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Some Problems of Modern Cosmology and Spinor Field

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In this report some burning problems of modern cosmology are discussed. The most popular models dealing with dark energy are also discussed in short. We specially stretch on the spinor model of fluid and dark energy. It is shown that the model with spinor field can resolve a number of standing problems of modern cosmology. It is noted that the non-trivial non-diagonal components of energy momentum tensor of spinor fields impose some severe restriction on the metric functions.

Key words and phrases: Bianchi type cosmological models, Energy momentum tensor.

1. Preface

In this conference we are going to celebrate 100 years anniversary of Prof. Yakov Petrovich Terletskii, who was the head of the Department of Theoretical Physics from its very foundation until his death in 1991. I came to Moscow in 1983 and after preparatory course joined the Faculty of Physics, Mathematics and Natural Science in 1984 as a student of first course. Beside me there were four from India: Partha Vadury, who left the University after first course and now a successful businessman in Germany; Sri Kumar, who finished the M.Sc in 1990 and now a social worker in Czech Republic, as far as I know got Gandhi medal for social work; Tamoghna Chattapadhya Rana, a brilliant student from West Bengal, after M.Sc. went to the USA and now a professor of management in NY University and Simanti Guha, who together with me finished the M.Sc. with red diploma and now working in Kerala as a school teacher. Every time when we saw Prof. Terletskii in the corridor, we looked at him with great admiration and bowed our heads as we did in India looking at the temple or divine person. Yes, for us he was someone like god. In 1984 when my senior compatriot Dr. Amal Halder, who is now a professor of Mathematics at the Dhaka University, left Moscow, I asked him who should I choose as a supervisor. He advised me to work with Yuri Petrovich Rybakov. And in 1987, being q student of third course, I approached Yuri Petrovich to be my supervisor. He had already three, Grisha Fomin (now works in VAK), Anna Zadunaeva (Living and working in Yaroslavl) and Alexei Upornikov (no idea where he is now). So Yuri Petrovich suggested me to work under Yakov Petrovich, but I was adamant and we decided to work together. The problem Yuri Petrovich set before me was related to the Einstein - de Broglie concept of quantum mechanics where we model the hydrogen atom substituting the electron with a soliton. Yakov Petrovich was also interested in this problem. So time and again when we found some results or we faced some trouble together with Yuri Petrovich we met Yakov Petrovich to discussed it. After completing M.Sc. in 1989 I stayed back to have my Ph.D. I was then working on soliton model is general relativity under Rybakov and Shikin, who were both the students of Terletski. Time and again he asked me about the research. And it was very sad that he died just one month before my defence. I still remember his active participation in pre-defence. He is still one of the best lecturer I have ever met. Though not directly, I believe, along with Rybakov and Shikin, he was my supervisor as well and I learnt a lot, from him.

In 2010 PFU celebrated 50 years anniversary. Last year our Faculty celebrated 50 years as well. During that ceremony I had the chance to ask our Rector to arrange this conference to commemorate Prof. Terletski on his hundredth birthday. 50 years is quite a good time to have own history. We had a lot of people to remember in the University. I think it is time to have the avenue of or heroes with Prof. Terletski being one of the first to have his statue there.

2. Introduction

Being related to almost all stable elementary particles such as proton, electron and neutrino, spinor field, especially Dirac spin-1/2 play a principal role at the microlevel. However, in cosmology, the role of spinor field was generally considered to be restricted. Only recently, after some remarkable works by different authors [1–9], showing the important role that spinor fields play on the evolution of the Universe, the situation began to change. This change of attitude is directly related to some fundamental questions of modern cosmology: (i) problem of initial singularity; (ii) problem of isotropization and (iii) late time acceleration of the Universe.

(i) Problem of initial singularity:

One of the problems of modern cosmology is the presence of initial singularity, which means the finiteness of time.

The first prediction for the expanding Universe has been made by Russian physicist A. A. Friedmann in 1922. But it remained unnoticed for a long time, as that time it was thought that the Universe is static. This model has gained big popularity only after the works of Robertson and Walker and became known as FRW model. This model describes a homogeneous and isotropic Universe.

It should be emphasized that the model of expanding Universe was experimentally supported by Hubble's 1929 findings, which in fact has buried idea of static character of the Universe forever. The model of expanding Universe suggested that it was once in an extremely hot and dense state which expanded rapidly. This rapid expansion caused the young Universe to cool and resulted in its present continuously expanding state. According to the most recent measurements and observations, this original state existed approximately 14 billion years ago, which is considered the age of the Universe and the time the Big Bang occurred.

(ii) Problem of isotropization:

Although the Universe seems homogenous and isotropic at present, it does not necessarily mean that it is also suitable for a description of the early stages of the development of the Universe and there are no observational data guaranteeing the isotropy in the era prior to the recombination. Moreover, there are two small deviations from isotropy.

The first, dipole anisotropy. It is related to the fact that we observe the relic radiation from Solar system.

The second one is quadrupole anisotropy. It is connected with the fact that for the formation of the future galaxies in the remote past in homogeneous and isotropic plasma there should exist some small heterogeneity which broke the general uniformity and isotropy.

The observations from Cosmic Background Explorer's (COBE) differential radiometer have detected and measured cosmic microwave background anisotropies in different angular scales.

In 1993 COBE received the first experimental acknowledgement of temperature fluctuation in some sites of the sky. WMAP (The Wilkinson Microwave Anisotropy Probe) image of the (extremely tiny) anisotropies in the cosmic background radiation.

These anisotropies are supposed to hide in their fold the entire history of cosmic evolution dating back to the recombination era and are being considered as indicative of the geometry and the content of the universe.

More about cosmic microwave background anisotropy is expected to be uncovered by the investigations of microwave anisotropy probe. There is widespread consensus among the cosmologists that cosmic microwave background anisotropies in small angular scales have the key to the formation of discrete structure.

In this connection it is natural to consider the models, which initially anisotropic, but in due course become isotropic.

(iii) Late time acceleration of the Universe:

Even some 15 years ago it was believed that the Universe is expanding with deceleration. So when in 1998 it was found that the Universe is expanding with acceleration, it comes like a bolt from the blue.

Detection and further experimental reconfirmation of current cosmic acceleration pose to cosmology a fundamental task of identifying and revealing the cause of such phenomenon. This fact can be reconciled with the theory if one assumes that the Universe is mostly filled with so-called dark energy. This form of matter (energy) is not observable in laboratory and it does not interact with electromagnetic radiation.

These facts played decisive role in naming this object. In contrast to dark matter dark energy is uniformly distributed over the space; does not intertwine under the influence of gravity in all scales; it has a strong negative pressure of the order of energy density.

Based on these properties, cosmologists have suggested a number of dark energy models, those are able to explain the current accelerated phase of expansion of the Universe.

What Dark Energy is?

More is unknown than is known. We know how much dark energy there is because we know how it affects the Universe's expansion. Other than that, it is a complete mystery. But it is an important mystery. It turns out that roughly 70% of the Universe is dark energy. Dark matter makes up about 25%. The rest - everything on Earth, everything ever observed with all of our instruments, all normal matter - adds up to less than 5% of the Universe.

One explanation for dark energy is that it is a property of space. Albert Einstein was the first person to realize that empty space is not nothing. Space has amazing properties, many of which are just beginning to be understood. The first property that Einstein discovered is that it is possible for more space to come into existence. Then one version of Einstein's gravity theory, the version that contains a cosmological constant, makes a second prediction: "empty space" can possess its own energy. Because this energy is a property of space itself, it would not be diluted as space expands. As more space comes into existence, more of this energy-of-space would appear. As a result, this form of energy would cause the Universe to expand faster and faster. Unfortunately, no one understands why the cosmological constant should even be there, much less why it would have exactly the right value to cause the observed acceleration of the Universe.

Another explanation for how space acquires energy comes from the quantum theory of matter. In this theory, "empty space" is actually full of temporary ("virtual") particles that continually form and then disappear. But when physicists tried to calculate how much energy this would give empty space, the answer came out wrong - wrong by a lot. The number came out 10^{120} times too big. That's a 1 with 120 zeros after it. It's hard to get an answer that bad. So the

Another explanation for dark energy is that it is a new kind of dynamical energy fluid or field, something that fills all of space but something whose effect on the expansion of the Universe is the opposite of that of matter and normal energy. Some theorists have named this "quintessence," after the fifth element of the Greek philosophers. But, if quintessence is the answer, we still don't know what it is like, what it interacts with, or why it exists. So the mystery continues.

Another possibility is that Einstein's theory of gravity is not correct. That would not only affect the expansion of the Universe, but it would also affect the way that normal matter in galaxies and clusters of galaxies behaved. This fact would provide a way to decide if the solution to the dark energy problem is a new gravity theory or not: we could observe how galaxies come together in clusters. But if it does turn out that a new theory of gravity is needed, what kind of theory would it be? How could it correctly describe the motion of the bodies in the Solar System, as Einstein's theory is known to do, and still give us the different prediction for the Universe that we need? There are candidate theories, but none are compelling. So the mystery continues.

3. Dark Energy Models

Λ term

To allow a steady state cosmological solution to the gravitational field equations Einstein introduced a fundamental constant, known as cosmological constant or Λ

term, into the system. Soon after E. Hubble had experimentally established that the Universe is expanding, Einstein returned to the original form of his equations citing his temporary modification of them as the biggest blunder of his life. Λ term made a temporary comeback in the late 60's. Finally after the pioneer paper by A. Guth on inflationary cosmology researchers began to study the models with Λ term with growing interest. With its introduction the Einstein field equations take the form:

$$G_\mu^\nu = R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R = -\varkappa T_\mu^\nu - \delta_\mu^\nu \Lambda. \tag{1}$$

Quintessence

The discovery that the expansion of the Universe is accelerating has promoted the search for new types of matter that can behave like a cosmological constant by combining positive energy density and negative pressure. This type of matter is often called *quintessence*. It was shown that "tracker field", a form of quintessence, may explain the coincidence, adding new motivation for the quintessence scenario. The quintessence obeys the equation of state

$$p_{\rm q} = w_{\rm q}\varepsilon_{\rm q}, \tag{2}$$

where the constant $w_{\rm q} \in [-1,\, 0]$.

K-essence

A key challenge for theoretical physics is to address the cosmic consequence problem: why does the dark energy component have a tiny energy density compared to the expectation based on the quantum field theory and why does the cosmic acceleration begin at such a late stage in the evolution of the universe. Most dark energy candidates require extraordinary fine-tuning of the initial energy.

The purpose of introducing k-essence is to provide a dynamical explanation which does not require the fine-tuning of initial condition or mass parameters and which is decidedly non-anthropic.

A further property of k-essence is that, because of the dynamical attractor behavior, cosmic evolution is insensitive to initial conditions.

The k-essence component has the property that it only behaves as a negative pressure component after the matter-radiation equality, so that it can only overtake the matter density and induce cosmic acceleration after the matter has dominated the universe for certain period. In general k-essence is defined as a scalar field with non-canonical kinetic energy and can be given by the Lagrangian

$$L_{\rm k} = K(\varphi)p(X), \quad X = (1/2)\nabla_\mu\varphi\nabla^\mu\varphi, \tag{3}$$

where $K(\varphi) > 0$.

Chaplygin Gas

An alternative model for the dark energy density was used by Kamenshchik *et al.*, where the authors suggested the use of some perfect fluid but obeying "exotic" equation of state:

$$p = -A/\varepsilon^\gamma. \tag{4}$$

with A being a positive constant. This type of matter is known as *Chaplygin gas.*

Modified Chaplygin Gas

Modified Chaplygin gas is the generalization of generalized Chaplygin gas $p = -B/\varepsilon^\gamma$ with the addition of a barotropic term $p = A\varepsilon$ and given by the EoS

$$p = A\varepsilon - B/\varepsilon^\alpha. \tag{5}$$

with A and B are the positive constants and $0 \leqslant \alpha \leqslant 1$. The MCG parameters α and B have been constrained by the cosmic microwave background CMB data. The MCG is able to explain the cosmic accelerated expansion and the EoS of MCG is valid from radiation era to Λ CDM model.

Phantom type dark energy

Until recently it was assumed that the standard cosmological source of dark energy must have a small negative pressure, such that $-\varepsilon < p < 0$, and under no condition the pressure should exceed the mysterious barrier $p = -\varepsilon = -\Lambda$, which corresponds to cosmological constant. In this case only strong energy condition may be violated

$$\varepsilon + 3p > 0, \quad \varepsilon + p > 0, \tag{6}$$

and the subsequent could follow one of the two scenarios: the Asymptotic Emptiness and the Big Crunch.

The phantom is dark energy with a strong negative pressure. It can be modeled by a scalar field with a negative kinetic energy given by the Lagrangian

$$L = \frac{l}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi), \tag{7}$$

where $l = -1$ corresponds to the phantom, while $l = 1$ to the standard scalar field. Here $V(\varphi)$ is a potential. The most striking result that is attributed to the phantom is that the energy density grows proportionally to the scale factor. Thus in contrast to the standard sources, when the increase of the energy density corresponds to the reduction of the scale factor, in this case the energy density's increase is accompanied by the Universe expansion. This leads to the appearance of singularities in the future known as **Big Rip**. In this case the Universe becomes infinite during a finite time.

Oscillating dark energy

The discovery of positive accelerations gives rise to a number of problems. One of the most baffling of those is the problem of eternal acceleration. There are many different approaches to eliminate this problem. A cosmological model of a cyclic Universe experiencing periodical expansions and contractions are proposed. Every cycle begins with a Big Bang, terminates with a Big Crunch only to begin with a Big Bang again. In these cases a negative cosmological constant is introduced together with a quintessence. Cosmological model with a quintessence with a modified equation of state

$$p = W(\varepsilon - \varepsilon_{\rm cr}), \quad W \in (-1,\, 0), \tag{8}$$

is also free from eternal acceleration. Here $\varepsilon_{\rm cr}$ some critical energy density. Setting $\varepsilon_{\rm cr} = 0$ one obtains ordinary quintessence.

Model with interaction between dark energy and dark matter

Experimental checks conducted within the solar system impose strict constrains on the possibility of nonminimal interaction between dark energy and dark matter. Nevertheless, a possibility of additional (non-gravitational) interaction between them without a contradiction with the experimental data, appears due to the unknown nature of the dark matter as the main fraction of that background. Moreover, it was established that the models with interacting dark energy are in good agreement with the modern observation data.

Scalar-tensor models of dark energy

Scalar-tensor theory of gravitation is an alternative to or generalization of Einstein's theory of gravitation, where a scalar field is present in addition to the tensor field. It was proposed almost half a century ago, and even at present time remains important for explaining the accelerating expansion phase, especially in the inflation and quintessence scenarios. The main assumption of this theory is a connection between the matter, and the scalar and gravitational fields φ and $g_{\mu\nu}$ via some effective metrics $\tilde{g}_{\mu\nu} = A^2(\varphi)g_{\mu\nu}$

Models with Tachyon Matter

Though were not observed experimentally, the tachyon models found their application is cosmology, mainly those with a very interesting equation of state, where the tachyon parameters exhibit smooth variations within the interval $(-1,\, 0)$ There are several models of tachyon dark energy. It is defined by using cosmological diagnostic pairs $[r,\, s]$ called the Statefinders:

$$r = \frac{\partial^3 a/\partial t^3}{aH^3}, \quad s = \frac{r-1}{3(q-1/2)}, \tag{9}$$

where q is the deceleration parameter, and a is the scale factor of FRW space-time. Since different cosmological models related to the dark energy yields qualitatively different trajectories on the $r-s$ plane, the proposed diagnostics can help to distinguish between these models.

Nonlinear Spinor field in BI Cosmology

Let us study the evolution of an anisotropic spacetime given by a Bianchi type-I model filled with spinor field. BI model is the simplest anisotropic cosmological model and gives an excellent scope to take into account the initial anisotropy of the Universe. Given the importance of BI model to study the effects of initial anisotropy in the evolution of he Universe, we study this models in details.

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a_1^2\mathrm{d}x^2 - a_2^2\mathrm{d}y^2 - a_3^2\mathrm{d}z^2. \tag{10}$$

Here metric functions a_1, a_2, a_3 depends on t only. In case of $a_1 = a_2 = a_3$ it transforms into a FRW model.

We consider the spinor field Lagrangian in the form

$$\mathcal{L} = \frac{i}{2}\left[\bar\psi\gamma^\mu\nabla_\mu\psi - \nabla_\mu\bar\psi\gamma^\mu\psi\right] - m\bar\psi\psi - F, \tag{11}$$

where the nonlinear term $F = F(I, J)$ describes the self-action of a spinor field and can be presented as some arbitrary functions of invariants generated from the real bilinear forms of a spinor field $I = S^2 = (\bar\psi\psi)^2$ and $J = P^2 = (i\bar\psi\gamma^5\psi)^2$. For simplicity we consider the case when $F = F(S)$ with $S = \bar\psi\psi$. Here ∇_μ is the covariant derivative of spinor field:

$$\nabla_\mu\psi = \frac{\partial\psi}{\partial x^\mu} - \Gamma_\mu\psi, \quad \nabla_\mu\bar\psi = \frac{\partial\bar\psi}{\partial x^\mu} + \bar\psi\Gamma_\mu, \tag{12}$$

with Γ_μ being the spinor affine connection.

Varying (11) with respect to $\bar\psi(\psi)$ one finds the spinor field equations:

$$i\gamma^\mu\nabla_\mu\psi - m\psi + (\mathrm{d}F/\mathrm{d}S)\psi = 0, \tag{13a}$$
$$i\nabla_\mu\bar\psi\gamma^\mu + m\bar\psi - (\mathrm{d}F/\mathrm{d}S)\bar\psi = 0, \tag{13b}$$

The energy-momentum tensor of the spinor field is given by

$$T_\mu^\rho = \frac{i}{4}g^{\rho\nu}\left(\bar\psi\gamma_\mu\nabla_\nu\psi + \bar\psi\gamma_\nu\nabla_\mu\psi - \nabla_\mu\bar\psi\gamma_\nu\psi - \nabla_\nu\bar\psi\gamma_\mu\psi\right) - \delta_\mu^\rho L_{\mathrm{sp}} \tag{14}$$

where L_{sp} in account of spinor field equations (13a) and (13b) takes the form

$$L_{\mathrm{sp}} = S(\mathrm{d}F/\mathrm{d}S) - F(S). \tag{15}$$

We consider the case when the spinor field depends on t only. In this case for the components of energy-momentum tensor we find

$$T_0^0 = mS + F = \varepsilon, \tag{16a}$$
$$T_1^1 = T_2^2 = T_3^3 = -S\frac{\mathrm{d}F}{\mathrm{d}S} + F = -p. \tag{16b}$$

In what follows, we study the evolution of the Universe for different types of nonlinearity.

The system of Einstein equations in this case reads

$$\frac{\ddot a_2}{a_2} + \frac{\ddot a_3}{a_3} + \frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3} = \kappa T_1^1, \tag{17a}$$

$$\frac{\ddot a_3}{a_3} + \frac{\ddot a_1}{a_1} + \frac{\dot a_3}{a_3}\frac{\dot a_1}{a_1} = \kappa T_2^2, \tag{17b}$$

$$\frac{\ddot a_1}{a_1} + \frac{\ddot a_2}{a_2} + \frac{\dot a_1}{a_1}\frac{\dot a_2}{a_2} = \kappa T_3^3, \tag{17c}$$

$$\frac{\dot a_1}{a_1}\frac{\dot a_2}{a_2} + \frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3} + \frac{\dot a_3}{a_3}\frac{\dot a_1}{a_1} = \kappa T_0^0. \tag{17d}$$

Solving the Einstein equation for the metric functions one finds

$$a_i = D_i V^{1/3} \exp\Bigl(X_i \int \frac{\mathrm{d}t}{V}\Bigr), \quad \prod_{i=1}^{3} D_i = 1, \quad \sum_{i=1}^{3} X_i = 0, \tag{18}$$

with D_i and X_i being the integration constants and $V = a_1 a_2 a_3$.

Summation of (17a), (17b), (17c) and 3 times (17d) gives the equation for V:

$$\ddot V = \frac{3\kappa}{2}[T_0^0 + T_1^1]V = \frac{3\kappa}{2}[mS + (2F - SF_S)]V. \tag{19}$$

From (13) it can be shown that

$$S = \frac{C_0}{V}, \quad C_0 = \text{const}. \tag{20}$$

Hence we may rewrite (19)

$$\ddot V = \mathcal{F}(V). \tag{21}$$

Before going to the detailed study of the corresponding equations, let us first consider the problems of initial singularity and isotropization.

Initial singularity

To study the initial singularity we note that it can be performed analyzing the Ricchi scalar and Kretschmann scalar. For BI metric we have

$$R = -2\Bigl[\sum_{i=1}^{3}\frac{\ddot a_i}{a_i} + \frac{\dot a_1}{a_1}\frac{\dot a_2}{a_2} + \frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3} + \frac{\dot a_3}{a_3}\frac{\dot a_1}{a_1}\Bigr], \tag{22}$$

and

$$\mathcal{K} = 4\Bigl[\sum_{i=1}^{3}\Bigl(\frac{\ddot a_i}{a_i}\Bigr)^2 + \Bigl(\frac{\dot a_1}{a_1}\frac{\dot a_2}{a_2}\Bigr)^2 + \Bigl(\frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3}\Bigr)^2 + \Bigl(\frac{\dot a_3}{a_3}\frac{\dot a_1}{a_1}\Bigr)^2\Bigr], \tag{23}$$

The metric functions and their derivatives for the BI take the following form:

$$\frac{\dot a_i}{a_i} = \frac{\dot V}{3V} + \frac{X_i}{3V}, \tag{24a}$$

$$\frac{\ddot a_i}{a_i} = \frac{\ddot V}{3V} - \frac{2}{9}\Bigl(\frac{\dot V}{V}\Bigr)^2 - \frac{X_i}{9}\frac{\dot V}{V^2} + \frac{X_i^2}{9V^2}. \tag{24b}$$

We study the singularity on the basis of Kretschmann scalar:

(i) For any finite t some $a_i \to 0$. (23) shows that if more than one scale factor becomes trivial at finite t, then it is a singularity. As is seen from (24) as $V \to 0$ (i) all $a_i \to 0$ if $X_1 = X_2 = X_3 = 0$, i.e., it is a singularity; (ii) more than one $a_i \to 0$ if more than one $X_i < 0$ and in this case we have singularity; (iii) only one $a_i \to 0$ if only one $X_i < 0$, i.e., the space-time can be non-singular.

Note that this criteria of singularity always fulfills at the point where $V = 0$.

As far as $t \to \infty$ is concerned, the corresponding asymptote can be singular if at least one a_i vanishes faster than exponentially. As $X_1 + X_2 + X_3 = 0$, it means one or more X_i is negative, and from (24) it follows that at least one function a_i vanishes faster than exponentially. Hence it is a singularity.

Thus we see that $V = 0$ is a singular space-time point, i.e., at this point the metric functions, the space-time invariants as well as the spinor field invariants such as spin current etc. becomes infinity due to the fact that $S = S_0/V$.

Isotropization

Since the present-day Universe is surprisingly isotropic, it is important to see whether our anisotropic BI model evolves into an isotropic FRW model. Isotropization means that at large physical times t, when the volume factor V tends to infinity, the three scale factors $a_i(t)$ grow at the same rate. Two wide-spread definition of isotropization read

$$\mathcal{A} = \frac{1}{3}\sum_{i=1}^{3}\frac{H_i^2}{H^2} - 1 \to 0, \tag{25a}$$

$$\Sigma^2 = \frac{1}{2}\mathcal{A}H^2 \to 0. \tag{25b}$$

Here $\mathcal{A}$ and Σ^2 are the average anisotottropy and shear, respectively. $H_i = \dot{a}_i/a_i$ is the directional Hubble parameter and $H = \dot{a}/a$ average Hubble parameter, where $a(t) = V^{1/3}$ is the average scale factor.

Here we exploit the isotropization condition

$$\frac{a_i}{a}\Big|_{t\to\infty} \to \text{const.} \tag{26}$$

Then by rescaling some of the coordinates, we can make $a_i/a \to 1$, and the metric will become manifestly isotropic at large t.

Taking into account that

$$\frac{a_i}{a} = \frac{a_i}{V^{1/3}} = D_i \exp\Big(X_i \int \frac{\mathrm{d}t}{V}\Big). \tag{27}$$

As is seen from (18) in our case $a_i/a \to D_i = \text{const}$ as $V \to \infty$. Recall that the isotropic FRW model has same scale factor in all three directions, i.e., $a_1(t) = a_2(t) = a_3(t) = a(t)$. So for the BI universe to evolve into a FRW one the constants D_i's are likely to be identical, i.e., $D_1 = D_2 = D_3 = 1$.

Moreover, the isotropic nature of the present Universe leads to the fact that the three other constants X_i should be close to zero as well, i.e., $|X_i| \ll 1$, $(i = 1, 2, 3)$, so that $X_i \int [V(t)]^{-1}\mathrm{d}t \to 0$ for $t < \infty$ (for $V(t) = t^n$ with $n > 1$ the integral tends to zero as $t \to \infty$ for any X_i). It can be concluded that the spinor field Lagrangian describing a perfect fluid leads to the isotropization of the Universe as $t \to \infty$, moreover, in case of dark energy the system undergoes an earlier isotropization.

Late time acceleration

Let us now see, if spinor field can explain the phenomenon of late time acceleration. For simplicity we consider a massless spinor field. Let us note that in the unified nonlinear spinor theory the mass term remains absent, and according to Heisenberg the particle mass should be obtained as a result of quantization of spinor prematter. In the nonlinear generalization of classical field equations, the massive term does not possess the significance it does in classical field theory, as by no means it defines the total energy (or mass) of the nonlinear system.

In what follows we study the evolution of the BI Universe for some concrete choice of spinor field non linearities.

Let us first consider the case with the nonlinear term being

$$F = \left[\frac{A}{1+W} + \lambda S^{(1+\alpha)(1+W)}\right]^{1/(1+\alpha)}. \tag{28}$$

The expression (28) describes a modified generalized Chaplygin gas. In case of $W = 0$ we have

$$F = \left(A + \lambda S^{(1+\alpha)}\right)^{1/(1+\alpha)}, \tag{29}$$

that describes a generalized Chaplygin gas. Further setting $\alpha = 1$ we find the spinor description of ordinary Chaplygin gas:

$$F = \sqrt{A + \lambda S^2}. \tag{30}$$

Setting $A = 0$ from (28) one finds

$$F = \lambda S^{1+W}. \tag{31}$$

Depending on the value of W (31) describes

$$
\begin{aligned}
W &= 0, && \text{(dust)}, && \text{(32a)}\\
W &= 1/3, && \text{(radiation)}, && \text{(32b)}\\
W &\in (1/3,\, 1), && \text{(hard Universe)}, && \text{(32c)}\\
W &= 1, && \text{(stiff matter)}, && \text{(32d)}\\
W &\in (-1/3,\, -1), && \text{(quintessence)}, && \text{(32e)}\\
W &= -1, && \text{(cosmological constant)}, && \text{(32f)}\\
W &< -1, && \text{(phantom matter)}, && \text{(32g)}\\
W &> 1, && \text{(ekpyrotic matter)}. && \text{(32h)}
\end{aligned}
$$

The discovery of positive accelerations gives rise to a number of problems. One of the most baffling of those is the problem of eternal acceleration. There are many different approaches to eliminate this problem. A cosmological model of a cyclic Universe experiencing periodical expansions and contractions are proposed. Every cycle begins with a Big Bang, terminates with a Big Crunch only to begin with a Big Bang again. Here we propose a spinor field nonlinearity that gives rise to a cyclic or periodical Universe and corresponds to a quintessence with a modified equation of state:

$$F = \lambda S^{1+W} + \frac{W}{1+W}\varepsilon_0, \tag{33}$$

where ε_0 is some critical energy density. The corresponding EoS has the form

$$p = W(\varepsilon - \varepsilon_{\rm cr}), \quad W \in (-1,\, 0). \tag{34}$$

4. Discussions and Conclusion

Within the scope of a Bianchi type-I anisotropic cosmological model we have studied the role of a nonlinear spinor field on the evolution of the Universe. It is shown that a suitable choice of nonlinearity, spinor field

(i) allows solutions those are free from space-time singularities;

(ii) plays a major role in the isotropization of an initially anisotropic Universe;

explains the phenomenon of the late time accelerated mode of the expansion of the Universe.

A spinor field nonlinearity can also simulate different kinds of matters such as perfect fluid and dark energy.

The energy-momentum tensor defined by (14) can be rewritten as

$$
\begin{aligned}
T_{\mu}{}^{\rho} = \frac{i}{4} g^{\rho\nu}\Big(\bar\psi\gamma_\mu\partial_\nu\psi + \bar\psi\gamma_\nu\partial_\mu\psi - \partial_\mu\bar\psi\gamma_\nu\psi - \partial_\nu\bar\psi\gamma_\mu\psi\Big) - \\
- \frac{i}{4} g^{\rho\nu}\bar\psi\Big(\gamma_\mu\Gamma_\nu + \Gamma_\nu\gamma_\mu + \gamma_\nu\Gamma_\mu + \Gamma_\mu\gamma_\nu\Big)\psi - \delta_\mu^\rho L = \\
= g^{\rho\nu}\bar T_{\nu\mu} - g^{\rho\nu}\tilde T_{\nu\mu} - \delta_\mu^\rho L. \quad (35)
\end{aligned}
$$

Since $\Gamma_1 \neq \Gamma_2 \neq \Gamma_3 \neq 0$ for the BI metric, the energy-momentum tensor of spinor field for BI space time possesses non-diagonal elements, namely

$$
\begin{aligned}
T_2^1 &= \frac{i}{4}\frac{a_2}{a_1}\Big(\frac{\dot a_1}{a_1} - \frac{\dot a_2}{a_2}\Big)\bar\psi\bar\gamma^1\bar\gamma^2\bar\gamma^0\psi, && (36a)\\
T_3^1 &= \frac{i}{4}\frac{a_3}{a_1}\Big(\frac{\dot a_3}{a_3} - \frac{\dot a_1}{a_1}\Big)\bar\psi\bar\gamma^3\bar\gamma^1\bar\gamma^0\psi, && (36b)\\
T_3^2 &= -\frac{i}{4}\frac{a_3}{a_2}\Big(\frac{\dot a_2}{a_2} - \frac{\dot a_3}{a_3}\Big)\bar\psi\bar\gamma^2\bar\gamma^3\bar\gamma^0\psi. && (36c)
\end{aligned}
$$

Since the non-diagonal elements of the Einstein tensor for BI metric are trivial, from (36) one duly finds

$$
\frac{\dot a_1}{a_1} = \frac{\dot a_2}{a_2} = \frac{\dot a_3}{a_3} \quad (37)
$$

which means the inclusion of spinor field imposes severe restriction on the metric functions. We plan to explore this study in some forthcoming paper.

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Некоторые проблемы современной космологии и спинорное поле

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В настоящем обзоре обсуждаются некоторые злободневные проблемы современной космологии. Также кратко обсуждаются наиболее популярные модели, использующие темную энергию. Показывается, что модель со спинорным полем может разрешить ряд первоочередных задач современной космологии. Отмечается, что нетривиальность недиагональных компонент тензора энергии-импульса спинорных полей накладывает некоторые сильные ограничения на метрические функции.

Ключевые слова: космологические модели типа Бианки, тензор энергии–импульса.