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Marginal asymptotic diffusion analysis of two-class retrial queueing system with probabilistic priority as a model of two-modal communication networks

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Abstract. In the paper, a retrial queueing system of $M_2/M_2/1$ type with probabilistic priority and interruptions is considered as a model of a two-modal communication network. Two classes of customers come to the system according Poisson arrival processes. There is one service device (or channel). If a customer finds the server occupying by a customer of the same class, it goes to an orbit and makes a repeated attempt after a random delay. If an arrival customer finds the other class customer on the server, it can interrupt its service with the given probability and start servicing itself. Customers from the orbit behave the same way. There is a multiply access for customers in the orbit. Service times and inter-retrial times have exponential distributions. Customers are assumed heterogeneous, so the parameters of the distributions are different for each class. In the paper, we propose the original marginal asymptotic-diffusion method for finding of the stationary probability distributions of the number of each class customers under the long delays condition.

Key words and phrases: two-class retrial queueing system, probabilistic priority, interruptions, marginal asymptotic-diffusion analysis

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1. Introduction

Heterogeneous of information is characteristic of modern telecommunication networks. Transmitted data may contain text, sound, image, service information, etc. Thus, we observe several arrival processes in networks with different characteristics (required job, quality of service, permissible latency, possibility of losses). In the field of robotics and telemedicine, multimedia networks are called as multimodal communication [1]. Interest in multimodal systems is increasing with the development of multimodal interfaces. By modality it is called physically recorded elements of communication (human-machine and/or human-human), including both the transmitted data (message) and individual information. The set of multimodal data and their size may vary depending on the task. So, in speech recognition systems based on audio recordings, it is sufficient 70–80 Kb for a speech modality [2], while for a sign modality (i.e. in Russian sign language), one modality record

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can require 125 Mb [3]. One of solutions to ensure the quality of data transmission is to distribute user application data into several sub-streams and to provide multi-stream data transmission using several available communication interfaces. Often some modalities may be different by priority [4]. It means that some information is more significant than other in a particular task (i.e. urgent emergency messages must be delivered immediately that leads to interruption of lower priority information transmission).

Most of the works devoted to the problems of multi-stream data transmission are based on the analysis of data transmission quality parameters using simulation models. Here, the mathematical modelling is applied for analyzing and optimization of multimodal data transmission systems, taking into account the priorities, interruptions and the existence of repeated attempts of information transmission after fails. In this way, we propose the mathematical model of multimodal communication system in the form of a retrial queueing system with two classes of arrivals and opportunity of interruptions in the case of probabilistic priority.

Retrial queueing systems (or queueing system with repeated calls) [5, 6] are new class of queuing models widely applied in various communication systems (call-centers, cellular networks, LANs, etc. [7, 8]). In retrial queues, there is a some virtual place (the orbit) for repeated calls, where unserved calls wait during random time before an attempt to receive service again.

In spite of the large number of studies of retrial queueing systems of various configurations, heterogeneous models are weak investigated. Retrial queues with several types of customers (and several orbits too) are called as multiclass RQs and considered in [9–15]. Most of cited papers are devoted only stability analysis, while probability distributions or even means of processes under study are hardly investigated. Queuing models with interruption are proposed in [16, 17]. Different types of service interruptions are described in [18]. Queuing systems with probabilistic priority are presented in [19, 20]. Retrial queues with different types of priority are studied in [8, 21–23]. The most close study of retrial queues with two classes and priority are considered in [24, 25].

The rest of the paper is organized as follows. In Section 2, the mathematical model is described, the process under study is denoted and a system of differential Kolmogorov equations is written. In Section 3, we propose the original marginal asymptotic-diffusion analysis method for the two-class retrial queueing system studying. We derived the formula for the marginal asymptotic stationary probability distribution of number of each class calls in the orbit under the long delays limit condition. Section 5 consists some conclusions.

2. Mathematical model

Let us consider a retrial queueing system with two classes of customers. A customer of the n-th class comes to the system according Poisson arrival process with parameter λ_n, where $n = 1, 2$. There is one server. If the server is idle, the n-th class customer starts its servicing during the exponentially distributed random time with rate μ_n. We assume that customers have probabilistic priority. If an arrival customer finds the servicing customer of the same class, it goes to the orbit. If a customer finds the other class customer on the server, it can: a) with probability s_n interrupt the servicing and starts servicing itself (and the displaced customer goes to the orbit); b) with probability $1 - s_n$ joins to the orbit. After a random time distributed exponentially with rate σ_n, a customer from the orbit makes a repeated attempt to get service. The customers from the orbit behave in the same way (and with same probabilities). So, in the model we assume that there are no loses of customers.

Note that it does not matter to consider one common orbit for both classes of customers or two orbits for each class. It is important to distinguish a number of customers of each class in the system at some time moment. The model structure is presented on Figure 2.

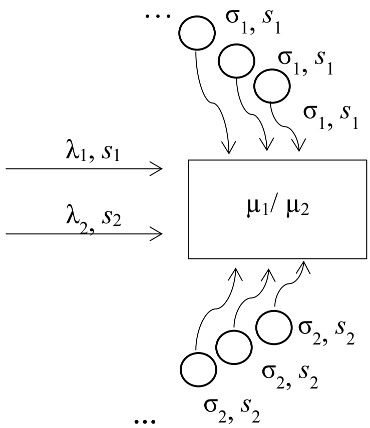

Figure 1. Two-class retrial queueing system with probabilistic priority

Note that in real life, the most common situation is when one class of customers is prioritized ($s_1 = 1$ and $s_2 = 0$). The considered mathematical model and obtained further results are general and can be applied for different cases.

Let us denote random processes of the number of the n-th class calls in the orbit by $i_n(t)$, where $n = 1, 2$. Process $k(t)$ determines states of the server as follows:

$$k(t) = \begin{cases} 0, & \text{if the server is free,} \\ 1, & \text{if the 1-st class customer is servicing,} \\ 2, & \text{if the 2-nd class customer is servicing.} \end{cases}$$

Denote by $P\{k(t) = k, i_1(t) = i_1, i_2(t) = i_2\} = P(k, i_1, i_2, t)$ the probability that the server has state k and there are i_1 customers of the first class and i_2 customers of the second class in the orbit at time t. Process $\{k(t), i_1(t), i_2(t)\}$ is three-dimensional continuous-time Markov chain. Let us write the following system of Kolmogorov equations for probability distribution $P(k, i_1, i_2, t)$:

$$\begin{cases} \dfrac{\partial P(0,i_1,i_2,t)}{\partial t} = -(\lambda_1 + \lambda_2 + i_1\sigma_1 + i_1\sigma_2)P(0,i_1,i_2,t) + \\ \qquad + \mu_1 P(1,i_1,i_2,t) + \mu_2 P(2,i_1,i_2,t), \\ \dfrac{\partial P(1,i_1,i_2,t)}{\partial t} = -(\lambda_1 + \lambda_2 + \mu_1 + i_2 s_2 \sigma_2)P(1,i_1,i_2,t) + \lambda_1 P(0,i_1,i_2,t) + \\ \qquad + (i_1+1)\sigma_1 P(0,i_1+1,i_2,t) + \lambda_1 s_1 P(2,i_1,i_2-1,t) + \\ \qquad + \lambda_2(1-s_2)P(1,i_1,i_2-1,t) + \lambda_1 P(1,i_1-1,i_2,t) + \\ \qquad + (i_1+1)\sigma_1 s_1 P(2,i_1+1,i_2-1,t), \\ \dfrac{\partial P(2,i_1,i_2,t)}{\partial t} = -(\lambda_1 + \lambda_2 + \mu_2 + i_1 s_1 \sigma_1)P(2,i_1,i_2,t) + \lambda_2 P(0,i_1,i_2,t) + \\ \qquad + (i_2+1)\sigma_2 P(0,i_1,i_2+1,t) + \lambda_2 s_2 P(1,i_1-1,i_2,t) + \\ \qquad + \lambda_1(1-s_1)P(2,i_1-1,i_2,t) + \lambda_2 P(2,i_1,i_2-1,t) + \\ \qquad + (i_2+1)\sigma_2 s_2 P(1,i_1-1,i_2+1,t). \end{cases} \tag{1}$$

Let us introduce the partial characteristic functions:

$$H(k,u_1,u_2,t)=\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}e^{ju_1i_1}\cdot e^{ju_2i_2}P(k,i_1,i_2,t).$$

Then we rewrite Equations (1) for the characteristic functions as follows

$$\begin{cases}\frac{\partial H(0,u_1,u_2,t)}{\partial t}=-(\lambda_1+\lambda_2)H(0,u_1,u_2,t)+j\sigma_1\frac{\partial H(0,u_1,u_2,t)}{\partial u_1}+\\ \qquad +j\sigma_2\frac{\partial H(0,u_1,u_2,t)}{\partial u_2}+\mu_1H(1,u_1,u_2,t)+\mu_2H(2,u_1,u_2,t),\\ \frac{\partial H(1,u_1,u_2,t)}{\partial t}=-(\lambda_1+\lambda_2+\mu_1)H(1,u_1,u_2,t)+\lambda_1H(0,u_1,u_2,t)+\\ \qquad +j\sigma_2s_2\frac{\partial H(1,u_1,u_2,t)}{\partial u_2}-j\sigma_1e^{-ju_1}\frac{\partial H(0,u_1,u_2,t)}{\partial u_1}+\\ \qquad +\lambda_1s_1e^{ju_2}H(2,u_1,u_2,t)+\lambda_2(1-s_2)e^{ju_2}H(1,u_1,u_2,t)+\\ \qquad +\lambda_1e^{ju_1}H(1,u_1,u_2,t)-j\sigma_1s_1e^{-ju_1}e^{ju_2}\frac{\partial H(2,u_1,u_2,t)}{\partial u_1},\\ \frac{\partial H(2,u_1,u_2,t)}{\partial t}=-(\lambda_1+\lambda_2+\mu_2)H(2,u_1,u_2,t)+\lambda_2H(0,u_1,u_2,t)+\\ \qquad +j\sigma_1s_1\frac{\partial H(2,u_1,u_2,t)}{\partial u_1}-j\sigma_2e^{-ju_2}\frac{\partial H(0,u_1,u_2,t)}{\partial u_2}+\\ \qquad +\lambda_2s_2e^{ju_1}H(1,u_1,u_2,t)+\lambda_1(1-s_1)e^{ju_1}H(2,u_1,u_2,t)+\\ \qquad +\lambda_2e^{ju_2}H(2,u_1,u_2,t)-j\sigma_2s_2e^{-ju_2}e^{ju_1}\frac{\partial H(1,u_1,u_2,t)}{\partial u_2}.\end{cases}\tag{2}$$

Introduce matrix form of the characteristic functions:

$$\mathbf{H}(u_1,u_2,t)=\{H(0,u_1,u_2,t),H(1,u_1,u_2,t),H(2,u_1,u_2,t)\}.$$

So Equations (2) are transformed in the following matrix equation:

$$\begin{aligned}\frac{\partial\mathbf{H}(u_1,u_2,t)}{\partial t}&=\mathbf{H}(u_1,u_2,t)(\mathbf{A}+e^{ju_1}\mathbf{B}_1+e^{ju_2}\mathbf{B}_2)+\\ &+j\sigma_1\frac{\partial\mathbf{H}(u_1,u_2,t)}{\partial u_1}(\mathbf{I}_1-e^{-ju_1}\mathbf{I}_2-e^{-ju_1}e^{ju_2}\mathbf{I}_3)+\\ &+j\sigma_2\frac{\partial\mathbf{H}(u_1,u_2,t)}{\partial u_2}(\mathbf{I}_4-e^{-ju_2}\mathbf{I}_5-e^{ju_1}e^{-ju_2}\mathbf{I}_6),\end{aligned}\tag{3}$$

where

$$\mathbf{A}=\begin{pmatrix}-(\lambda_1+\lambda_2)&\lambda_1&\lambda_2\\ \mu_1&-(\lambda_1+\lambda_2+\mu_1)&0\\ \mu_2&0&-(\lambda_1+\lambda_2+\mu_2)\end{pmatrix},$$

$$\mathbf{B}_1=\begin{pmatrix}0&0&0\\0&\lambda_1&\lambda_2s_2\\0&0&\lambda_1(1-s_1)\end{pmatrix},\quad \mathbf{B}_2=\begin{pmatrix}0&0&0\\0&\lambda_2(1-s_2)&0\\0&\lambda_1s_1&\lambda_2\end{pmatrix},$$

$$\mathbf{I}_1=\begin{pmatrix}1&0&0\\0&0&0\\0&0&s_1\end{pmatrix},\quad \mathbf{I}_2=\begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix},\quad \mathbf{I}_3=\begin{pmatrix}0&0&0\\0&0&0\\0&s_1&0\end{pmatrix},$$

$$\mathbf{I}_4=\begin{pmatrix}1&0&0\\0&s_2&0\\0&0&0\end{pmatrix},\quad \mathbf{I}_5=\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix},\quad \mathbf{I}_6=\begin{pmatrix}0&0&0\\0&0&s_2\\0&0&0\end{pmatrix}.$$

3. Asymptotic Analysis

The further analysis can be devided on several stages:

1. Derivation of “marginal” asymptotic equations for each process $i_1(t)$ and $i_2(t)$ under a long delay limit condition.
2. Finding of the asymptotic stationary means of numbers of calls of each class and stationary probabilities of the server states.
3. Implementation of asymptotic-diffusion analysis for “marginal” asymptotic equations. Finding of the asymptotic stationary probability distribution of numbers of customers of each class.

3.1. Marginal asymptotic equations

Let us derive marginal asymptotic equations for process $i_1(t)$ when parameter $\sigma_2 \to 0$. First of all, we introduce infinitesimal parameter ε and substitutions

$$\sigma_2=\gamma_2\sigma,\quad \sigma=\varepsilon,\quad u_2=\varepsilon w,$$

$$\mathbf{H}(u_1,u_2,t)=\mathbf{F}(u_1,w,t,\varepsilon).$$

From System (3), we obtain the following asymptotic matrix equation:

$$\begin{aligned}\frac{\partial\mathbf{F}(u_1,w,t,\varepsilon)}{\partial t}&=\mathbf{F}(u_1,w,t,\varepsilon)(\mathbf{A}+e^{ju_1}\mathbf{B}_1+e^{jw\varepsilon}\mathbf{B}_2)+\\+j\sigma_1&\frac{\partial\mathbf{F}(u_1,w,t,\varepsilon)}{\partial u_1}(\mathbf{I}_1-e^{-ju_1}\mathbf{I}_2-e^{-ju_1}e^{jw\varepsilon}\mathbf{I}_3)+\\+j\gamma_2&\frac{\partial\mathbf{F}(u_1,w,t,\varepsilon)}{\partial w}(\mathbf{I}_4-e^{-jw\varepsilon}\mathbf{I}_5-e^{ju_1}e^{-jw\varepsilon}\mathbf{I}_6).\end{aligned}$$

Under limit $\varepsilon\to 0$, we have

$$\begin{aligned}\frac{\partial\mathbf{F}(u_1,w,t)}{\partial t}&=\mathbf{F}(u_1,w,t)(\mathbf{A}+e^{ju_1}\mathbf{B}_1+\mathbf{B}_2)+\\+j\sigma_1&\frac{\partial\mathbf{F}(u_1,w,t)}{\partial u_1}(\mathbf{I}_1-e^{-ju_1}\mathbf{I}_2-e^{-ju_1}\mathbf{I}_3)+\\+j\gamma_2&\frac{\partial\mathbf{F}(u_1,w,t)}{\partial w}(\mathbf{I}_4-\mathbf{I}_5-e^{ju_1}\mathbf{I}_6).\end{aligned}\tag{4}$$

Let the solution have the following form:

$$\mathbf{F}(u_1,w,t)=\mathbf{H}_1(u_1,t)e^{jw\cdot x_2}.$$

By substituting in Equation (4), we have

$$\begin{aligned}\frac{\partial \mathbf{H}_1(u_1,t)}{\partial t} &= \mathbf{H}_1(u_1,t)(\mathbf{A}+\mathbf{B}_2+\gamma_2 x_2(\mathbf{I}_5-\mathbf{I}_4)+e^{ju_1}(\mathbf{B}_1+\gamma_2 x_2 \mathbf{I}_6))+\\ &+j\sigma_1\frac{\partial \mathbf{H}_1(u_1,t)}{\partial u_1}(\mathbf{I}_1-e^{-ju_1}(\mathbf{I}_2+\mathbf{I}_3)).\end{aligned} \tag{5}$$

In the same way, the following marginal asymptotic equation for process $i_2(t)$ (when $\sigma_1 \to 0$) can be derived:

$$\begin{aligned}\frac{\partial \mathbf{H}_2(u_2,t)}{\partial t} &= \mathbf{H}_2(u_2,t)(\mathbf{A}+\mathbf{B}_1+\gamma_1 x_1(\mathbf{I}_2-\mathbf{I}_1)+e^{ju_2}(\mathbf{B}_2+\gamma_1 x_1 \mathbf{I}_3))+\\ &+j\sigma_2\frac{\partial \mathbf{H}_2(u_2,t)}{\partial u_2}(\mathbf{I}_4-e^{-ju_2}(\mathbf{I}_5+\mathbf{I}_6)).\end{aligned} \tag{6}$$

3.2. Asymptotic means

The next step of the study is finding of parameters x_n, $n=1,2$. Let us write Equation (3) in the steady state

$$\begin{aligned}&\mathbf{H}(u_1,u_2)(\mathbf{A}+e^{ju_1}\mathbf{B}_1+e^{ju_2}\mathbf{B}_2)+j\sigma_1\frac{\partial \mathbf{H}(u_1,u_2)}{\partial u_1}(\mathbf{I}_1-e^{-ju_1}\mathbf{I}_2-e^{-ju_1}e^{ju_2}\mathbf{I}_3)+\\ &+j\sigma_2\frac{\partial \mathbf{H}(u_1,u_2)}{\partial u_2}(\mathbf{I}_4-e^{-ju_2}\mathbf{I}_5-e^{ju_1}e^{-ju_2}\mathbf{I}_6)=\mathbf{0}.\end{aligned} \tag{7}$$

For obtaining an additional scalar equation, we multiply (7) by unit vector $\mathbf{e}$. Taking into account the form of matrix $\mathbf{A}$, $\mathbf{B}_1$, $\mathbf{B}_2$ and $\mathbf{I}_v$ $(v=\overline{1,6})$, we obtain

$$\begin{aligned}&\mathbf{H}(u_1,u_2)((e^{ju_1}-1)\mathbf{B}_1+(e^{ju_2}-1)\mathbf{B}_2)\mathbf{e}+\\ &+j\sigma_1\frac{\partial \mathbf{H}(u_1,u_2)}{\partial u_1}((1-e^{-ju_1})\mathbf{I}_2+(1-e^{-ju_1}e^{ju_2})\mathbf{I}_3)\mathbf{e}+\\ &+j\sigma_2\frac{\partial \mathbf{H}(u_1,u_2)}{\partial u_2}((1-e^{-ju_2})\mathbf{I}_5+(1-e^{ju_1}e^{-ju_2})\mathbf{I}_6)\mathbf{e}=0.\end{aligned} \tag{8}$$

Let us use the following substitutions:

$$\sigma_n=\gamma_n\sigma,\ \sigma=\varepsilon,\ u_n=\varepsilon w_n,\ \mathbf{H}(u_1,u_2)=F(w_1,w_2,\varepsilon),\ n=1,2.$$

From Equations (7)–(8), we obtain the following system:

$$\begin{cases}\mathbf{F}(w_1,w_2,\varepsilon)(\mathbf{A}+e^{jw_1\varepsilon}\mathbf{B}_1+e^{jw_2\varepsilon}\mathbf{B}_2)+\\ +j\gamma_1\dfrac{\partial \mathbf{F}(w_1,w_2,\varepsilon)}{\partial w_1}(\mathbf{I}_1-e^{-jw_1\varepsilon}\mathbf{I}_2-e^{-jw_1\varepsilon}e^{jw_2\varepsilon}\mathbf{I}_3)+\\ +j\gamma_2\dfrac{\partial \mathbf{F}(w_1,w_2,\varepsilon)}{\partial w_2}(\mathbf{I}_4-e^{-jw_2\varepsilon}\mathbf{I}_5-e^{jw_1\varepsilon}e^{-jw_2\varepsilon}\mathbf{I}_6)=\mathbf{0},\\ \mathbf{F}(w_1,w_2,\varepsilon)((e^{jw_1\varepsilon}-1)\mathbf{B}_1+(e^{jw_2\varepsilon}-1)\mathbf{B}_2)\mathbf{e}+\\ +j\gamma_1\dfrac{\partial \mathbf{F}(w_1,w_2,\varepsilon)}{\partial w_1}((1-e^{-jw_1\varepsilon})\mathbf{I}_2+(1-e^{-jw_1\varepsilon}e^{jw_2\varepsilon})\mathbf{I}_3)\mathbf{e}+\\ +j\gamma_2\dfrac{\partial \mathbf{F}(w_1,w_2,\varepsilon)}{\partial w_2}((1-e^{-jw_2\varepsilon})\mathbf{I}_5+(1-e^{jw_1\varepsilon}e^{-jw_2\varepsilon})\mathbf{I}_6)\mathbf{e}=0.\end{cases} \tag{9}$$

Using Maclaurin series, Equations (9) are written under $\varepsilon \to 0$ as follows

$$\begin{cases} \mathbf{F}(w_1, w_2)(\mathbf{A} + \mathbf{B}_1 + \mathbf{B}_2) + j\gamma_1 \dfrac{\partial \mathbf{F}(w_1, w_2)}{\partial w_1}(\mathbf{I}_1 - \mathbf{I}_2 - \mathbf{I}_3) + \\ + j\gamma_2 \dfrac{\partial \mathbf{F}(w_1, w_2)}{\partial w_2}(\mathbf{I}_4 - \mathbf{I}_5 - \mathbf{I}_6) = \mathbf{0}, \\ \mathbf{F}(w_1, w_2)(w_1\mathbf{B}_1 + w_2\mathbf{B}_2)\mathbf{e} + j\gamma_1 \dfrac{\partial \mathbf{F}(w_1, w_2)}{\partial w_1}(w_1\mathbf{I}_2 + (w_1 - w_2)\mathbf{I}_3)\mathbf{e} + \\ + j\gamma_2 \dfrac{\partial \mathbf{F}(w_1, w_2)}{\partial w_2}(w_2\mathbf{I}_5 + (w_2 - w_1)\mathbf{I}_6)\mathbf{e} = 0. \end{cases} \tag{10}$$

Let us find the solution in the following form:

$$\mathbf{F}(w_1, w_2) = \mathbf{R} \cdot \exp\{jw_1x_1 + jw_2x_2\}.$$

By substituting into (10), we get

$$\begin{cases} \mathbf{R}(\mathbf{A} + \mathbf{B}_1 + \mathbf{B}_2) - \gamma_1 x_1 \mathbf{R}(\mathbf{I}_1 - \mathbf{I}_2 - \mathbf{I}_3)\mathbf{e} - \gamma_2 x_2 \mathbf{R}(\mathbf{I}_4 - \mathbf{I}_5 - \mathbf{I}_6) = \mathbf{0}, \\ \mathbf{R}\mathbf{B}_1\mathbf{e} - \gamma_1 x_1 \mathbf{R}(\mathbf{I}_2 + \mathbf{I}_3)\mathbf{e} + \gamma_2 x_2 \mathbf{R}\mathbf{I}_6\mathbf{e} = 0, \\ \mathbf{R}\mathbf{B}_2\mathbf{e} + \gamma_1 x_1 \mathbf{R}\mathbf{I}_3\mathbf{e} - \gamma_2 x_2 \mathbf{R}(\mathbf{I}_5 + \mathbf{I}_6)\mathbf{e} = 0. \end{cases}$$

By solving the system above and taking into account the normalization condition $\mathbf{Re} = 1$, we can obtain the values of stationary probabilities of server states R_k and asymptotic stationary means $M\{i_1(t)\} = \gamma_1 x_1/\sigma_1$ and $M\{i_2(t)\} = \gamma_2 x_2/\sigma_2$.

3.3. Asymptotic-diffusion analysis

Let us solve asymptotic Equations (5)–(6). First of all, we write this equations in general form as follows

$$\frac{\partial \mathbf{H}(u,t)}{\partial t} = \mathbf{H}(u,t)(\mathbf{A}_1 + e^{ju}\mathbf{A}_2) + j\sigma \frac{\partial \mathbf{H}(u,t)}{\partial u}(\mathbf{J}_1 - e^{-ju}\mathbf{J}_2), \tag{11}$$

where matrices $\mathbf{A}_1, \mathbf{A}_2, \mathbf{J}_1, \mathbf{J}_2$ have the following values for corresponding classes of customers:

– for the first class of customers (process $i_1(t)$):

$$\begin{aligned} \mathbf{A}_1^{(1)} &= \mathbf{A} + \mathbf{B}_2 + \gamma_2 x_2(\mathbf{I}_5 - \mathbf{I}_4), \\ \mathbf{A}_2^{(1)} &= \mathbf{B}_1 + \gamma_2 x_2 \mathbf{I}_6, \\ \mathbf{J}_1^{(1)} &= \mathbf{I}_1, \\ \mathbf{J}_2^{(1)} &= \mathbf{I}_2 + \mathbf{I}_4; \end{aligned} \tag{12}$$

– for the second class of customers (process $i_2(t)$):

$$\begin{aligned} \mathbf{A}_1^{(2)} &= \mathbf{A} + \mathbf{B}_1 + \gamma_1 x_1(\mathbf{I}_2 - \mathbf{I}_1), \\ \mathbf{A}_2^{(2)} &= \mathbf{B}_2 + \gamma_1 x_1 \mathbf{I}_3, \\ \mathbf{J}_1^{(2)} &= \mathbf{I}_4, \\ \mathbf{J}_2^{(2)} &= \mathbf{I}_5 + \mathbf{I}_6. \end{aligned} \tag{13}$$

By multiplying Equation (11) by unit vector $\mathbf{e}$, we have an additional equation:

$$\frac{\partial \mathbf{H}(u,t)\mathbf{e}}{\partial t} = (e^{ju} - 1)\left(\mathbf{H}(u,t)\mathbf{A}_2 + j\sigma \frac{\partial \mathbf{H}(u,t)}{\partial u} e^{-ju}\mathbf{J}_2\right)\mathbf{e}. \tag{14}$$

3.3.1. First asymptotics

Let us denote

$$\sigma = \varepsilon,\ \sigma t = \varepsilon t = \tau,\ u = \varepsilon w,\ \mathbf{H}(u,t) = \mathbf{F}(w,\tau,\varepsilon).$$

Then equations (11) and (14) are rewritten as

$$\varepsilon\frac{\partial \mathbf{F}(w,\tau,\varepsilon)}{\partial \tau} = \mathbf{F}(w,\tau,\varepsilon)(\mathbf{A}_1 + e^{jw\varepsilon}\mathbf{A}_2) + j\frac{\partial \mathbf{F}(w,\tau,\varepsilon)}{\partial w}(\mathbf{J}_1 - e^{-jw\varepsilon}\mathbf{J}_2), \tag{15}$$

$$\varepsilon\frac{\partial \mathbf{F}(w,\tau,\varepsilon)\mathbf{e}}{\partial \tau} = (e^{jw\varepsilon} - 1)\left(\mathbf{F}(w,\tau,\varepsilon)\mathbf{A}_2 + j\frac{\partial \mathbf{F}(w,\tau,\varepsilon)}{\partial w}e^{-jw\varepsilon}\mathbf{J}_2\right)\mathbf{e}. \tag{16}$$

Under $\varepsilon \to 0$, Equation (15) has the form

$$\mathbf{F}(w,\tau)(\mathbf{A}_1 + \mathbf{A}_2) + j\frac{\partial \mathbf{F}(w,\tau)}{\partial w}(\mathbf{J}_1 - \mathbf{J}_2) = \mathbf{0}. \tag{17}$$

We assume that the solution has the following form

$$\mathbf{F}(w,\tau) = \mathbf{R}\cdot e^{jwx(\tau)}, \tag{18}$$

where values of vector $\mathbf{R}$ depend on $x(\tau)$ too.

By substituting (18) into Equation (17) and taking into account normalization condition, we obtain the following system for $\mathbf{R}$ finding:

$$\begin{cases} \mathbf{R}(\mathbf{A}_1 + \mathbf{A}_2) - x(\tau)\mathbf{R}(\mathbf{J}_1 - \mathbf{J}_2) = \mathbf{0}, \\ \mathbf{Re} = 1. \end{cases} \tag{19}$$

By using Maclaurin series in Equation (16), we have the following equation under $\varepsilon \to 0$

$$\frac{\partial \mathbf{F}(w,\tau)\mathbf{e}}{\partial \tau} = jw\left(\mathbf{F}(w,\tau)\mathbf{A}_2 + j\frac{\partial \mathbf{F}(w,\tau)}{\partial w}\mathbf{J}_2\right)\mathbf{e}. \tag{20}$$

Substituting (18) into (20), we obtain that

$$\frac{dx(\tau)}{d\tau} = (\mathbf{RA}_2 - x(\tau)\mathbf{RJ}_2)\,\mathbf{e}.$$

A derived equation is a differential equation for $x(\tau)$ with a transfer coefficient:

$$a(x) = \mathbf{RA}_2\mathbf{e} - x(\tau)\mathbf{RJ}_2\mathbf{e}. \tag{21}$$

3.3.2. Second asymptotics

Let us make the following substitution:

$$\mathbf{H}(u,t) = \mathbf{H}^{(2)}(u,t)\exp\left\{\frac{ju}{\sigma}x(\sigma t)\right\}. \tag{22}$$

By substituting (22) into Equations (11) and (14), we have:

$$\begin{gathered} \frac{\partial \mathbf{H}^{(2)}(u,t)}{\partial t} + jua(x)\mathbf{H}^{(2)}(u,t) = j\sigma\frac{\partial \mathbf{H}^{(2)}(u,t)}{\partial u}(\mathbf{J}_1 - e^{-ju}\mathbf{J}_2) + \\ + \mathbf{H}^{(2)}(u,t)\left(\mathbf{A}_1 + e^{ju}\mathbf{A}_2 - x(\sigma t)(\mathbf{J}_1 - e^{-ju}\mathbf{J}_2)\right), \end{gathered} \tag{23}$$

$$\frac{\partial \mathbf{H}^{(2)}(u,t)\mathbf{e}}{\partial t} + jua(x)\mathbf{H}^{(2)}(u,t)\mathbf{e} = \\ = (e^{ju}-1)\left(\mathbf{H}^{(2)}(u,t)(\mathbf{A}_2 - e^{-ju}x(\sigma t)\mathbf{J}_2) + j\sigma e^{-ju}\frac{\partial \mathbf{H}^{(2)}(u,t)}{\partial u}\mathbf{J}_2\right)\mathbf{e}. \tag{24}$$

Let us denote

$$\sigma = \varepsilon^2, \quad \sigma t = \varepsilon^2 t = \tau, \quad u = \varepsilon w, \quad \mathbf{H}^{(2)}(u,t) = \mathbf{F}^{(2)}(w,\tau,\varepsilon).$$

Thus, Equations (23)–(24) are rewritten as follows

$$\varepsilon^2\frac{\partial \mathbf{F}^{(2)}(w,\tau,\varepsilon)}{\partial \tau} + jw\varepsilon a(x)\mathbf{F}^{(2)}(w,\tau,\varepsilon) = j\varepsilon\frac{\partial \mathbf{F}^{(2)}(w,\tau,\varepsilon)}{\partial w}(\mathbf{J}_1 - e^{-jw\varepsilon}\mathbf{J}_2) + \\ + \mathbf{F}^{(2)}(w,\tau,\varepsilon)\left(\mathbf{A}_1 + e^{jw\varepsilon}\mathbf{A}_2 - x(\tau)(\mathbf{J}_1 - e^{-jw\varepsilon}\mathbf{J}_2)\right), \tag{25}$$

$$\varepsilon^2\frac{\partial \mathbf{F}^{(2)}(w,\tau,\varepsilon)\mathbf{e}}{\partial \tau} + jw\varepsilon a(x)\mathbf{F}^{(2)}(w,\tau,\varepsilon)\mathbf{e} = \\ = (e^{jw\varepsilon}-1)\left(\mathbf{F}^{(2)}(w,\tau,\varepsilon)(\mathbf{A}_2 - e^{-jw\varepsilon}x(\tau)\mathbf{J}_2) + j\varepsilon e^{-jw\varepsilon}\frac{\partial \mathbf{F}^{(2)}(w,\tau,\varepsilon)}{\partial w}\mathbf{J}_2\right)\mathbf{e}. \tag{26}$$

For simplicity of notation, in further expressions we will write $x(\tau) = x$.

Let the solution be in the following form

$$\mathbf{F}^{(2)}(w,\tau,\varepsilon) = \Phi(w,\tau)(\mathbf{R} + jw\varepsilon\mathbf{f}) + \mathbf{O}(\varepsilon^2). \tag{27}$$

By substituting (27) into Equation (25), after some transforms we obtain

$$\varepsilon\frac{\partial \Phi(w,\tau)}{\partial \tau}(\mathbf{R} + jw\varepsilon\mathbf{f}) + jw\varepsilon a(x)\Phi(w,\tau)(\mathbf{R} + jw\varepsilon\mathbf{f}) = \\ = \Phi(w,\tau)\mathbf{R}(jw\mathbf{A}_2 - jwx\mathbf{J}_2) + jw\Phi(w,\tau)\mathbf{f}(\mathbf{A}_1 + \mathbf{A}_2 + x(\mathbf{J}_2 - \mathbf{J}_1)) + \\ + j\frac{\partial \Phi(w,\tau)}{\partial w}(\mathbf{R} + jw\varepsilon\mathbf{f})(\mathbf{J}_1 - (1 - jw\varepsilon)\mathbf{J}_2) - \\ - \varepsilon\Phi(w,\tau)\mathbf{f}(\mathbf{J}_1 - (1 - jw\varepsilon)\mathbf{J}_2) + \mathbf{O}(\varepsilon^2).$$

Under $\varepsilon \to 0$, we have

$$\mathbf{f}(\mathbf{A}_1 + \mathbf{A}_2 + x(\mathbf{J}_2 - \mathbf{J}_1)) = a(x)\mathbf{R} - \mathbf{R}(\mathbf{A}_2 - x\mathbf{J}_2) - \frac{\partial \Phi(w,\tau)/\partial w}{w\Phi(w,\tau)}\mathbf{R}(\mathbf{J}_1 - \mathbf{J}_2).$$

It is obvious that vector **f** can be written as the sum

$$\mathbf{f} = C\mathbf{R} + \mathbf{g} - \mathbf{q}\frac{\partial \Phi(w,\tau)/\partial w}{w\Phi(w,\tau)}, \tag{28}$$

where $C = const$, vectors **g** and **q** are defined by the following equations:

$$\begin{cases} \mathbf{g}(\mathbf{A}_1 + \mathbf{A}_2 + x(\mathbf{J}_2 - \mathbf{J}_1)) = a(x)\mathbf{R} - \mathbf{R}(\mathbf{A}_2 - x\mathbf{J}_2), \\ \mathbf{q}(\mathbf{A}_1 + \mathbf{A}_2 + x(\mathbf{J}_2 - \mathbf{J}_1)) = \mathbf{R}(\mathbf{J}_1 - \mathbf{J}_2), \\ \mathbf{ge} = 0, \\ \mathbf{qe} = 0. \end{cases}$$

By comparing the second equation with (19), we can note that $\mathbf{q} = \mathbf{R}'(x)$.

For finding function $\Phi_n(w,\tau)$, we substitute Expression (27) into Equation (26)

$$\varepsilon^2\frac{\partial\Phi(w,\tau)}{\partial\tau}+jw\varepsilon a(x)\Phi(w,\tau)(1+jw\varepsilon\mathbf{fe})=$$
$$=jw\varepsilon\Phi(w,\tau)(\mathbf{R}+jw\varepsilon\mathbf{f})(\mathbf{A}_2-(1-jw\varepsilon)x(\tau)\mathbf{J}_2)\mathbf{e}+$$
$$+\frac{(jw\varepsilon)^2}{2}\Phi(w,\tau)\mathbf{R}(\mathbf{A}_2-x(\tau)\mathbf{J}_2)\mathbf{e}+(j\varepsilon)^2w\frac{\partial\Phi(w,\tau)}{\partial w}\mathbf{RJ}_2\mathbf{e}+O(\varepsilon^3).$$

After some transforms, we obtain

$$\frac{\partial\Phi(w,\tau)}{\partial\tau}=(jw)^2\Phi(w,\tau)\times$$
$$\times\left(-a(x)\mathbf{fe}+\mathbf{f}(\mathbf{A}_2-x\mathbf{J}_2)\mathbf{e}+x\mathbf{RJ}_2\mathbf{e}+\frac{\partial\Phi(w,\tau)/\partial w}{w\Phi(w,\tau)}\mathbf{RJ}_2\mathbf{e}+\frac{a(x)}{2}\right)+O(\varepsilon).$$

Taking into account (28), under $\varepsilon\to 0$ we finally have

$$\frac{\partial\Phi(w,\tau)}{\partial\tau}=(jw)^2\Phi(w,\tau)\times$$
$$\times\left(\mathbf{g}(\mathbf{A}_2-x\mathbf{J}_2)\mathbf{e}+x\mathbf{RJ}_2\mathbf{e}+\frac{\partial\Phi(w,\tau)/\partial w}{w\Phi(w,\tau)}(\mathbf{RJ}_2\mathbf{e}-\mathbf{q}(\mathbf{A}_2-x\mathbf{J}_2)\mathbf{e})+\frac{a(x)}{2}\right).$$

So, we obtain the following equation

$$\frac{\partial\Phi(w,\tau)}{\partial\tau}=w\frac{\partial\Phi(w,\tau)}{\partial w}a'(x)+\frac{(jw)^2}{2}\Phi(w,\tau)b(x),$$

where

$$b(x)=a(x)+2\left[\mathbf{g}(\mathbf{A}_2-x\mathbf{J}_2)+x\mathbf{RJ}_2\right]\mathbf{e}. \tag{29}$$

Let us introduce probability distribution density function

$$P(y,\tau)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{-jwy}\Phi(w,\tau)dw$$

of diffusion process $dy(\tau)=y(\tau)a'(x)d\tau+\sqrt{b(x)}dw(\tau)$, where $w(\tau)$ is a Wiener process. We can write the following Fokker-Planck equation for $P(y,\tau)$:

$$\frac{\partial P(y,\tau)}{\partial\tau}=-\frac{\partial}{\partial y}\left(P(y,\tau)ya'(x)\right)+\frac{1}{2}\frac{\partial^2}{\partial y^2}(P(y,\tau)b(x)).$$

Combining the results of two asymptotics, we introduce process $z(\tau)=x(\tau)+\varepsilon y(\tau)$, such as $dz(\tau)=a(z)d\tau+\sqrt{\sigma b(z)}dw(\tau)$. which is diffusion random process satisfying the following Fokker-Planck equation

$$\frac{\partial P(z,\tau)}{\partial\tau}=-\frac{\partial\left(P(z,\tau)a(z)\right)}{\partial z}+\frac{1}{2}\frac{\partial^2(P(z,\tau)\sigma b(z))}{\partial z^2}.$$

In steady state, it is easy to obtain the following expression for probability distribution density of process $z(\tau)$:

$$P(z)=\frac{C}{b(z)}\exp\left\{\frac{\sigma}{2}\int_0^z\frac{a(x)}{b(x)}dx\right\}.$$

Returning to processes $i_n(t)$, we conclude that stationary probabilities of numbers of each class customers in the orbit are calculated as follows:

$$P_n(i_n)=\frac{C}{b(\sigma_n i_n)}\exp\left\{\frac{\sigma_n}{2}\int_0^{\sigma_n i_n}\frac{a_n(x)}{b_n(x)}dx\right\}, \tag{30}$$

where $C=$ const obtained from the normalization condition, parameters $a_n(x)$ and $b_n(x)$ are defined by expressions (21) and (29) with corresponding matrices (12)–(13).

4. Results and discussion

In this way, it was shown that asymptotic marginal distributions of the number of calls of each class in the orbit have the form (30). Undoubtedly, in real stochastic processes of the numbers of k-class calls $i_k(t)$ (where $k = 1, 2$) are correlated, but the obtaining a two-dimensional distribution is difficult (or may be impossible). The asymptotic marginal method let us analyze marginal distributions with enough high accuracy. In addition we obtained expression for transfer coefficients (21), which can help us to define stability conditions of the system (if all $a_k(\infty) < 0$) or the condition of partial stability (if only $a_1(\infty) < 0$ or only $a_2(\infty) < 0$).

5. Conclusions

In the paper, we have proposed an original method of the marginal asymptotic-diffusion analysis of the two-class retrial queueing systems studying. We have considered non-classical model of retrial queues – systems with interruptions of servicing, furthermore, customers have probabilistic priority. Such model has not yet been studied analytically. The proposed model and analytical results allow to evaluate the effectiveness of various scenarios of multimodal data transmission with possibility priority e.g. by setting of interruption probabilities or delay rates.

Note that in real life (i.e. in communication networks) the most common situation is when one class of customers is prioritized ($s_1 = 1$ and $s_2 = 0$). The considered mathematical model is more general and the obtained results can be applied in different cases.

In further research, we plan to study more complex models of multimodal comunication networks, such as multiclass retrial queues with probabilistic priority, multiclass retrial queues with constant retrial rate and models with non-Poisson arrivals.

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Маргинальный асимптотически-диффузионный анализ двуклассовой RQ-системы с вероятностным приоритетом как математической модели сети связи с двумодальной информацией

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Аннотация. В работе исследуется RQ-система $M_2/M_2/1$ с вероятностным приоритетом и вытеснением заявок как модель двумодальной сети связи. На вход системы поступает два класса заявок, т.е. два потока. В системе имеется одно обслуживающее устройство (канал связи). Если входящая заявка застает прибор занятым заявкой того же класса, она идет на орбиту и осуществляет случайную задержку через экспоненциально распределенное случайное время. Если же на приборе находится заявка другого типа, то с некоторой вероятностью возможно прерывание обслуживания (вытеснение заявки). Необслуженная заявка уходит на орбиту. Обращаясь к прибору с орбиты, заявки действуют тем же образом. Время обслуживания каждой заявки распредлено экспоненциально. На орбите реализован протокол множественного доступа. В статье предложен оригинальный метод маргинального асимптотически-диффузионного анализа в условии большой задержки заявок на орбите для нахождения стационарных распределений вероятностей числа заявок каждого типа в системе.

Ключевые слова: RQ-системы, теория массового обслуживания, вероятностный приоритет, вытеснение, асимптотически-диффузионный анализ