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Profile thickness synthesis of thin-film waveguide Luneburg lens

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Abstract. In the work the link between the focusing inhomogeneity of the effective refractive index of waveguide Luneburg lens and the irregularity of the waveguide layer thickness generating this inhomogeneity is demonstrated. For the dispersion relation of irregular thin-film waveguide in the model of adiabatic waveguide modes the problem of mathematical synthesis and computer-aided design of the waveguide layer thickness profile for the Luneburg thin-film generalized waveguide lens with a given focal length is being solved. The calculations are carried out in normalized (in a special way) coordinates to adapt the used relations to computer calculations. The obtained solution is compared with the same solution within the cross-section's method. The performance of the algorithm implemented in Delphi, was demonstrated by plotting the dispersion curves and plotting a family of dispersion curves, demonstrating a critical convergence. As an additional result, the thickness profiles of additional (irregular in thickness) waveguide layer, forming a thin film generalized waveguide Luneburg lens were synthesized. This result generalizes Southwell's results.

Key words and phrases: Luneburg waveguide lens, effective refractive index inhomogeneity, section method, waveguide adiabatic mode model

1. Model of adiabatic waveguide modes

Using the example of a thin film generalized waveguide Luneburg (TGWL) lens (see figure 1), which performs a two-dimensional Fourier transform with a finite aperture, the application of the adiabatic mode model is demonstrated. The inverse problem of synthesizing a thin film generalized waveguide Luneburg (TGWL) lens is solved within the framework of the model of adiabatic waveguide modes.

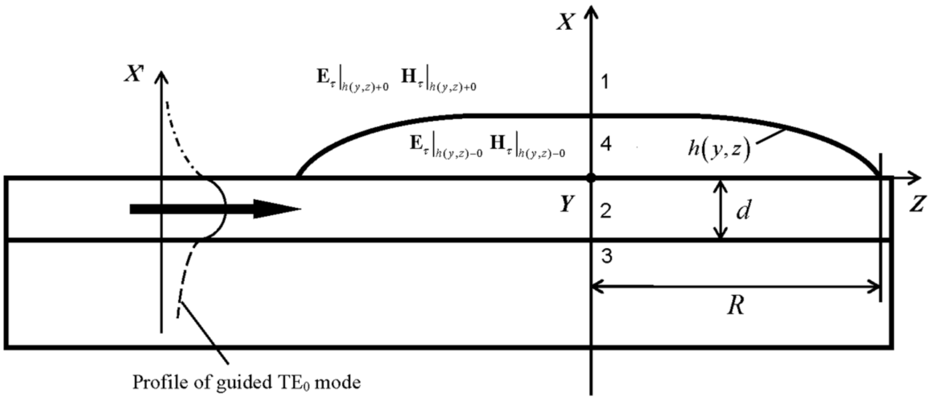

Figure 1. Cross section of the TGWL under consideration

Solutions of the Maxwell's equations for adiabatic waveguide modes are sought in the form [1]:

$$\begin{Bmatrix} \mathbf{E}(x,y,z,t) \\ \mathbf{H}(x,y,z,t) \end{Bmatrix} = \begin{Bmatrix} \tilde{\mathbf{E}}(x,y,z,t) \\ \tilde{\mathbf{H}}(x,y,z,t) \end{Bmatrix} \frac{\exp\{i\omega t - i\varphi(y,z)\}}{\sqrt{\beta(y,z)}},$$

where

$$\beta_y(y,z) = \frac{1}{k_0}\left(\frac{\partial\varphi}{\partial y}\right), \quad \beta_z(y,z) = \frac{1}{k_0}\left(\frac{\partial\varphi}{\partial z}\right),$$
$$\beta(y,z) = \sqrt{\beta_y(y,z)^2 + \beta_z(y,z)^2}.$$

In a multilayer waveguide, in the zeroth approximation, the equations for the leading components of the electromagnetic field of adiabatic waveguide modes take the form [2–5]:

$$\frac{d^2\tilde{E}_z^0}{d\tilde{x}^2} + (\tilde{\varepsilon}\tilde{\mu} - \tilde{\beta}^2)\tilde{E}_z^0 = 0, \quad \frac{d^2\tilde{H}_z^0}{d\tilde{x}^2} + (\tilde{\varepsilon}\tilde{\mu} - \tilde{\beta}^2)\tilde{H}_z^0 = 0.$$

The expressions for the remaining four components of the electromagnetic field take the form:

$$\tilde{H}_x^0 = \frac{1}{\tilde{\varepsilon}\tilde{\mu} - \tilde{\beta}_z^2}\left[-i\tilde{\beta}_z\frac{d\tilde{H}_z^0}{d\tilde{x}} + \tilde{\varepsilon}\tilde{\beta}_y\tilde{E}_z^0\right], \ \tilde{H}_y^0 = \frac{1}{\tilde{\varepsilon}\tilde{\mu} - \tilde{\beta}_z^2}\left[-i\tilde{\varepsilon}\frac{d\tilde{E}_z^0}{d\tilde{x}} - \tilde{\beta}_z\tilde{\beta}_y\tilde{H}_z^0\right],$$
$$\tilde{E}_x^0 = \frac{1}{\tilde{\varepsilon}\tilde{\mu} - \tilde{\beta}_z^2}\left[-i\tilde{\beta}_z\frac{d\tilde{E}_z^0}{d\tilde{x}} - \tilde{\mu}\tilde{\beta}_y\tilde{H}_z^0\right], \ \tilde{E}_y^0 = \frac{1}{\tilde{\varepsilon}\tilde{\mu} - \tilde{\beta}_z^2}\left[i\tilde{\mu}\frac{d\tilde{H}_z^0}{d\tilde{x}} - \tilde{\beta}_z\tilde{\beta}_y\tilde{E}_z^0\right].$$

In subdomains (waveguide layers) with constant refractive indices $n_j^2 = \varepsilon_j\mu_j$, solutions from the fundamental system of solutions are written as linear combinations of exponentials. Substituting them all into the boundary equations, after reducing similar terms, we obtain homogeneous linear equations for indefinite amplitude coefficients $\tilde{A}_s$, $\tilde{B}_s$, $\tilde{A}_f^{\pm}$, $\tilde{B}_f^{\pm}$, $\tilde{A}_l^{\pm}$, $\tilde{B}_l^{\pm}$, $\tilde{A}_c$, $\tilde{B}_c$. This system with a matrix admits non-trivial solutions under the condition [1]:

$$\det M(\beta(r)) = 0.$$

The solution of given nonlinear PDE of the first order with respect to $h(r)$ synthesizes the proper profile of Luneburg TGWL.

2. Mathematical synthesis of a thin film generalized waveguide Luneburg lens

In the works of Southwell [6, 7] the thickness profile of Luneburg TGWL with parameters $n_s = 1.47$, $n_f = 1.565$, $n_l = 2.10$, $n_c = 1.0$, $d = 1.0665\mu$, $\lambda = 0.5\mu$ was synthesized by the cross-section method in which precise boundary conditions for the Maxwell's equations are replaced by their approximations — projections on the horizontal plane [1]. The supporter of irregularity region of TGWL in the method of comparison waveguides automatically coincides with the supporter of $n_{\text{eff}}^j(r) = \beta_j(r)/\beta_j$ inhomogeneity of the two-dimensional TGWL of Luneburg: $\operatorname{supp} h = \operatorname{supp} n = \operatorname{supp} \beta$.

The method of adiabatic waveguide modes does not require the matching of these two supporters. However, the fact that the irregularity of TGWL forces to focus a family of curves locally orthogonal to the focused wave front of the waveguide mode, that the region of waveguide irregularity does not exceed the circle $Q(F)$ of the radius F: $\operatorname{supp} h \subseteq Q(F)$. Following this reasoning, we seek the thickness profile of the irregular waveguide layer $h(r)$ in a circle of radius F.

The equation [1] of the synthesis of the thickness profile of the irregular waveguide layer of Luneburg TGWL depends on the trajectories of the rays (parallel at the entrance to the irregularity region) and on their velocities derived from the equations for the rays that pass through the region of inhomogeneity. From ODE systems for $(y, z)^T(s)$ rays' equations

$$\frac{d}{ds}\left(\beta(y,z)\frac{dy}{ds}\right) = \frac{\partial\beta}{\partial y}(y,z), \quad \frac{d}{ds}\left(\beta(y,z)\frac{dz}{ds}\right) = \frac{\partial\beta}{\partial z}(y,s)$$

using substitution $\dfrac{dy}{dz} = V$, $A = \dfrac{1}{\beta}\dfrac{\partial\beta}{\partial z}$, $B = \dfrac{1}{\beta}\dfrac{\partial\beta}{\partial y}$, we obtain the equivalent system of ODEs other realization:

$$\frac{dy}{ds} = V, \quad \frac{dV}{dz} = (1 + V^2)(B - AV). \tag{1}$$

The Cauchy problem for the system (1) with the initial conditions

$$y(z_0) = y_0, \quad V(z_0) = 0, \tag{2}$$

we solve by the Runge–Kutta–Fehlberg method of the 6th order with automatically step selection [8] resulting in the family of "data" $y_j^F(z_k^j)$, $V_j^F(z_k^j)$ at the family of points, automatically generated while solving the problem (1), (2) for any beforehand defined focal distance F.

Data from the files $y_j^F(z_k^j)$, $V_j^F(z_k^j)$ allow calculating the components of the vector field of the phase delay $\vec{\beta} = \left(\beta_z, \beta_y\right)^T$ of AWM, for which $\beta(y,z) = \sqrt{\beta_y^2 + \beta_z^2}$ according to the formulas

$$\beta_y(z_k^j) = \frac{\beta(z_k^j) V(z_k^j)}{\left(1 + V^2(z_k^j)\right)^{1/2}}, \quad \beta_z(z_k^j) = \frac{\beta(z_k^j)}{\left(1 + V^2(z_k^j)\right)^{1/2}}. \tag{3}$$

The result of the calculations allows to complement the previously generated data file to $y_j^F(z_k^j)$, $V_j^F(z_k^j)$, $\beta_y^F(z_k^j)$, $\beta_z^F(z_k^j)$. Now we have at our disposal all the necessary data for the formulation of the problem of mathematical synthesis $h^F(r)$. The matrix $M(\beta^F)$ depends on the following variables

$$M(\beta^F) = M\begin{pmatrix} n_s, n_f, n_l, n_c, d; \{z_k^j, y_k^j, V_k^j\}; \beta^F(y_k^j, z_k^j), \gamma_s^F(y_k^j, z_k^j), \\ \gamma_c^F(y_k^j, z_k^j), \chi_f^F(y_k^j, z_k^j), \chi_l^F(y_k^j, z_k^j), \beta_y^F(y_k^j, z_k^j), \\ \beta_z^F(y_k^j, z_k^j); h^F(z_k^j, y_k^j), \dfrac{\partial h^F}{\partial y}(z_k^j, y_k^j), \dfrac{\partial h^F}{\partial z}(z_k^j, y_k^j), F \end{pmatrix}. \tag{4}$$

For the matrix $M(*; z_k^j, y_k^j; ..., F)$ at each point of «phase-ray mesh» $\{z_k^j, y_k^j\}^F$ the condition $\det M(*; z_k^j, y_k^j; ..., F) = 0$ must be fulfilled.

The approximation $h_N^F(r)$ of the function $h^F(r)$, which defines the thickness profile of the irregular waveguide layer, are sought in the form:

$$h_N^F(z,y) = \sum_{i=1}^{N} K_i \exp\left\{-\frac{(y-y_c)^2 + (z-z_c)^2}{C_i^2}\right\}, \tag{5}$$

which allows calculating the explicit form of derivatives (see figure 2).

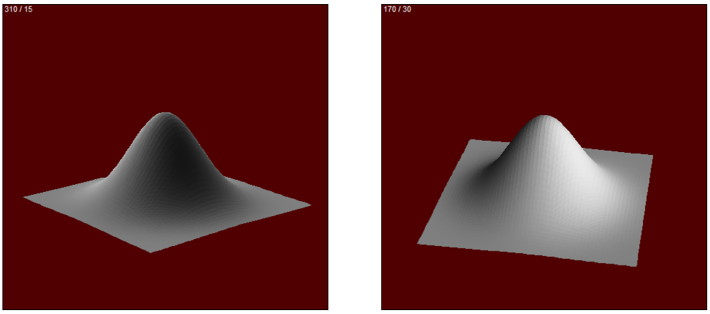

Figure 2. Plot of the approximate solution $h_N^F(z,y)$ of the thickness profile of the irregular waveguide layer of Luneburg TGWL

The search for the unknown coefficients (K_i, C_i) to construct an approximate solution $h^F(r)$ in (5) carried out by minimizing the objective function

$$F_{arg}(K_i, C_i) = \sum_{z_k^j, y_k^j} \left| \det M \left(h_N^F(z,y), \frac{\partial h_N^F(z,y)}{\partial y}, \frac{\partial h_N^F(z,y)}{\partial z} \right) \right|^2, \qquad (6)$$

by the Nelder–Mead method [9, 10].

3. Conclusion

The paper discusses the problems of numerical implementation of the cross-section method to calculate an evolution of the mode in a smooth transition from one planar regular open waveguide to another. In the class of multi-layer thin-film dielectric waveguides the problem on eigenvalues and eigen guided modes can be reduced (by expansion in the fundamental system of solutions in separate layers) to solving a system of linear algebraic equations for the coefficients of expansion in the fundamental system of solutions. The condition of non-triviality of the resulting solution in this case is the condition for the vanishing of the determinant of the corresponding system of linear algebraic equations.

The paper describes the classical and generalized Luneburg lens in bulk and waveguide implementation. Then the link between the focusing inhomogeneity of the effective refractive index of waveguide Luneburg lens and the irregularity of the waveguide layer thickness generating this inhomogeneity demonstrated. For the dispersion relation of irregular thin-film waveguide in the model of AWM the problem of mathematical synthesis and computer-aided design of the waveguide layer thickness profile for Luneburg TGWL with a given focal length is being solved.

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Синтез толщины профиля тонкоплёночной волноводной линзы Люнеберга

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Аннотация. В работе показана связь между фокусирующей неоднородностью эффективного показателя преломления волноводной линзы Люнеберга и неравномерностью толщины волноводного слоя, порождающей эту неоднородность. Для закона дисперсии нерегулярного тонкоплёночного волновода в модели адиабатических мод волновода решается задача математического синтеза и компьютерного проектирования профиля толщины волноводного слоя для тонкоплёночной обобщённой волноводной линзы Люнеберга с заданным фокусным расстоянием. Расчёты ведутся в нормированных специальным образом координатах для адаптации используемых соотношений к компьютерным расчётам. Полученное решение сравнивается с таким же решением в рамках метода сечений. Работоспособность алгоритма, реализованного в Delphi, была продемонстрирована путём построения дисперсионных кривых и семейства дисперсионных кривых, показывающих критическую сходимость. В качестве дополнительного результата были синтезированы профили толщины дополнительного нерегулярного по толщине волноводного слоя, образующего тонкоплёночную обобщённую волноводную линзу Люнеберга. Этот результат обобщает результаты Саутвелла.

Ключевые слова: волноводная линза Люнеберга, неоднородность эффективного показателя преломления, метод сечений, модель адиабатических мод волновода