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On cyclotron damping of longitudinal wave

Sergey P. Karnilovich[1], Konstantin P. Lovetskiy[1], Leonid A. Sevastianov[1,2],
Svetlana B. Strashnova[1], Yahya N. Shaar[1]

[1] *RUDN University, 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation*
[2] *Joint Institute for Nuclear Research, 6 Joliot-Curie St, Dubna, 141980, Russian Federation*

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Abstract. Average equations of motion of relativistic charged particles in the field of HF (high frequency) wave packets are obtained in the range of cyclotron resonance in the case of strong LF (low frequency) electric field. Strong electric field means that the characteristic velocity of the particle comparable with the electric drift velocity ($v \sim v_E$). It is shown that with taking into account the electric drift velocity, new mechanisms of damping of longitudinal waves become possible. The effect of a strong electrostatic field on the resonant interaction of relativistic particles with high-frequency waves, as well as the relativistic effect, on cyclotron resonance for a longitudinal wave, is analyzed. The analytical solution of the averaged system of equations in the quasi-relativistic approximation is analyzed, as well as a numerical experiment for the Langmuir wave under the condition of cyclotron resonance in the case of a strong electric field.

Key words and phrases: Electric drift velocity, damping, relativistic charged particles, strong electric field, longitudinal waves, high frequency wave packets, cyclotron resonance

1. Introduction

In the drift theory of the motion of charged particles in electromagnetic fields, two cases are distinguished: a “weak” electric field, when the velocity of electric drift is $V_E \sim \varepsilon V$, and a “strong” electric field, when $V_E \sim V$. Here V is the characteristic velocity of the particle, ε is a small parameter equal to the ratio of the gyroradius of the particle to the characteristic scale of inhomogeneity of a strong magnetic field. The case of a strong electrostatic field is fraught with certain difficulties even in the non-relativistic approximation [1]. In [1–4], a theory of the motion of charged particles in the field of wave packets in crossed electric and magnetic fields was constructed taking into account weakly relativistic effects.

In this paper, the interaction of cyclotron resonance for longitudinal waves propagating along a strong magnetic field is considered. It is assumed that the rate of electric drift is small compared to the speed of light in vacuum. Such a proposal is quite sufficient for solving many applied problems. The equations of motion of an advertising charged particle averaged over fast oscillations are obtained, taking into account the effects of quasi-stationary electric drift under the condition of cyclotron resonance.

2. Basic equations

The motion of a particle with charge q and rest mass m under the influence of HF (high frequency) electromagnetic field $\vec{E}_\sim,\vec{B}_\sim$ and LF fields $\vec{E}_0,\vec{B}_0$ is described by the equations [1]

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \alpha\vec{F} - \frac{1}{c^2}\vec{v}(\vec{v}' \cdot \vec{F}) + \alpha[\vec{\Omega} \cdot \vec{v}], \tag{1}$$

where $\vec{F}_{0,s} = \dfrac{q}{m}\vec{E}_{0,s}$, $\vec{\Omega}_{0,s} = \dfrac{q\vec{B}_{0,s}}{mc}$, $\alpha = 1 - \dfrac{v^2}{2c^2}$, $\vec{F} = \vec{F}_0 + \sum \vec{F}_s$, $\vec{\Omega} = \vec{\Omega}_0 + \sum \vec{\Omega}_s$, c – the speed of light.

The velocity vector has the form

$$\vec{v}' = v_{||}\vec{e}_1 + \vec{v}_E + \vec{v}_\perp(\vec{e}_2 \cos\theta_0 + \vec{e}_3 \sin\theta_0), \tag{2}$$

where Θ_0 is the gyrophase, $\vec{e}_1(\vec{r},t) = \vec{B}_0/B_0$, $\vec{e}_2(\vec{r},t)$, $\vec{e}_3(\vec{r},t)$ are the unit vectors. $\vec{v}_E = c[\vec{E}_0 \cdot \vec{e}_1]/B_0$ is the electric drift velocity. The electromagnetic fields $\vec{E}_\sim$,$\vec{B}_\sim$, are considered in eikonal approximation as:

$$\vec{E}_s = \sum \vec{\mathcal{E}}_s e^{i\Theta_s} + c.c., \quad \vec{B}_s = \sum \vec{\mathcal{B}}_s e^{i\Theta_s} + c.c., \quad 1 \leqslant s \leqslant m. \tag{3}$$

Here $\vec{\mathcal{E}}_s$,$\vec{\mathcal{B}}_s$ are slowly varying complex amplitudes and Θ_s is the fast phase (eikonal) of the s^{th} wave packets ($s = 1, 2, 3, ..., M$).

Phases Θ_s are considered as the independent variables which are described by the equations:

$$\frac{\mathrm{d}\Theta_s}{\mathrm{d}t} = \omega_s + \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} \cdot \vec{k}_s = v_s + \frac{v_\perp}{2}(\vec{e}\_e^{i\Theta_0} + c.c.)\vec{k}_s + \vec{k}_s\vec{v}_E. \tag{4}$$

The quantities

$$\omega_s(\vec{r},t) = -\frac{\mathrm{d}\Theta_s}{\mathrm{d}t}, \quad \vec{k}_s(\vec{r},t) = \nabla\Theta_s, \tag{5}$$

are the local frequency and the wave vector of the s^{th} wave packets, respectively.

Equations (1)–(4) together with Eq. (3) constitute a multi–periodic system, which can be simplified by smoothing over fast and nonresonant phases [1].

3. Average equations

In the range of cyclotron resonance, the corresponding combination of the phases $\Psi_{\mathrm{res}} = \Theta_0 + \Theta_s$, shold be corresponded as an "semifast" variable and an equation for resonant phases Ψ_{res} should be added to the equations for slow dynamic variables of particles. Smoothed equations of motion for a single particle interacting with the arbitrary s^{th} wave packet at the condition of the cyclotron resonance $v + \omega \cong 0$ have the form:

$$\frac{\mathrm{d}\vec{R}}{\mathrm{d}t} = \vec{v}_E + \vec{e}_1 v_{||} \equiv \vec{u}_0, \tag{6}$$

$$\begin{aligned}\frac{\mathrm{d}v_{||}}{\mathrm{d}t} &= \vec{v}_E\vec{e}_1' + \frac{v_\perp}{2}\mathrm{div}\vec{e}_1 + \left(\Gamma - \frac{v_{||}^2}{c^2}\right)F_{0||} - \\ &- \left\{\frac{i}{2}v_\perp\vec{\Omega} + \frac{v_\perp}{2c^2}(F_{s||} + \vec{e}_1[\vec{\Omega}_{s1}\vec{v}_E])\vec{v}_E + \frac{v_\perp v_{||}}{2c^2}\vec{F}_s\right\}\vec{e}\_e^{i\psi_{\mathrm{res}}} + c.c.,\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{\mathrm{d}v_\perp}{\mathrm{d}t} &= \frac{v_\perp}{2}(\mathrm{div}\vec{u}_0 - (\vec{e}_1(\vec{e}_1\nabla)\vec{u}_0)) + \frac{v_\perp v_{||}}{c^2}F_{0||} + \\ &+ \left\{\frac{1}{2}\Gamma\left(\vec{F}_s + \frac{i}{\omega_s}\left(v_{||}[\vec{k}_s\vec{F}_s] + i\left[\vec{v}_E[\vec{k}_s\vec{F}_s]\right]\right)\right) - \frac{1}{2c^2}\left(v_\perp^2\vec{F}_s + (\vec{u}_0 F_s)\vec{v}_E\right)\right\}\vec{e}\_e^{i\psi_{\mathrm{res}}} + c.c.,\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{\mathrm{d}\psi_{\mathrm{res}}}{\mathrm{d}t} &= \omega_0 + v - \frac{1}{2}\vec{e}_1 \cdot rot\vec{u}_0 - \frac{i}{2}\vec{e}_- \cdot \vec{e}_+' - \Omega_0\frac{v_E^2}{2c^2} + \vec{k}_s \cdot \vec{v}_E + \\ &+ \left\{\frac{i}{v_\perp}\left(\frac{1}{2}\Gamma\left(\vec{F}_s + \frac{i}{\omega_s}\left(v_{||}[\vec{k}_s,\vec{F}_s] + i\left[\vec{v}_E[\vec{k}_s,\vec{F}_s]\right]\right)\right)\right) - \frac{v_\perp}{2c^2}\left(\frac{[\vec{e}_1,\vec{k}_s]}{\omega_s} + i\vec{\omega}_0\right)\vec{F} \cdot \vec{v}_E\right\}\vec{e}\_e^{i\psi_{\mathrm{res}}} + c.c.,\end{aligned} \tag{9}$$

where

$$\omega_0 = \Gamma\Omega_0, \quad \Gamma = 1 - \frac{v_{||}^2 + v_\perp^2 + v_E^2}{2c^2}, \quad F_{s,0||} = \vec{F}_{s,0}\cdot\vec{e}_1, \quad k_{s||} = \vec{k}_s\vec{e}_1,$$

$$v_s = -\omega_s + k_{s||}v_{||}, \quad \vec{e}_\pm = \vec{e}_2 \pm i\vec{e}_3, \quad (...)' = \left(\frac{\partial}{\partial t} + v_{||}\vec{e}_1\cdot\nabla + \vec{v}_E\cdot\nabla\right)(...)$$

note, that Eqs. (6)–(9) take place only in the case of quasilongitudinal propagation of the wave with respect to $\vec{B}_0$.

4. Cyclotron resonance for a longitudinal wave

For simplicity let us consider Eqs. (6)–(9) in the case of the cyclotron resonance $\Psi_{\text{res}} = \Theta_0 + \Theta_s$ for a longitudinal wave $\vec{\mathcal{E}}_s||\vec{k}_s||\vec{B}_0$, $\psi_\Gamma = \theta_0 - \theta_s$, $\vec{B}_0 = const$, $\vec{E}_0 = const$, $\vec{k}_0 = const$, $\vec{\mathcal{E}}_s = \vec{\mathcal{E}}e^{i\alpha}$, $\vec{\mathcal{E}} = (0,0,\mathcal{E})$.

Then

$$\frac{d\vec{R}}{dt} = \vec{u}_0, \quad \frac{dv_{||}}{dt} = \frac{e\mathcal{E}v_E v_\perp}{2me^2}\sin(\psi_r + \alpha), \quad \frac{dv_\perp}{dt} = \frac{e\mathcal{E}v_E v_{||}}{2me^2}\sin(\psi_r + \alpha),$$
$$\frac{d\psi_r}{dt} = \Omega_0\left(\Gamma - \frac{v_E^2}{2c^2}\right) + \omega - kv_{||} + \frac{v_{||}v_E}{2c^2 v_\perp}\cdot\frac{e\mathcal{E}}{m}\cos(\psi_\Gamma + \alpha). \tag{10}$$

System (10) shows that the cyclotron resonance is possible, when the relativistic effects and the $\vec{v}_E$ drift velocity are taken into account, such a resonance is impossible in the case of a weak electric field $v \gg v_E$. To explain physical mechanism of this resonance it is necessary to use a new system, which is moving with a drift velocity. One can get the energy integral from Eqs. (10):

$$\varkappa^{-2} = (\acute{\xi}^2\tau^2 + \sin^2\xi)^{-\frac{1}{2}}, \quad \xi = \frac{1}{2}(\Psi_r + \alpha), \tag{11}$$

where

$$\varkappa^{-2} = \frac{2U}{H+U}, \quad \frac{1}{\tau^2} = 2U, \quad U = \frac{ke\mathcal{E}v_{0\perp}v_E}{2mc^2}\cdot v_{0\perp}$$

is an initial velocity of the transversal velocity $v_\perp$, H is the Hamilton function of the system (10).

If $|\varkappa| > 1$, the particle is trapped by the wave and if $|\varkappa| < 1$, the particle is untrapped by the wave. The sign of $\varkappa$ is chosen to coincide with the sign of $\acute{\xi}$.

The equation (11) has the same form as the equation for resonant particles in the case of electrostatic wave [2].

Then by standard methods [1–6] one can calculate the coefficient of cyclotron damping of the longitudinal wave under consideration:

$$\gamma(t) = \gamma_L\sum_n \frac{64}{\pi}\int_0^1 d\mathcal{X}\left\{\frac{-2\pi n\cdot\sin\left(\frac{\pi nt}{\mathcal{X}k\tau}\right)}{\mathcal{X}^5k^2(1-q^{2n})(1+q^{-2n})} + \frac{(2n+1)\pi^2\sin\left(\frac{(2n+1)\pi t}{\mathcal{X}k\tau}\right)}{k^2(1+q^{2n+1})(1+q^{-2n-1})}\right\}, \tag{12}$$

where

$$q = \exp\left(\frac{\pi K}{K}\right), \quad K = K(\varkappa)(1-\varkappa^2)^{\frac{1}{2}}, \quad K(\varkappa) = F\left(\frac{\pi}{2},\varkappa\right)$$

is the complete of elliptic integral of the first kind, γ_L is Landau damping coefficient.

5. Conclusion

Numerical solving the equation system with initial conditions and parameters match the ones in works [7–11], Langmuir wave was selected with frequency in the range from $\omega = 1.38\times10^9 s^{-1}$ to $2.39\times10^{11}s^{-1}$ and girophase $\Omega_0 = 4.60\times10^9 s^{-1}$ to $4.60\times10^{11}s^{-1}$ with a wavelength of $\lambda = 2.28$ cm to 2.5 cm. The electric drift velocity between the values $v_E = 3.0\times10^5\frac{\text{cm}}{\text{s}}$ and $5.0\times10^8\frac{\text{cm}}{\text{s}}$.

The results of the numerical solutions of the averaged equations for cyclotron resonance confirm the possibility of cyclotron resonance in longitudinal wave in the case of a relativistic particle. The development of energy the graph in figure 1, shows the resonant leg with growth of energy ($|\varkappa| > 1$), which is unstable so the particle goes shortly out of resonance ($|\varkappa| < 1$) with partly retention of the gained energy.

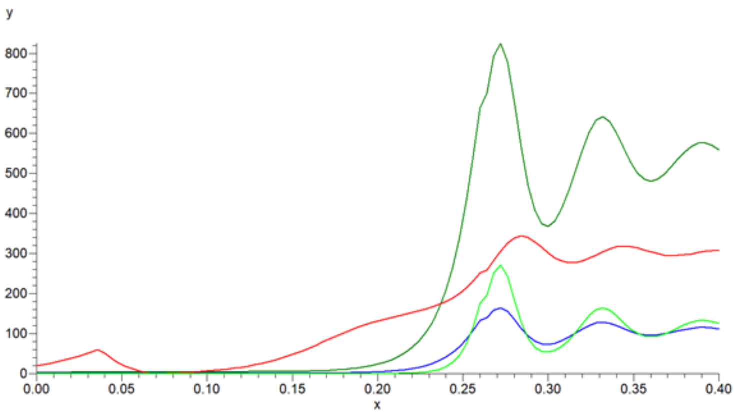

Figure 1. The graphs show the behavior of the particle's energy (bright green line), as well as the longitudinal and transverse velocities of the particle along the x axes (green and blue lines, respectively). The phase ψ_r is shown in graph by red line. The X-axis for velocity, energy, and resonant phase has dimension $t\omega_0$. The Y-axis for velocity has dimension v/c and for energy has dimension v^2/c^2

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Information about the authors

Karnilovich, Sergey P.—Candidate of Sciences in Physics and Mathematics, Assistant professor of Institute of Physical Research and Technology of Peoples' Friendship University of Russia named after Patrice Lumumba (RUDN University) (e-mail: karnilovich-sp@rudn.ru, ORCID: https://orcid.org/0000-0002-7669-7030)

Lovetskiy, Konstantin P.—Candidate of Sciences in Physics and Mathematics, Associate Professor of Department of Computational Mathematics and Artificial Intelligence of Peoples' Friendship University of Russia named after Patrice Lumumba (RUDN University) (e-mail: lovetskiy-kp@rudn.ru, phone: +7(495)952-25-72, ORCID: https://orcid.org/0000-0002-3645-1060)

Sevastianov, Leonid A.—Professor, Doctor of Sciences in Physics and Mathematics, Professor at the Department of Computational Mathematics and Artificial Intelligence of Peoples' Friendship University of Russia named after Patrice Lumumba (RUDN University), Leading Researcher of Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research (e-mail: sevastianov-la@rudn.ru, phone: +7(495)952-25-72, ORCID: https://orcid.org/0000-0002-1856-4643)

Strashnova, Svetlana B.—Candidate of Chemical Sciences, Associate Professor at the Department of General and Inorganic Chemistry of Peoples' Friendship University of Russia named after Patrice Lumumba (RUDN University) (e-mail: strashnova-sb@rudn.ru, ORCID: https://orcid.org/0000-0002-2588-504X)

Yahya N. Shaar—PhD in Physics and Mathematics, Assistant professor of Institute of Physical Research and Technology of Peoples' Friendship University of Russia named after Patrice Lumumba (RUDN University) (e-mail: al-shaar-ya@rudn.ru, ORCID: https://orcid.org/0000-0003-4951-5657)

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О циклотронном затухании продольной волны

С. П. Карнилович[1], К. П. Ловецкий[1], Л. А. Севастьянов[1,2], С. Б. Страшнова[1], Я. Н. Шаар[1]

[1] *Российский университет дружбы народов, ул. Миклухо-Маклая, д. 6, Москва, 117198, Российская Федерация*
[2] *Объединённый институт ядерных исследований, ул. Жолио-Кюри, д. 6, Дубна, 141980, Российская Федерация*

Аннотация. Выведены усредненные уравнения движения для релятивистских заряженных частиц в ВЧ поле (высокочастотных) волновых пакетов в диапазоне циклотронного резонанса в случае НЧ (низкочастотного) сильного электрического поля, где сильное электрическое поле означает, что характерная скорость частицы сравнима со скоростью электрического дрейфа ($v \sim v_E$). Показано, что при учете скорости электрического дрейфа становятся возможными новые механизмы затухания продольных волн. Проведен анализ влияния сильного электростатического поля на резонансное взаимодействие релятивистских частиц с высокочастотными волнами, а также влияние релятивизма на циклотронный резонанс для продольной волны. Получено аналитическое решение усредненной системы уравнений в квазирелятивистском приближении, а также проведен численный эксперимент для Ленгмюровской волны в случае циклотронного резонанса с учетом сильного электрического поля.

Ключевые слова: Скорость электрического дрейфа, затухание, релятивистские заряженные частицы, сильное электрическое поле, продольные волны, высокочастотные волновые пакеты, циклотронный резонанс