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Laser Acceleration of Channeled Light Particles in Periodically Modulated Crystal

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An approach for the laser acceleration of the charged particles channeled in a periodically bent crystal have been investigated under the accounting for the transverse energy leveles quantization.

Key words and phrases: channeling, laser acceleration, metacrystallic structures.

1. Introduction

At the present time, particle accelerators are widely used in a number of researching fields both of the fundamental and applied science. However, the modern acceleration mechanisms are limited in terms of the practically attainable acceleration gradients. This limitation causes an enormous increase in accelerators length that is appearing to be a significant challenge. It gives a reason to investigate and develop some possible aproaches for channeled particles acceleration, alternative to conventional ones.

As early as at the end of the previous century a number of ideas for such alternative acceleration technologies has emerged [1]. Among the basic concepts that should be emphazised are the possibility to utilize a wave in plasma [2, 3], and diverse ways to make use of superpower laser radiation [4,5]. The latter ones are namely the direct laser acceleration (DLA) in a lens focus [6], and the acceleration in photonic crystals (both two-dimensional [7] and three-dimensional [8]), and also the inverse free electron laser (IFEL) acceleration [9–11]. A number of these ideas has been successfully implemented in an experiment [12–14]. The approaches have in common the concept of creation of an extremely high power accelerating field by some means or other. On the other hand, these fields already exist within the crystal structures. It is concerned with the channeling phemonena that hence can be used in a particle accelerator.

Channeling is a phenomena of guiding of a charged particle along the major crystallographic directions called channels. Provided that the particle velocity is large enough and an angle between the motion direction and the crystallographic planes, or the atomic strings, is small enough (smaller than so called critical Lindhard angle [15, 16]), it experiences the multiple scattering from the crystallographic planes or strings (in the cases of the planar or axial channeling, respectively). The particle interaction with the planes or strings of atoms in a crystal lattice nodes is described by the continuous averaged Lindhard potential [15, 16]. A positively charged particle, being channeled in a crystal, moves in a region of the minimal electron density, consequently it penetrates at much larger distance than in the absence of the channeling.

An important feature of the channeling phenomena is the preserving of this motion mode in a slightly bent crystal. Therefore one of the channeling applications is the ability of the deviation of charged high energy particles with the help of the bent crystals. Notably, the deflection angle achieved by such a way could be much larger than its maximal possible magnitude enabled by the use of a magnetic field. The concept was first proposed in [17]. Subsequent works have shown the capability of this effect to be more widely used for the steering of the high energy particles beams, namely the beams focusing, splitting, collimation, extraction from the accelerators and even for the measurements of short-lived particle magnetic moments [18]. In the

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number of cases it is the only possibility to carry out the experiments in the collider mode and with a fixed target as well.

The first concepts of the possibility to achieve any advantages in accelerators design by means of the channeling features usage were also suggested as far back as in 80th years of the last century [5, 19, 20]. Since a bent crystal may be used instead of a magnetic field for the charged particle deflection, an idea to utilize a crystal with the periodically bent channels as an undulator in a FEL have emerged rather long time ago. The next step in this technique development appeared to be the acceleration on the principle of IFEL with the aid of a crystalline undulator. That is exactly what has been put forward in the Bogacz work [21]. He suggested to use an effect of particles channeling in the periodically bent channels under the laser field having a polarization transverse to a channel direction for the purpose of particles acceleration. This method is based upon the laser pumpling of the particle transverse degree of freedom and its subsequent transfer to the longitudinal one due to a crystal periodical bending.

Since the Bogacz paper [21] publication techniques of metacrystallic structures production and utilizing have been developed extensively. In particular, a paradigm of the crystalline undulators construction in the form of the crystals with the channels periodically bent by means of the mechanical stress has already been realized experimentally [22] (it implies the deformation creating through the making use of the regular tranches on the surface of a thin crystal silicium plate). Other schemes for the obtaining of such structures are widely investigated [23]. Moreover, practically feasible magnitudes of the laser radiation power have increased significantly. In total, it gives a convincing reason to continue the theoretical research of the subject in this field.

In the Bogacz work [21] completely classical approach was used which is rather applicable in the context of the heavy particle channeling. But for the light ones the quantum effects, namely the transverse energy levels quantization, cannot be negliged in the course of the transverse degree of freedom considering. Meanwhile, the energy levels quantization may lead to the acceleration regimes discretization. The advantage can be taken from this in order to produce the coherent particle beams. In the current work a method of applying the quantum treatment for the transverse degree of freedom and the classical approach for the longitudinal one is explicated. This method will be used to examine a feasibility of the laser acceleration of a light charged particle channeled in a crystal with the periodically bent channels, and also a possibility of the particle filtering for the aid of the monochromatic beams extraction.

The work structure is as follows. In the section 2 the quantum-classical equations describing a particle motion in a crystal with the periodically bent channels under the complementary external laser field are derived from the Schrödinger equation. Next, in the section 3 we obtain in terms of the two level approximation the solution to these equations yielding a dependence of a particle acceleration on the parameters of the laser field and a channel bending, and also on the initial conditions. This is followed by the numerical calculation results illustrating the obtained theoretical estimations in the section 4. The section 5 contains the numerical evaluations of the above-stated acceleration parameters for the different kinds of particles and crystals. Also, the relevant possibilities of the proposed method application are discussed. Finally, in the conclusion the general summary is presented.

Atomic units will be used throughout the paper.

2. The quantum-classical approximation

The motion of a quantum particle is described by the Schrödinger equation

$$i\frac{\partial\Psi(\mathbf{r},t)}{\partial t}=\left[-\frac{1}{2m}\nabla^2+U(\mathbf{r},t)\right]\Psi(\mathbf{r},t), \tag{1}$$

where $\Psi(\mathbf{r},t)$ is the particle wavefunction and $U(\mathbf{r},t)$ is a potential affecting it. The latter consists of a crystal lattice potential and the external laser field summed

$$U(\mathbf{r},t) = U_0[\boldsymbol{\rho} - \boldsymbol{\rho}_0(z)] + V(\boldsymbol{\rho},t), \tag{2}$$

where ρ denotes a transversal coordinate, z being a longitudinal one, and $U_0(\boldsymbol{\rho})$ is an effective potential of a crystal channel, $V(\boldsymbol{\rho},t)$ is a potential of the laser field, where $\boldsymbol{\rho}_0(z)$ is a position of the center of the channel bent for instance by a shear sound wave. The potential might be rewritten as

$$U(\mathbf{r},t) = U_0(\boldsymbol{\rho}) + V(\boldsymbol{\rho},t) + W(\boldsymbol{\rho},z), \tag{3}$$

where $U_0(\boldsymbol{\rho} - \boldsymbol{\rho}_0(z)) - U_0(\boldsymbol{\rho}) \simeq -\frac{\mathrm{d}U_0}{\mathrm{d}\boldsymbol{\rho}}(\boldsymbol{\rho})\boldsymbol{\rho}_0(z) \simeq -\frac{\mathrm{d}^2U_0}{\mathrm{d}\boldsymbol{\rho}^2}(0)(\boldsymbol{\rho}_0(z)\cdot\boldsymbol{\rho})$ is a potential approximately linear over ρ.

Further, one can obtain the solutions to the stationary Schrödinger equation for the transversal coordinates having the form

$$\hat{h}_\perp \varphi_n(\boldsymbol{\rho}) = \epsilon_n \varphi_n(\boldsymbol{\rho}), \quad \text{where} \quad \hat{h}_\perp = -\frac{1}{2m}\nabla_\perp^2 + U_0(\boldsymbol{\rho})$$

is a transversal Hamiltonian, ϵ_n are the eigenenergies of the stationary transversal states, and $\varphi_n(\boldsymbol{\rho})$ are the wavefunctions of these states that may be used subsequently for the deriving of the required quantities.

Next, a problem appraisal could be made for the purpose of achieving any simplifications to the equation being solved. First, for the fast particles the quantum effects are likely to appear only for the transverse degree of freedom. In contrast, for the longitudinal one the wavelength magnitude is much less than the typical system size gauge, namely the channel width. As a consequence, it makes a sense to handle the longitudinal degree of freedom approximately. Specifically, it can be described in terms of the Gaussian wavepacket. In order to obtain the motion equations in the classical approximation for the particle's longitudal degree of freedom from the general Schrödinger equation, one may use the Petrov-Galerkin approach.

First, let us assume a trial function for the solution of the Eq.(1) to be expressed as

$$\Psi(\mathbf{r},t) = \frac{1}{\sqrt{a\sqrt{\pi}}}\sum_n c_n(t)\exp\left[ik_n(t)[z - z_n(t)] - \frac{[z - z_n(t)]^2}{2a^2} - i\int_0^t E_n(t')\mathrm{d}t'\right]\varphi_n(\boldsymbol{\rho}).$$

Here amplitudes of transversal levels population denoted by $c_n(t)$, longitudinal positions of the wave packet center labelled as $z_n(t)$, average longitudinal packet momenta written as $k_n(t)$ and effective energies $E_n(t)$ are the unknown parameters. In order to evaluate them, the Petrov–Galerkin method will be applied. For this purpose, let us introduce the test functions in the following way:

$$\Phi_{n0}(\mathbf{r},t) = \frac{1}{\sqrt{a\sqrt{\pi}}}\exp\left[ik_n(t)[z - z_n(t)] - \frac{[z - z_n(t)]^2}{2a^2} - i\int_0^t E_n(t')\mathrm{d}t'\right]\varphi_n(\boldsymbol{\rho});$$

$$\Phi_{n1}(\mathbf{r},t) = \sqrt{\frac{2}{a^3\sqrt{\pi}}}[z - z_n(t)]\times$$

$$\times \exp\left[ik_n(t)[z - z_n(t)] - \frac{[z - z_n(t)]^2}{2a^2} - i\int\limits_0^t E_n(t')\mathrm{d}t'\right]\varphi_n(\boldsymbol{\rho}).$$

The Petrov–Galerkin method is confined in the orthogonalization of the discrepancy to the test function, namely

$$\left\langle \Phi_{n\alpha} \middle| i\frac{\partial}{\partial t} - \left[-\frac{1}{2}\frac{\partial^2}{\partial z^2} + \hat{h}_\perp(\boldsymbol{\rho}) + V(\boldsymbol{\rho}, t) + W(\boldsymbol{\rho}, z)\right] \middle| \Psi \right\rangle = 0;\ n = 1, 2;\ \alpha = 0, 1. \tag{4}$$

Further, by collecting the terms before $\frac{i}{\sqrt{2}a}$ in equations from (4) obtained through the projection on Φ_{n1}, one arrives to

$$\dot{z}_n = k_n.$$

In other words, as expected, the position of the wave packet "'center of mass"' and its average momentum are related to each other in the same manner as a position and a momentum of a classical particle. Besides, the term containing c_n which is given by projection on Φ_{n0} might be removed from equations on the assumption that

$$E_n = -\frac{k_n^2}{2} + \frac{1}{4a^2} + \epsilon_n.$$

Next, as mentioned above, the method is applicable only on the condition that the positions of packets in the different transversal states are much less then the packet width, namely $(z_{n'} - z_n \ll a$. Furthermore, the difference of momenta $\Delta k_{n'n} = k_{n'} - k_n$ should also be small, viz $\Delta k_{n'n}/k_n \ll 1$. Consequently, the approximated expressions for the potentials matrix elements can be obtained in this way:

$$V_{nn'}(\mathbf{z}, \mathbf{k}, t) = \langle \Phi_{n0}|V|\Phi_{n0}\rangle \simeq \langle n|V|n'\rangle e^{-i\int_0^t \omega_{n'n}(t')\mathrm{d}t' - i(k_{n'}z_{n'} - k_n z_n)};$$
$$W_{nn'}(\mathbf{z}, \mathbf{k}, t) = \langle \Phi_{n0}|W|\Phi_{n0}\rangle \simeq \langle n|W|n'\rangle(\bar{z}_{nn'})e^{-i\int_0^t \omega_{n'n}(t')\mathrm{d}t' - i(k_{n'}z_{n'} - k_n z_n)},$$

where the denotations $\omega_{n'n}(t) = E_{n'}(t) - E_n(t)$ and $\bar{z}_{nn'} = (z_{n'} + z_n)/2$ are used. In turn, the expanding of $W(\boldsymbol{\rho}, z)$ and $\exp(i\Delta k_{n'n}z)$ in Taylor series by z up to the first order yields the following:

$$\langle \Phi_{n1}|V|\Phi_{n0}\rangle \simeq i\Delta k_{n'n}\frac{a}{\sqrt{2}}\langle n|V|n'\rangle e^{-i\int_0^t \omega_{n'n}(t')\mathrm{d}t' - i(k_{n'}z_{n'} - k_n z_n)}$$

$$\langle \Phi_{n1}|W|\Phi_{n0}\rangle \simeq$$
$$\simeq \frac{a}{\sqrt{2}}\left[\frac{\partial\langle n|W|n'\rangle}{\partial z}(\bar{z}_{nn'}) + i\Delta k_{n'n}\langle n|W|n'\rangle(\bar{z}_{nn'})\right]e^{-i\int_0^t \omega_{n'n}(t')\mathrm{d}t' - i(k_{n'}z_{n'} - k_n z_n)}$$

Having the terms of the same order of a collected, one arrives to the system of equations

$$\dot{\mathbf{c}} = -i[\mathbf{V}(\mathbf{z}, \mathbf{k}, t) + \mathbf{W}(\mathbf{z}, \mathbf{k}, t)]\mathbf{c}, \tag{5}$$

$$\dot{z}_n = k_n, \tag{6}$$

$$|c_n|^2\dot{k}_n = \sum_{n'} c_n^* F_{nn'}(\mathbf{z}, \mathbf{k}, t)c_{n'} - \sum_{n'} c_n^*[V_{nn'}(\mathbf{z}, \mathbf{k}, t) + W_{nn'}(\mathbf{z}, \mathbf{k}, t)]i(k_{n'} - k_n)c_{n'}, \tag{7}$$

where

$$F_{nn'}(\mathbf{z}, \mathbf{k}, t) = -\frac{\partial \langle n|W|n'\rangle}{\partial z}(\bar{z}_{nn'}) e^{-i\int_0^t \omega_{n'n}(t')\mathrm{d}t' - i(k_{n'}z_{n'} - k_n z_n)}$$

may be interpreted as a force affecting the particle due to the presence of the spatial field W.

The further problem simplification can then be achieved through the introducing of the average longitudinal position and the momentum of the particle

$$\langle z\rangle = \sum_n c_n^* z_n c_n, \quad \langle k\rangle = \sum_n c_n^* k_n c_n.$$

Next, let us multiply the equation (6) by $|c_n|^2$ and subsequently perform the summation over n in the Eq.(6) and the Eq.(7). Thereupon, considered that $z_n \approx \langle z\rangle$ and $k_n \approx \langle k\rangle$ in the right hand sides of equations (6) and (5), the final result is

$$\begin{gathered} \dot{\mathbf{c}} = -i[\mathbf{V}(t) + \mathbf{W}(\langle z\rangle, t)] \cdot \mathbf{c}; \\ \dot{\langle z\rangle} = \langle k\rangle; \\ \dot{\langle k\rangle} = \sum_{n,n'} c_n^* F_{nn'}(\langle z\rangle, t) c_{n'}, \end{gathered} \tag{8}$$

which appear to be much easier to solve than the common Schrödinger equation. Here the coefficients of the wavefunction expansion in the terms of the transversal states $\varphi_n(\boldsymbol{\rho})$ having a physical meaning of population amplitudes and denoted by c_n, the wavepacket center position along the z axis labelled as $\langle z\rangle$, and $\langle k\rangle$ standing for the averaged longitudinal momentum of the particle are the system variables. Further, the system contains the following values:

$$V_{nn'}(t) = \langle n|V(\boldsymbol{\rho}, t)|n'\rangle e^{i\omega_{nn'}t}; \tag{9}$$

$$W_{nn'}(z, t) = \langle n|W(\boldsymbol{\rho}, z)|n'\rangle e^{i\omega_{nn'}t} \tag{10}$$

being the matrix elements of the summand potentials from (3) and

$$F_{nn'}(z, t) = -\frac{\partial W_{nn'}(z, t)}{\partial z} \tag{11}$$

is the one of a longitudinal force F defined as the force corresponding to the spatial potential affecting the particle. Besides, $\omega_{nn'} = \epsilon_n - \epsilon_{n'}$ are the transition frequencies, n and n' stand for the stationary states wavefunctions $\varphi_n(\boldsymbol{\rho})$ referred to above.

3. Analitycal solution by means of the two-level approximation

Provided that the potentials (9,10,11) explicit form is specified, the solution of the system (8) might be obtained. So, let us assume that

$$V(\boldsymbol{\rho}, t) = -\boldsymbol{\mathcal{E}_0}\boldsymbol{\rho}\sin(\omega t); \tag{12}$$

$$W(\boldsymbol{\rho}, z) = -\boldsymbol{\mathcal{D}_0}\boldsymbol{\rho}\sin(\varkappa z), \tag{13}$$

and also the fields amplitudes $\mathcal{E}_0 \uparrow\downarrow \mathcal{D}_0$. Here ω denotes the external field frequency and $\varkappa$ means the periodical spatial potential wavenumber. In this case

$$
\begin{aligned}
V_{nn'}(t) &= -\mathcal{E}_0 d_{nn'} \sin(\omega t) e^{i\omega_{nn'}t};\\
W_{nn'}(z,t) &= \mathcal{D}_0 d_{nn'} \sin(\varkappa z) e^{i\omega_{nn'}t};\\
F_{nn'}(z,t) &= -\varkappa \mathcal{D}_0 d_{nn'} \cos(\varkappa z) e^{i\omega_{nn'}t},
\end{aligned}
\tag{14}
$$

where $d_{nn'} = \langle\varphi_n|x|\varphi_{n'}\rangle$ is the dipole transition matrix element, at that $Ox \uparrow\uparrow \mathcal{E}_0$. On condition that the channel potential $U_0(\boldsymbol{\rho})$ is even the diagonal element $d_{nn} = 0$.

Next, consider the particular case of $\omega = \omega_{21}$. Granted that $\langle k\rangle$ is large and changes rather slightly, true is the expression $z \approx \langle k\rangle t + z_0$. Thereupon, if $\langle k\rangle = k_{res}$, where

$$
k_{res} = \omega_{21}/\varkappa, \tag{15}
$$

then the z-dependent potential affects the particle with an effective frequency $\varkappa k_{res} = \omega_{21}$. In such instance the transitions between the levels 1 and 2 are the most probable, hence it is possible to use the two-level approximation. When the quickly oscillating parts of the matrix elements (14) are omitted one arrives to

$$
\begin{aligned}
V_{21}(t) &\approx \frac{1}{2i}\mathcal{E}_0 d_{21};\\
W_{21}(k_{res}t,t) &\approx -\frac{1}{2i}\mathcal{D}_0 d_{21} e^{-i\delta_0};\\
F_{21}(k_{res}t,t) &\approx -\frac{1}{2}\varkappa\mathcal{D}_0 d_{21} e^{-i\delta_0},
\end{aligned}
$$

where $\delta_0 = \varkappa z_0$ is the spatial potential initial phase. In that event, namely the two-level approximation, the solution for the population amplitudes is obtained in the form

$$
\begin{aligned}
c_1(t) &= \cos\Omega t\cos\delta_1 - \sin\Omega t\sin\delta_1 e^{i(\delta_2-\gamma)};\\
c_2(t) &= -\cos\Omega t\sin\delta_1 e^{i\delta_2} - \sin\Omega t\cos\delta_1 e^{i\gamma},
\end{aligned}
$$

where

$$
\Omega = \frac{1}{2}|(\mathcal{E}_0 - \mathcal{D}_0 e^{-i\delta_0})d_{21}|
$$

is the Rabi frequency,

$$
e^{i\gamma} = \frac{\mathcal{E}_0 - \mathcal{D}_0 e^{-i\delta_0}}{|\mathcal{E}_0 - \mathcal{D}_0 e^{-i\delta_0}|}\frac{d_{21}}{|d_{21}|}
$$

is the effective potential phase factor, δ_1 stands for an initial phase of Rabi oscillations, and δ_2 denotes an initial phase shift between the population amplitudes. In particular, on the assumption that

$$
\mathcal{E}_0 = \mathcal{D}_0, \tag{16}
$$

and $\delta_0 = 0$, the Rabi frequency $\Omega = 0$. Consequently, the coefficients $c_{1,2}(t)$ are constant; therefore the longitudinal force

$$
\langle F\rangle = \sum_{n,n'} c_n^* F_{nn'}(k_{res}t,t)c_{n'} = \varkappa\frac{\mathcal{D}_0 d_{21}}{2}\sin(2\delta_1)\cos\delta_2,
$$

is constant too. It is easy to see that the maximal acceleration is achieved on the condition that $\delta_1 = \pi/4$ and $\delta_2 = 0$, i.e., when $c_1 = -c_2$.

Now turn back to the more general case of $\Omega \neq 0$. The force might be averaged by time over a single Rabi period in the following manner:

$$\overline{\langle F\rangle} = \frac{\Omega}{2\pi}\int\limits_0^{2\pi/\Omega}\sum_{n,n'} c_n^* F_{nn'}(k_{res}t,t)c_{n'} =$$
$$= \frac{\varkappa\mathcal{D}_0 d_{21}}{4}\sin 2\delta_1\left[\cos(\delta_2+\delta_0)-\cos(\delta_2-\delta_0-2\gamma)\right].$$

As long as the condition (16) is satisfied, $\gamma = (\pi-\delta_0)/2$ and thus

$$\overline{\langle F\rangle} = \frac{\varkappa\mathcal{D}_0 d_{21}}{4}\sin 2\delta_1\left[\cos(\delta_0+\delta_2)+\cos\delta_2\right].$$

It is clear from the last expression that the average force is maximal in the event of $\delta_1 = \pi/4$ and $\delta_2 = \delta_0 = 0$. Besides, its magnitude is equal to

$$\overline{\langle F\rangle}_{\max} = \frac{\varkappa\mathcal{D}_0 d_{21}}{2}.$$

In such a manner, the conditions of resonance (15) and amplitudes equality (16) have been met, the equation for the particle momentum k from the system under consideration (8) can be derived in the form

$$\dot{k} = \frac{\omega_{12}}{k}\frac{\mathcal{E}_0 d_{21}}{4}\sin(2\delta_1)\left[\cos(\delta_2+\delta_0)+\cos\delta_2\right].$$

Its solution is

$$k(t) = \sqrt{k_0^2+\alpha t};$$
$$z(t) = \frac{2}{3\alpha}(k_0^2+\alpha t)^{3/2} - \frac{2}{3\alpha}k_0^3 + z_0,$$

where denotation $\alpha = \omega_{12}\mathcal{E}_0 d_{21}\sin(2\delta_1)\left[\cos(\delta_2+\delta_0)+\cos\delta_2\right]/4$ is used. From these expressions one can obtain the momentum dependence on z as follows:

$$k(z) = \left[k_0^3 + \frac{3\alpha}{2}(z-z_0)\right]^{1/3}.$$

In turn, it may be substituted to the resonance condition (15). Therefore, in order to supply the resonance of the transversal transition with z–dependent potential its “wave number” should be equal to

$$\varkappa(z) = \omega_{21}\left(\kappa^3 + \frac{3\alpha}{2}z\right)^{-1/3},$$

where $\kappa = k_0$ is the particle initial momentum. Thereat, instead of (13), the spatial potential should have the form

$$W(\boldsymbol{\rho},z) = -\mathcal{D}_{\mathbf{0}}\boldsymbol{\rho}\sin\left(\int\limits_0^z \varkappa(z')dz'\right) = -\mathcal{D}_{\mathbf{0}}\boldsymbol{\rho}\sin\frac{\omega_{21}}{\alpha}\left[\left(\kappa^3+\frac{3\alpha}{2}z\right)^{2/3}-\kappa^2\right], \quad (17)$$

where now κ and α denote the metacrystallic structure parameters. Besides, it is easy to see that the parameter α should meet the condition $0 < \alpha \leqslant \omega_{12}\mathcal{D}_0 d_{21}/2$ to provide the acceleration of particles with the initial momentum $k_0 = \kappa$. Thuswise we have obtained the potential yielding the resonance and, as a consequence, enabling the acceleration.

Now let us turn to the exploring of the resonance mode stability. To this end consider the phase difference between z-dependent potential and transversal oscillations; it might be expressed in this way:

$$\delta_0(t) = \frac{\omega_{21}}{\alpha}\left[\left(\kappa^3 + \frac{3\alpha}{2}\bar{z}(t)\right)^{2/3} - \kappa^2\right] - \omega_{21}t \simeq \frac{\omega_{21}z_0}{\sqrt{\alpha t}} = \frac{\kappa}{\sqrt{\alpha t}}\delta_0(0).$$

So, as long as $\delta_0(0)$ is small enough to keep the acceleration during rather long time period, δ_0 converges to zero and the acceleration becomes stable. Thus, the acceleration mode is stable while δ_0 detunes slightly from zero.

Finally, the energy of the particle having the maximal acceleration is equal to

$$E = E_0 + \frac{\mathcal{E}_0 d_{21}\omega_{12}}{2}t.$$

Thuswise, the maximal energy increase per a distance unit, i.e. the accelerating gradient, can be estimated as

$$\frac{\mathrm{d}E}{\mathrm{d}z} = \frac{d_{21}\omega_{21}}{\sqrt{8E}}\mathcal{E}_0. \tag{18}$$

4. Numerical calculations

In order to demonstrate the particle acceleration process, the system (8) was solved, the potentials have been assumed to have the forms (12), (17) proved above to provide the resonance mode. In the Fig.1 one can see the results of this numerical solution.

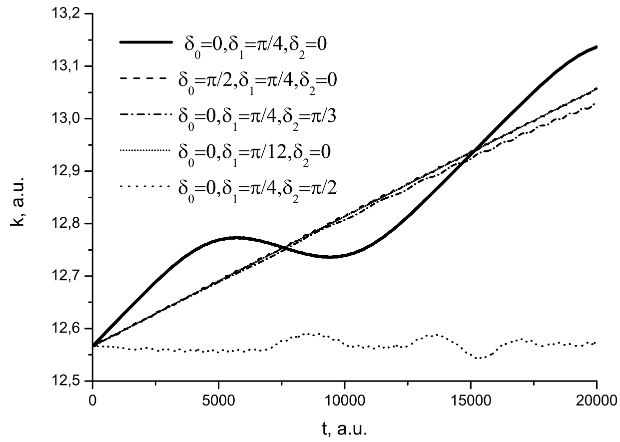

Figure 1. The particle momentum dependence on time for the different initial conditions: $\delta_1 = \pi/4$, $\delta_0 = \delta_2 = 0$ (solid curve), $\delta_0 = \pi/2$, $\delta_1 = \pi/4$, $\delta_2 = 0$ (dashed curve), $\delta_0 = 0$, $\delta_1 = \pi/4$, $\delta_2 = \pi/2$ (dotted curve), $\delta_0 = \pi/2$, $\delta_1 = \pi/4$, $\delta_2 = \pi/3$ (dash-dotted curve), $\delta_0 = \pi/2$, $\delta_1 = \pi/12$, $\delta_0 = \delta_2 = 0$ (short dotted curve)

The system parameters used in the calculations are as follows: $d_{12} = 1$, $\omega_{12} = 2\pi/50$, $\mathcal{E}_0 = \mathcal{D}_0 = 0.01$, $\kappa = 4\pi$, $\alpha = \omega_{12}\mathcal{E}_0 d_{21}/2$ at $k(t = 0) = \kappa$ and different initial conditions specified by the phases δ_0, δ_1, δ_2. The choice of the constant α standing for the nonuniformity of the spatial potential period provides the permanent resonance with the accelerating force magnitude being 2 times less than its maximal value. Therefore in the event of the initial conditions yielding the maximal force, namely $\delta_1 = \pi/4$, $\delta_0 = \delta_2 = 0$, the particle momentum periodically becomes larger than its resonance value, as a consequence the resonance vanishes and the particle decelerates, whereat it enters back into the resonance mode. In sum, the acceleration 2 times less than the maximum possible is produced. The population density evolution for this case is displayed in the Fig.2.

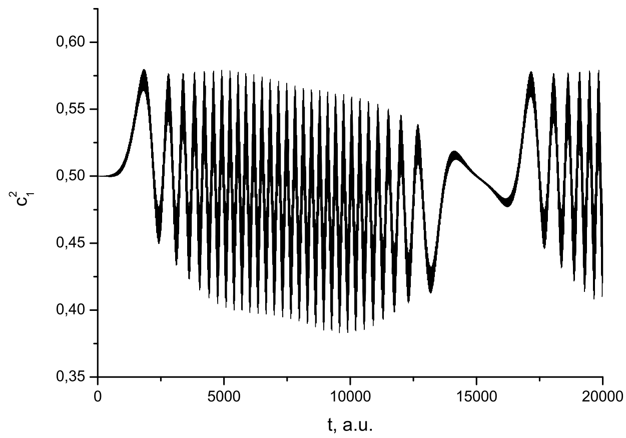

Figure 2. The lower transversal level population dependence on time for the case of the maximal accelerating force

It is apparent that in proximity of the resonance point the population density curve nearly reaches the plateau, while at the momentum magnitude strongly different from its resonance value it oscillates. The Fig.1 also presents the curves for several sets of initial conditions yielding the average force magnitude equal to the half of its maximal value. In other words, our choice for α should enable the permanent resonance observed, just as it is clear from the linearity of the momentum increase over time. The plot shows as well the curve corresponding to the initial conditions producing the average force magnitude equal to zero, i.e., as a consequence, the absence of the acceleration. Hereby the acceleration mode stability has been ensured, besides it is provided by the fact that the force magnitude is greater than some value corresponding to the chosen α at the initial acceleration stage.

The Fig.3 presents the dependence of the probability of the affecting at the initial acceleration stage of the force $\overline{\langle F\rangle}$, such that $\overline{\langle F\rangle}/\overline{\langle F\rangle}_{\max} > f$. At that the distribution over either of the phases δ_0, δ_1, δ_2 is assumed to be uniform. It is clear from the figure that the particle entering the crystal is rather likely to get into the regime such that the accelerating force magnitude would be close to the maximal possible value.

In the Bogacz work [21] the particle acceleration was investigated in the case that it is allowable to use the harmonic approximation for the channel transverse potential as well as the classical approach for the particle transverse motion. In terms of the quantum mechanics it implies a cascade of transitions between a number of equidistant transverse motion energy levels. In contrast, we examine the situation of the two-level approximation applicability. Therefore in this case one should take into consideration not only the resonance between the laser radiation and the periodically spatial potential which always occurs at the certain particle momentum. Also the resonance of the laser radiation with the frequency of the transition between the transverse levels requires taking into account.

The fig.4 demonstrates the evolution of the momentum of the particle channeled in a medium with a constant “wavelength” of the periodically spatial potential. At that, the particle is assumed to have an initial momentum

$$k_{res} = \omega/\varkappa, \tag{19}$$

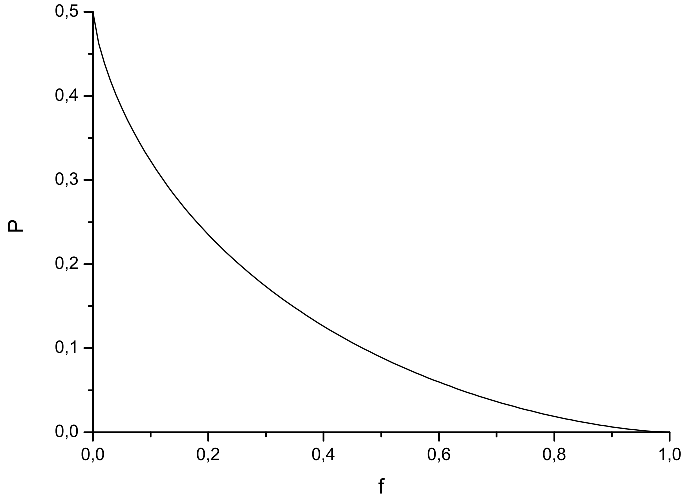

Figure 3. The probability for the particle to be affected by the initial conditions such that the normalized average accelerating force $\overline{\langle F\rangle}/\overline{\langle F\rangle}_{\max} > f$

providing the resonance between the laser radiation and the periodically spatial potential. Presented are curves corresponding to the various frequency ω detuning from the frequency of the tranverse transition ω_{12}. The acceleration range width in the frequency space is apparent to have the same order as the width of the resonance broadening due to the acceleration itself.

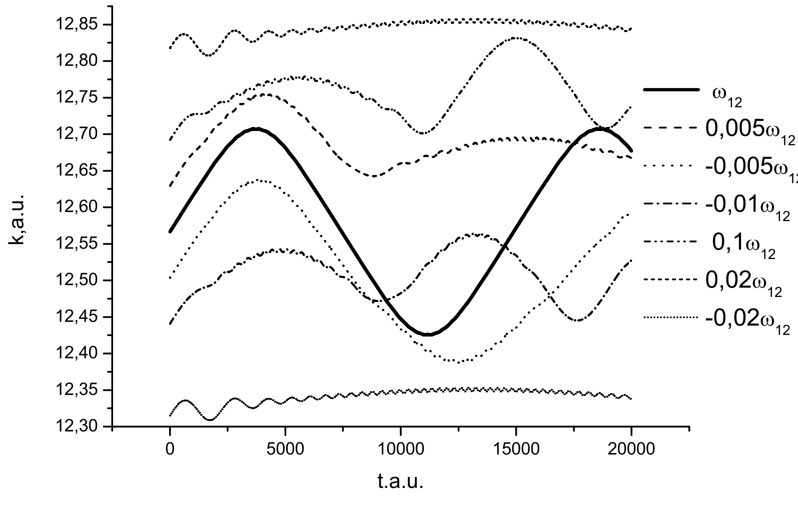

Figure 4. The accelerated particle longitudinal momentum dependence on time in the presence of the resonance with the periodically spatial potential for the different laser radiation frequency detuning from the frequency of the tranverse transition: $\omega = \omega_{12}$ (thick solid curve), $\omega = 0.005\omega_{12}$ (dashed curve), $\omega = -0.005\omega_{12}$ (dotted curve), $\omega = -0.01\omega_{12}$ (dash-dotted curve), $\omega = 0.1\omega_{12}$ (dash-dot-dotted curve), $\omega = 0.02\omega_{12}$ (short dashed curve), $\omega = -0.02\omega_{12}$ (short dotted curve)

All the previous cases have been considered under the neglecting of both ionizing and radiation energy losses of the particle travelling through the crystal. Meanwhile, these losses are necessary to be regarded in the common crystals. The paper [24] gives a half-emphirical dependence expression for the energy loss per the length unity for a positron passing through the matter. The channeling losses are some less than in the course of the propagation in a random direction. The value of the coefficient of the stopping power reducing in the channeling mode has been shown to be roughly equal to 0.7 [25]. So, in order to take the energy loss into account we have added in the last equation of the system (8) the stopping power calculated by the formula from [24] and additionally multiplied by 0.7.

The fig. 5 presents the accelerated particle longitudinal momentum dependence on the longitudinal coordinate meaning with provision for the stopping power for the constant “wavelength” of the periodically spatial potential and the different values of the external laser field intensity.

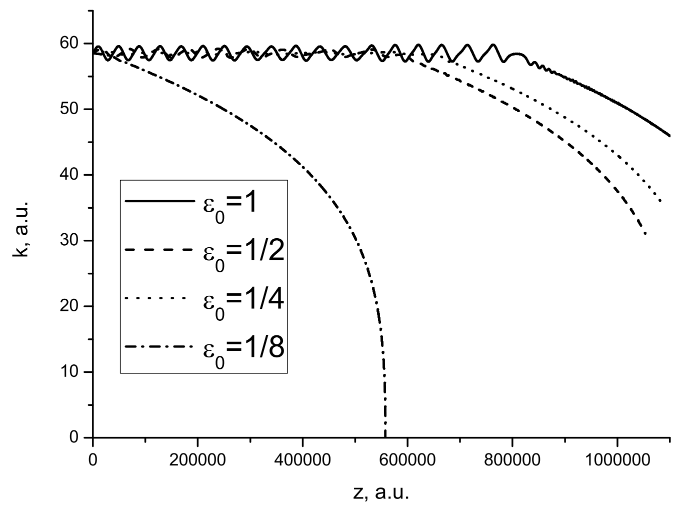

Figure 5. The accelerated particle longitudinal momentum dependence on the longitudinal coordinate meaning with provision for the stopping power for the different values of the external laser field intensity: $\mathcal{E}_0 = 1$ (solid curve), $\mathcal{E}_0 = 1/2$ (dashed curve), $\mathcal{E}_0 = 1/4$ (dotted curve), $\mathcal{E}_0 = 1/8$ (dash-dotted curve)

At that, the phases initial meanings have been set to grant the maximal laser acceleration. It is clear that the laser acceleration can be observed only at very large $\mathcal{E}_0$ magnitudes yielding its efficiency greater than the stopping power. Besides, after a number of momentum oscillations with respect to the resonance value at a certain moment system inevitably transgress to the clean deceleration mode in cosequence of the stopping power presence. For the case of the laser acceleration force less than the stopping power this transgression occurs immediately. One may conclude that for the small acceleration force, contrary to the apparent expectations, the laser acceleration efficiency is not “substracted” from the stopping power, but simply disappears. This effect arises as a consequence of the rapid departing out of the resonance with the periodically spatial potential.

Further, in order to compensate the resonance vanishing by virtue of the stoping power presence, the variable “wavelength” of the periodically spatial potential have been introduced. This have been done in such a manner that the resonance would nevertheless remain in the presence of the stopping power and the infinitesimal laser field as well. That is, the periodically spatial potential “wavelength” decreases with the z increasing in such a way that if the particle entering the crystal had the resonance momentum meaning then the momentum decreasing due to the stopping power would not result in the disappearance of the resonance with the periodically spatial potential. The relevant accelerated particle momentum dependencies on the distance from the crystal entering point for the different values of the external laser field intensity are displayed in the fig. 6.

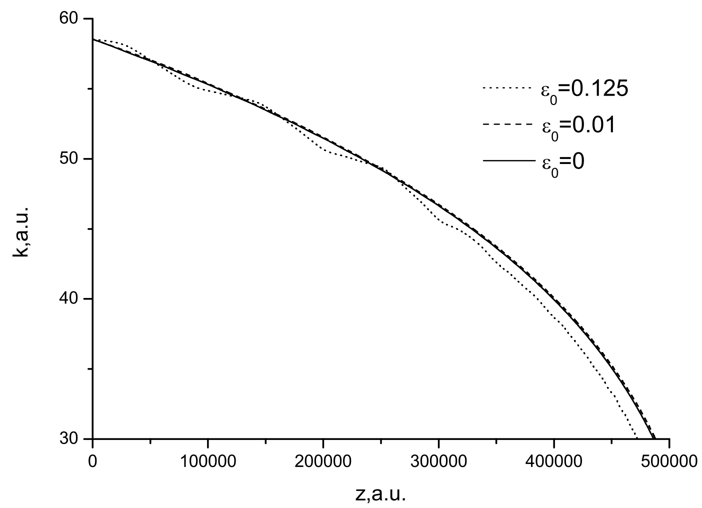

Figure 6. The accelerated particle longitudinal momentum dependence on the longitudinal coordinate meaning on the condition of the periodically spatial potential "wavelength" providing the continuous resonance in the absence of the laser field for the different values of the external laser field intensity: $\mathcal{E}_0 = 0.125$ (dotted curve), $\mathcal{E}_0 = 0.01$ (dashed curve), $\mathcal{E}_0 = 0$ (solid curve)

The plots have been obtained on the conditions that the periodically spatial potential has a variable "wavelength" supporting the continuous resonance under the absence of the laser field and also the stopping power is present. It is apparent that for the $\mathcal{E}_0$ values not leading to the crystal destroy (see the sec. 5) the laser acceleration provides some increase in the particle penetration depth. However, since this increase is extremely small, the idea of the monochromatic positrons producing by means of the laser acceleration appears to be practically unrealizeable. Hereafter, for rather large $\mathcal{E}_0$ values (but still less than the meanings yielding the laser acceleration efficiency exceeding the stopping power) the momentum oscillations with respect to the resonance values arise, just as well as in the case of the constant periodically spatial potential "wavelength" in the absence of the stopping power.

5. Estimations

Now it is necessary to discuss a matter of the practical aspects of the suggested acceleration method implementation. It is known that the optical damage threshold for a Si crystal for 1550 nm laser pulse lasting less than 1 ps is $I_{th} \sim 10^{11}$ W/cm^2 [26], that corresponds to $\mathcal{E}_0 = 1.7 \times 10^{-3}$[a.u.]= 0.9 GeV/m. Next, let us assume the transversal dipole transition matrix element to be $|d_{12}| \sim 1$. For a nonrelativistic proton in Si $\omega_0 \simeq 10^{-2}$ a.u. In the case of the proton energy value being equal to $E = 10$ MeV the proton velocity is $v = 20$ a.u., that yields $\dfrac{\mathrm{d}E}{\mathrm{d}z} = 0.1$ MeV/m. It is negligible with regard to the stopping power of 10 MeV–protons that is on order of ~ 10 GeV/m. Moreover, this quantity of $\frac{\mathrm{d}E}{\mathrm{d}z}$ is small in comparison with the one yielded by an usual linear accelerator, which acceleration rate may be estimated as ~ 10 MeV/m.

For the channeled nonrelativistic positron $\omega_0 \simeq 1$ a.u., and the velocity value equal to $v = 20$ a.u. (and, as a conqequence, the energy value of $E = 5.4$ KeV) implies $\frac{\mathrm{d}E}{\mathrm{d}z} = 10$ MeV/m. Though the latter energy increment is much greater than that for the proton case, it appears to be still unsufficient in terms of the direct application of the considered effect for an accelerator designing on this framework.

Furthermore, the usage of carbon nanotubes instead of the Si crystals strongly reduces the stopping power (due to much larger channel radius values). The effective transversal potential of the tube can be approximated by a circular well potential. If one considers the two lowest transversal states in the tube, its radius R increase would lead to the transition frequency ω_{12} decreasing in accordance with $\omega_{12} \sim 1/R^2$. On the other hand, the dipole matrix element of the transition between thses states would increase as $d_{12} \sim R$. Thus, in sum the acceleration gradient (18) depends on the tube radius as

$$\frac{\mathrm{d}E}{\mathrm{d}z} \sim \frac{\mathcal{E}_0}{mvR}.$$

However, since the stopping power is inversely related to the material density, true is the following:

$$\left.\frac{\mathrm{d}E}{\mathrm{d}z}\right|_{stop} \sim -\frac{1}{R},$$

hence there is no possibility to make the accelerating efficiency larger than the stopping power through the only increasing of the tube radius. Therefore the utilizing of the periodically bent carbon nanotubes instead of the crystals has the only advantage of the production easiness in contrast to a crystal with the periodically bent channels.

6. Conclusion

In this work we have studied an approach of the charged particles acceleration in a crystal with the periodically bent channels through the use of the laser radiation field. The concept has been originally suggested by Bogacz [21], and developed here for the case of the transverse energy levels quantization being significant. The technique as it stands have been demonstrated to be inefficient for the purpose of the particles acceleration upon using either the Si crystals or the carbon nanotubes. This appeared to be true at least for the laser radiation intensities meanings not causing a medium destroy. The considered scheme turned out to be nonapplicable in terms of its usage with the aim of the monochromatic particle beams obtaining as well.

Nevertheless, the scientific field of the particle acceleration, particularly using the channeling special features, is widely researched at the time. It comprises a number of perspectives and possibilities of its implementation. The development of particle acceleration approaches, especially non-conventional innovative ones, appears to be of crucial importance. So, further thorough investigations to the branch are greatly desireable.

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Лазерное ускорение легких частиц, каналированных в периодически модулированных кристаллах

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Рассмотрен метод лазерного ускорения заряженных частиц, каналированных в периодически искривлённых кристаллах, с учётом дискретности уровней поперечных энергий.

Ключевые слова: каналирование, лазерное ускорение, метакристаллические структуры.