = a$, $a \cup a = a$;
5) $a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$;
6) $a \leq a \cup b$, $b \leq a \cup b$;
7) if $a \leq c$ and $b \leq c$ then $a \cup b \leq c$;
8) $a \cap (a \to b) \leq b$;
9) if $a \cap c \leq b$ then $c \leq (a \to b)$;

10) $0 \leq a$, $a \leq 1$;

11) $x^0 \leq (\sim x)^0$, where $x^0 = \sim(x \cap \sim x)$;

12) $x \cup \sim x \leftrightarrow 1$, where $a \leftrightarrow b$ iff $a \leq b$ and $b \leq a$;

13) $\sim\sim x \leq x$, where $\sim\sim x$ abbreviates $\sim(\sim x)$;

14) $a^0 \leq (b \to a) \to ((b \to \sim a) \to \sim b)$;

15) $x^0 \cap \sim(x^0) \leftrightarrow 0$.

If there exists $x \in S$ such that it is not true that $x \cap \sim x \not\leftrightarrow 0$ the algebra A is said to be a *proper da Costa algebra.*

In order to obtain da Costa paraconsistent combined logic $PC^{\theta(C_1)}$ (cf. [13]) we enrich the axiom schemata of positive classic sentential logic and the rule *modus ponens* with the following schemes:

A1. $\theta a \wedge \theta b \equiv \theta(a \cap b)$;

A2. $\theta a \vee \theta b \equiv \theta(a \cup b)$;

A3. $\theta a \wedge \theta(a \to b) \supset \theta b$;

A4. $(\theta(a \cap c) \supset \theta b) \supset (\theta c \supset \theta(a \to b))$;

A5. $\theta(a^0) \supset \theta(\sim a)^0$, where $a^0 = \sim(a \cup \sim a)$;

A6. $\theta(\sim\sim a) \supset \theta a$;

A7. $\theta(a^0) \supset \theta((b \to a) \to ((b \to \sim a) \to \sim b))$;

A8. $\theta b \supset \theta(a \cup \sim a)$;

A9. $\theta(a^0 \cap \sim(a^0)) \supset \theta b$.

A10. $\alpha \supset \theta[\alpha]$

A11. $\theta[\alpha \vee \beta] \equiv \theta([\alpha] \cup [\beta])$

A12. $\theta[\alpha \wedge \beta] \equiv \theta([\alpha] \cap [\beta])$

An algebraic semantics of $PC^{\theta(C_1)}$ will be a an algebraic da Costa bundle which is 4-tuple $\langle \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{f}, \boldsymbol{g} \rangle$ where $\boldsymbol{A} = \langle A, +, \circ \rangle$ is a Boolean algebra (A contains two elements at least), $\boldsymbol{B} = \langle B, 0, 1, \leq, \cap, \cup, \to, \sim \rangle$ is a da Costa algebra (B contains three elements at least), $\boldsymbol{f} : \boldsymbol{B} \to \boldsymbol{A}$, $\boldsymbol{g}: \boldsymbol{A} \to \boldsymbol{B}$ are embedding functions. For $\boldsymbol{f}$, $\boldsymbol{g}$ the following conditions are fulfilled:

$\boldsymbol{f}(a \cup b) = \boldsymbol{f}(a) + \boldsymbol{f}(b)$,

$\boldsymbol{f}(a \cap b) = \boldsymbol{f}(a) \circ \boldsymbol{f}(b)$,

$\boldsymbol{f}(a) \circ \boldsymbol{f}(a \to l) \leq \boldsymbol{f}(b)$;

$(\boldsymbol{f}(a \cap c) \supset \boldsymbol{f}(b)) \leq (\boldsymbol{f}(c) \supset \boldsymbol{f}(a \to b))$;

$\boldsymbol{f}(a^0) \leq \boldsymbol{f}(\sim a)^0$, where $a^0 = \sim(a \cup \sim a)$;

$\boldsymbol{f}(\sim\sim a) \leq \boldsymbol{f}(a)$;

$\boldsymbol{f}(a^0) \leq \boldsymbol{f}((b \to a) \to ((b \to \sim a) \to \sim b))$;

$\boldsymbol{f}(b) \leq \boldsymbol{f}(a \cup \sim a)$;

$\boldsymbol{f}(a^0 \cap \sim(a^0)) \leq \boldsymbol{f}(b)$.

where $x \supset y = -x + y$ and $a, b, c \in B$;

$\boldsymbol{g}(x + y) = \boldsymbol{g}(x) \cup \boldsymbol{g}(y)$,

$\boldsymbol{g}(x \circ y) = \boldsymbol{g}(x) \cap \boldsymbol{g}(y)$,

$x \leq \boldsymbol{f}(\boldsymbol{g}(x))$,

where $x, y \in A$.

If F is a set of well-formed formulas and E is a set of events then a valuation $\boldsymbol{v}$ is defined by:

$\boldsymbol{v}: F \bigcup E \to A \bigcup B,$

$\boldsymbol{v}(\alpha\vee\beta) = \boldsymbol{v}(\alpha) + \boldsymbol{v}(\beta),$

$\boldsymbol{v}(\alpha\wedge\beta) = \boldsymbol{v}(\alpha)\circ \boldsymbol{v}(\beta),$

$\boldsymbol{v}(\neg\alpha) = -\boldsymbol{v}(\alpha)$

$\boldsymbol{v}([\alpha]) = \boldsymbol{g}\,(\boldsymbol{v}(\alpha)).$

(where α, β are wff and $\boldsymbol{v}(\alpha)$, $\boldsymbol{v}(\beta) \in A$),

$\boldsymbol{v}(a\cup b) = \boldsymbol{v}(a) \cup \boldsymbol{v}(b),$

$\boldsymbol{v}(a\cap b) = \boldsymbol{v}(a) \cap \boldsymbol{v}(b),$

$\boldsymbol{v}(a\to b) = \boldsymbol{v}(a) \to \boldsymbol{v}(b),$

$\boldsymbol{v}(\sim a) = \sim\boldsymbol{v}(a),$

$\boldsymbol{v}(\theta a) = \boldsymbol{f}\,(\boldsymbol{v}(a)),$

(where a, b are events and $\boldsymbol{v}(a)$, $\boldsymbol{v}(b) \in B$).

Theorem 3. Axioms PC+(A1—A12) are valid in any da Costa bundle $\langle \mathbf{A}, \mathbf{B}, \mathbf{f}, \mathbf{g}\rangle$.

Proof is straightforward ■.

Let us remind that we can introduce a valuation $\boldsymbol{v}_{C_1}: F_{C_1} \to \boldsymbol{B}$ of da Costa system C_1 by means of the following conditions:

$\boldsymbol{v}_{C_1}(\alpha \vee \beta) = \boldsymbol{v}_{C_1}(a) \cup \boldsymbol{v}_{C_1}(b),$

$\boldsymbol{v}_{C_1}(\alpha\wedge\beta) = \boldsymbol{v}_{C_1}(a) \cap \boldsymbol{v}_{C_1}(b),$

$\boldsymbol{v}_{C_1}(\neg\alpha) = \sim \boldsymbol{v}_{C_1}(a),$

$\boldsymbol{v}_{C_1}(\alpha\to\beta) = \boldsymbol{v}_{C_1}(a) \to \boldsymbol{v}_{C_1}(b),$

where α, β are wff of C_1 and $\boldsymbol{v}_c(\alpha)$, $\boldsymbol{v}_c(\beta) \in \boldsymbol{B}$.

If we split $PC^{\theta(C_1)}$-valuation $\boldsymbol{v}$ into a valuation $\boldsymbol{v}^{\boldsymbol{f}}_{PC^{\theta(C_1)}}$ which is defined on formulas and a valuation $\boldsymbol{v}^{\boldsymbol{e}}_{PC^{\theta(C_1)}}$ which is defined on events then this gives us the following diagram of valuations and embeddings:

$$\begin{tikzcd}
C_1 \arrow[r, shift left, "\boldsymbol{h}"] \arrow[d, "\boldsymbol{v}_{S5}"'] & PC^{\theta(C_1)} \arrow[l, shift left, "\boldsymbol{i}"] \arrow[dl, "\boldsymbol{v}^{e}_{PC^{\theta(C_1)}}"'] \arrow[d, "\boldsymbol{v}^{f}_{PC^{\theta(C_1)}}"] \\
\boldsymbol{B} \arrow[r, shift left, "\boldsymbol{g}"] & \boldsymbol{A} \arrow[l, shift left, "\boldsymbol{f}"]
\end{tikzcd}$$

where $\boldsymbol{h}$ is a translation $\boldsymbol{h}$: $PC^{\theta(C_1)} \to C_1$ defined by the conditions

$\boldsymbol{h}(a) = p_a,$

$\boldsymbol{h}(p_i) = p_i,$

$\boldsymbol{h}(\alpha\vee\beta) = \boldsymbol{h}(\alpha) \cup \boldsymbol{h}(\beta),$

$\boldsymbol{h}(\alpha\wedge\beta) = \boldsymbol{h}(\alpha) \cap \boldsymbol{h}(\beta),$

$\boldsymbol{h}(a\cup b) = \boldsymbol{h}(a) \cup \boldsymbol{h}(b),$

$\boldsymbol{h}(a \cap b) = \boldsymbol{h}(a) \cap \boldsymbol{h}(b)$,
$\boldsymbol{h}(a \to b) = \boldsymbol{h}(a) \to \boldsymbol{h}(b)$,
$\boldsymbol{h}(\sim a) = \sim \boldsymbol{h}(a)$,
$\boldsymbol{h}(\theta a) = \boldsymbol{h}(a)$,
$\boldsymbol{h}([\alpha]) = a_\alpha$

and $\boldsymbol{i}$ is a translation $\boldsymbol{i}\colon C_1 \to PC^{\theta(C_1)}$ defined by the condition $\boldsymbol{i}(\alpha) = \theta(\boldsymbol{v}^*_{C_1}(\alpha))$ where $\boldsymbol{v}^*_{C_1}(\alpha)$ means substitution for C_1-operators respective events-operators of $PC^{\theta(C_1)}$, i.e. we replace $\cup$, $\cap$, $\to$, $\sim$ with event-operators $\cup$, $\cap$, $\to$, $\sim$ respectively. Now it is easy to define $\Gamma \vDash_{PC^{\theta(C_1)}} \alpha$ and $\Gamma \vDash_{C_1} \alpha$ iff for any $\beta \in \Gamma$ and an every valuation $\boldsymbol{v}^{\theta(C_1)}_{PC}$ $(= \boldsymbol{v}^f_{PC^{\theta(C_1)}} \cup \boldsymbol{v}^e_{PC^{\theta(C_1)}}$ and $\boldsymbol{v}_{C_1}$ we have $\boldsymbol{v}_{PC^{\theta(C_1)}}(\beta) \le \boldsymbol{v}_{PC^{\theta(C_1)}}(\alpha)$ and $\boldsymbol{v}_{C_1}(\beta) \le \boldsymbol{v}_{C_1}(\alpha)$ respectively.

Introducing analogously to the case of *JVCD* PTS for $PC^{\theta(C_1)}$ we define a local forcing relation $\vDash^{i}_{PC^{\theta(C_1)}}$ for $PC^{\theta(C_1)}$ (in respect to $\boldsymbol{h}$) as

$$\Gamma \vDash^{\boldsymbol{h}}_{PC^{\theta(C_1)}} \alpha \text{ iff } \boldsymbol{h}[\Gamma] \vDash_{C_1} \boldsymbol{h}(\alpha)$$

for every set $\Gamma \cup \{\alpha\}$ of formulas of $PC^{\theta(C_1)}$. Generalizing we get a definition:

$\Gamma \vDash_{PC^{\theta(C_1)}} \alpha$ iff for any translation $\boldsymbol{h}$ it is the case that $\Gamma \vDash^{\boldsymbol{h}}_{PC^{\theta(C_1)}} \alpha$.

Since $\boldsymbol{i}(\alpha) = \theta(\boldsymbol{v}^*_{C_1}(\alpha))$ then substituting we analogously obtain $\Gamma \vDash^{\boldsymbol{v}_{C_1}}_{C_1} \alpha$ iff $\theta(\boldsymbol{v}^*_{C_1}([\Gamma]) \vDash^{\boldsymbol{h}}_{PC^{\theta(C_1)}} \theta(\boldsymbol{v}^*_{C_1}(\alpha))$ and $\Gamma \vDash_{C_1} \alpha$ iff for any valuation $\boldsymbol{v}_{C_1}$ we have $\theta(\boldsymbol{v}^*_{C_1}([\Gamma]) \vDash^{\boldsymbol{h}}_{PC^{\theta(C_1)}} \theta(\boldsymbol{v}^*_{C_1}(\alpha))$. The role of θ-operator would be depicted on the diagram above with the help of the function $\boldsymbol{j}\colon \boldsymbol{B} \to PC^{\theta(C_1)}$, i.e. as

$$\begin{tikzcd}
C_1 \arrow[r, shift left, "\boldsymbol{h}"] \arrow[d, "\boldsymbol{v}_{C_1}"'] & PC^{\theta(C_1)} \arrow[l, shift left, "\boldsymbol{i}"] \arrow[d, "\boldsymbol{v}^f_{PC^{\theta(C_1)}}"] \arrow[dl, shift left, "\boldsymbol{v}^f_{PC^{\theta(C_1)}}"] \\
\boldsymbol{B} \arrow[r, shift left, "\boldsymbol{g}"] \arrow[ur, shift left, "\boldsymbol{J}"] & \boldsymbol{A} \arrow[l, shift left, "\boldsymbol{f}"]
\end{tikzcd}$$

and we obtain $\boldsymbol{i}(\alpha) = \theta(\boldsymbol{v}_{C_1}(\alpha))$ where

$\boldsymbol{j}(a) = \theta a$
$\boldsymbol{j}(a \cup b) = \boldsymbol{j}(a) \cup \boldsymbol{j}(b)$
$\boldsymbol{j}(a \cap b) = \boldsymbol{j}(a) \cap \boldsymbol{j}(b)$
$\boldsymbol{j}(\sim a) = \theta(\sim a)$
$\boldsymbol{j}(a \to b) = \theta(a \to b)$

and respectively modified condition:

$\Gamma \vDash_{C_1} \alpha$ iff for any valuation $\boldsymbol{v}_{C_1}$ we have $\boldsymbol{j}(\boldsymbol{v}_{C_1}[\Gamma]) \vDash_{PC^{\theta(C_1)}} \boldsymbol{j}(\boldsymbol{v}_{C_1}(\alpha))$.

If as in case of *JVCD* we replace $\vDash$ with $\vdash$ then we can define local consequence relation $\vdash^{\boldsymbol{h}}_{PC^{\theta(C_1)}}$ for $PC^{\theta(C_1)}$ (relative to $\boldsymbol{h}$) as

$$\Gamma \vdash^{\boldsymbol{h}}_{PC^{\theta(C_1)}} \alpha \text{ iff } \boldsymbol{h}[\Gamma] \vdash_{C_1} \boldsymbol{h}(\alpha),$$

and we define consequence relation on $PC^{\theta(C_1)}$ via C_1 as

$\Gamma \vdash_{PC^{\theta(C_1)}} \alpha$ iff for any translation $\boldsymbol{h}$ it is the case that $\Gamma \vdash^{\boldsymbol{h}}_{PC^{\theta(C_1)}} \alpha$.

Our definition of $\Gamma \vdash_{PC^{\theta(C_1)}} \alpha$ provide us with the construction of coexponential of $PC^{\theta(C_1)}$ to C_1 which would be denoted $PC^{\theta(C_1)} \Leftarrow C_1$ since the last is the system in which the consequence relation is determinate by the consequence relation of C_1 depending of all translations from $PC^{\theta(C_1)}$ to C_1.

If as above $\boldsymbol{i}(\alpha) = \theta(\boldsymbol{v}_{C_1}(\alpha))$ then every translation of $\boldsymbol{i}$-type will be determined by the respective $\boldsymbol{v}_{C_1}$, i.e. $\Gamma \vdash^{v_{C_1}}_{C_1} \alpha$ iff $\boldsymbol{j}(\boldsymbol{v}_{C_1}[\Gamma]) \vdash_{PC^{\theta(C_1)}} \boldsymbol{j}(\boldsymbol{v}_{C_1}(\alpha))$ and $\Gamma \vdash_{C_1} \alpha$ iff for any valuation $\boldsymbol{v}_{C_1}$ we have $\boldsymbol{j}(\boldsymbol{v}_{C_1}[\Gamma]) \vdash_{PC^{\theta(C_1)}} \boldsymbol{j}(\boldsymbol{v}_{C_1}(\alpha))$. In this case we deal with the coexponential $C_1 \Leftarrow PC^{\theta(C_1)}$ of C_1 to $PC^{\theta(C_1)}$ where translations $\boldsymbol{i}$: $C_1 \to PC^{\theta(C_1)}$ are defined via $\boldsymbol{v}_{C_1}$.

If we replace *JVCD* with *PC* then $\boldsymbol{i}$ will not depends on θ-operator and a valuation $\boldsymbol{v}_{C_1}$ and hence instead of $\Gamma \vdash_{C_1} \alpha$ we will deal with $\Gamma \vdash_{C_1 \Leftarrow PC} \alpha$ where $C_1 \Leftarrow PC$ (*coexponential* of C_1 to *PC*) is a system in which $\Gamma \vdash_{C_1 \Leftarrow PC} \alpha$ iff for any translation $\boldsymbol{i}$: $C_1 \to PC$ it is the case that $\Gamma \vdash^{\boldsymbol{i}}_{C_1 \Leftarrow PC} \alpha$ i.e. iff for any $\boldsymbol{i}$: $C_1 \to PC$ we have $\boldsymbol{i}[\Gamma] \vdash_{PC} \boldsymbol{i}(\alpha)$.

Again, the problem arises that there might be the case that $\boldsymbol{i}_1(\alpha) = \boldsymbol{i}_2(\beta)$ i.e. $\Gamma \vdash_{C_1 \Leftarrow PC} \alpha$ and $\Gamma \vdash_{C_1 \Leftarrow PC} \beta$ would be determined by one and the same *PC*-formula. We as above employ analogous to $\boldsymbol{g}$ translation $\boldsymbol{k}$: $PC \to C_1$ and characterize a system $PC \Rightarrow C_1$ (of *constraint possible translating* of *PC* to C_1 or *exponential* of *PC* to C_1) with the help of condition $\Gamma \vdash_{PC \Rightarrow C_1} \alpha$ iff there are translations $\boldsymbol{i}$: $C_1 \to PC$, $\boldsymbol{k}$: $PC \to C_1$ such that $\Gamma \vdash^{\boldsymbol{ki}}_{C_1} \alpha$ i.e. $\boldsymbol{k}(\boldsymbol{i}([\Gamma])) \vdash_{C_1} \boldsymbol{k}(\boldsymbol{i}(\alpha))$. And the other way round, we can also consider the exponential $C_1 \Rightarrow PC$ of C_1 to *PC* where $\Gamma \vdash_{C_1 \Rightarrow PC} \alpha$ iff there is the case that $\Gamma \vdash^{\boldsymbol{ik}}_{PC} \alpha$ i.e. $\boldsymbol{i}(\boldsymbol{k}([\Gamma])) \vdash_{PC} \boldsymbol{i}(\boldsymbol{k}(\alpha))$.

REFERENCES

[1] *Carnielli W.A.* Possible-Translations Semantics for Paraconsistent Logics // D. Batens et al. (eds.), Frontiers of Paraconsistent Logic. — Research Studies Press Ltd., Baldock, Herfordshire, 2000. — P. 149—163.

[2] *Carnielli W.A., Alcantara L.P.* Paraconsistent algebras // Studia Logica. — XLIII. — № 1/2. — 1984. — P. 79—87.

[3] *Frege G.* Begriffschrift // Translations from the Philosophical Writings of Gottlob Frege / ed. By. P. Geach and M. Black. — Oxford, 1952.

[4] *Jaśkowski S.* Rachunek zdań dla systemów dedukcyjnych sprzecznych // Studia Soc. Sci. Torunensis. — Sectio A. — Vol. I. — № 5. — 1948 (English translation in [5]).
[5] *Jaśkowski S.* Propositional Calculus for Contradictory Deductive Systems // Studia Logica. — XXIV (1969). — P. 143—157.
[6] *Lemmon E.* Algebraic Semantics for Modal Logics I // Journ. Symb. Log. — 1966. — Vol. 31. — № 1. — P. 46—65.
[7] *Perzanowski J.* Towards Post-Tractatus Ontology // Wittgenstein — Towards Re-Evaluation / Eds. R. Haller & J. Brandl, Verlag Holder-Pichler-Tempsky. — Wien, 1990. — P. 185—199.
[8] *Słupecki J.* A Generalization of Modal Logic // Studia Logica. — V. 28. — 1971. — P. 7—17.
[9] *Smirnov V.A*. Internal and External Logics // Bull. Sect. Logic. — V. 17. — № 3/4. — 1988. — P.170—181.
[10] *Smirnov V.A.* Statement and Predication. A Combined Calculus of Sentences and Events. [In:] Intensional Logic, History of Philosophy and Methodology. — Budapest, 1988. — P. 101—108.
[11] *Smirnov V.A*. Combined Calculi of Sentences and Events and von Wright's Logic of Truth. [In:] Investigations on Non-Classical Logics, V.A. Smirnov (ed.). — Moscow, 1989. — P. 16—29 (in Russian).
[12] *Smirnov V.A.* Assertion and Predication. Combined Calculi of Sentences and Events. [In:] Syntactical and Semantic Investigations of Non-Extensional Logics, V.A. Smirnov (ed.). — Moscow, 1989. — P. 27—35 (in Russian).
[13] *Vasyukov V.L.* Combined da Costa's Logics (world according N.C.A. da Costa) // Logic et Analyse. — № 165—166 (1999). — P. 127—138.
[14] *Vasyukov V.L.* Formal Ontology. — Moscow, 2006 (in Russian).
[15] *Vasyukov V.L.* Structuring the Universe of Universal Logic // Logica Universalis. — Vol. 1. — № 2. — 2007. — P. 277—294.

КОМБИНИРОВАННЫЕ ПАРАНЕПРОТИВОРЕЧИВЫЕ ЛОГИКИ И ИХ (КО)ЭКСПОНЕНЦИАЛЫ

В.Л. Васюков

Институт философии РАН
Волхонка ул., 14, Москва, Россия, 119991

Комбинированные логики предложений и событий состоят из двух частей: внешней логики, зависящей от эпистемологических допущений, и внутренней логики, зависящей от онтологических допущений. Комбинированные логики были разработаны В.А. Смирновым следуя идеям Г. Фреге и Н. Васильева. Анализ структуры комбинированных логик обнаруживает, что фактически они используют связи между двумя логическими системами, постулируя в роли онтологической части алгебры, служащие моделями соответствующих логик. Это наводит на мысль рассмотреть системы, описывающие взаимоотношение двух логик на синтаксическом уровне, когда у нас есть доступ к этим логикам без посредничества их моделей. В роли подобных систем могут быть использованы так называемые коэкспоненциалы и экспоненциалы, разработанные в [15]. В статье рассматривается случай двух паранепротиворечивых комбинированных логик (с паранепротиворечивыми алгебрами в качестве их онтологических частей).

Ключевые слова: комбинированные логики, дискурсивная логика Яськовского, паранепротиворечивые логики да Косты, перевод, (не)ограниченная возможная переводимость.