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Two Approaches to Quantum Generalization of Thermodynamics

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For the describing of a system under the combined quantum and thermal influences, it is offered two approaches each of them is a thermofield analogue of Clausius'classical and Einstein's statistical thermodynamics accordingly. We call them as *Thermofield Thermodynamics* and *Thermofield Statistical Thermodynamics*. We start from the thermofield vacuum and make a consent between basic notions of the quantum mechanics and thermodynamics. We suppose to consider the thermofield vacuum as an effective thermobath and ground stationary state as an thermal equilibrium state. On this ground we introduce such notions as effective temperature and effective entropy and get possibility to describe systems under combined quantum and thermal influences.

Key words and phrases: effective temperature, effective entropy, stochastic influence, combined quantum and thermal influences, thermofield vacuum.

For the describing of a system under the combined quantum and thermal influences, it is offered, as the first step, to start from the thermofield vacuum as the main idea of Umezawa's thermofield dynamics (*TfD*) [1]. To formulate an entire theory of fluctuations it is necessary to make a consent between two ways of description: the quantum and thermodynamics languages. So we suppose to consider the thermofield vacuum as an effective thermobath (ETB) [2] and ground stationary state as an thermal equlibrium state [3].

Taking the first way, we use the Bogolyubov's (u, v)-transformation to find a wave function of quantum oscillator in the thermal correlated-coherent state (TCCS) in the form [4]

$$\psi(q,T) = \sqrt{\rho(q,T)}\exp\{i\varphi\,(q,T)\}.$$

Its amplitude and phase depend on frequency ω and temperature T. We note the phase plays an essential role even at the high temperatures when $\varphi(q) = m\omega q^2/2\hbar$. As a result Sroedinger's uncertainties relation "coordinate- momentum" takes the form of equality at any temperature

$$(\Delta p)^2(\Delta q)^2 = \frac{1}{4}\langle|\{\Delta\widehat{p},\Delta\widehat{q}\}|\rangle^2 + \frac{\hbar^2}{4} = \frac{\hbar^2}{4}\coth^2\frac{\hbar\omega}{2k_BT},$$

that corresponds to presence of a thermal noise in the pure state. Here we can formally enter the "effective quantum of action"

$$\hbar_{\text{ef}} = \hbar\coth\frac{\hbar\omega}{2k_BT} \geqslant \hbar.$$

Taking the second way we constructed a model of the system environment in the form of the ETB filled with the thermal radiation which is the most natural object with really infinite number of freedom degrees. It contains the infinite set of weakly bounded *quantum* oscillators, each of them corresponds to a harmonic mode of radiation at frequency $\widetilde{\omega}$ and average energy $\langle\varepsilon\rangle$, determined by Planck's formula

$$\langle\varepsilon\rangle = \frac{\hbar\widetilde{\omega}}{2}\coth\frac{\hbar\widetilde{\omega}}{2k_B\widetilde{T}}$$

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at any temperature. So this model is fundamentally differed from the standard model of thermobath used in quantum statistical mechanics (*QSM*) consisting from weakly bounded *classical* oscillators with

$$\langle \varepsilon \rangle = k_B \tilde{T}.$$

Thus, it is accepted that an effective temperature of an ETB mode (Bloch's temperature [5])

$$\widetilde{T}_{\text{ef}} = \frac{\langle \varepsilon \rangle}{k_B}.$$

Its limiting expressions are Kelvin's temperature $\widetilde{T}$ at $k_B\widetilde{T} \gg \hbar\widetilde{\omega}/2$, which is the same for all modes and a specific quantum temperature $\widetilde{T}_Q = \hbar\widetilde{\omega}/2k_B$ (Wigner's temperature [6]) at $k_B\widetilde{T} \ll \hbar\widetilde{\omega}/2$.

Essentially, that the initial imbedding of any system into the ETB allows consistently constructing two sections of the new theory each of them does not use such a notion as a particles number and, in a certain sense, is a thermofield analogue of Clausius'classical and Einstein's statistical thermodynamics accordingly. We call them as *Thermofield Thermodynamics* (*TfT*) and *Thermofield Statistical Thermodynamics* (*TfSD*).

In the *TfT* section (as it is accepted in thermodynamics) non concrete representations about a system concerning its mass and structure are required and the condition of thermal equilibrium of system with the ETB mode is kept its validity in the standard form of the Zero Law

$$T_{\text{ef}} = \widetilde{T}_{\text{ef}}$$

. The First and the Second Laws of equilibrium thermodynamics thus formally do not change, but we suggest generalizing the concept of transferred heat

$$\delta Q_{\text{ef}} = T_{\text{ef}} \Delta S_{\text{ef}} = (C_{\text{ef}})_V \Delta T_{\text{ef}}.$$

Here, a more general characteristic, such as energy capacity $(C_{\text{ef}})_V$, is entered. Its limiting values has the expressions C_V and $\frac{k_B}{\hbar/2}\rho_\omega$ where ρ_ω is the spectral density of radiation. Finally, the Third Law and efficiency of Carnot cycle accordingly have the forms

$$\lim_{\widetilde{T} \to 0} S_{\text{ef}} = S_0 \neq 0; \qquad \eta = 1 - \frac{(\widetilde{T}_{\text{ef}})_2}{(\widetilde{T}_{\text{ef}})_1}.$$

In the *TfST* section both types of uncontrollable influences on a system are considered correspondingly to an simultaneously account of *quantum* and *thermal* fluctuations of system macroparameters, including its effective temperature as well. The thermal equilibrium is defined now by the generalized Zero Law

$$T_{\text{ef}} = \widetilde{T}_{\text{ef}} \pm \Delta T_{\text{ef}}.$$

According to Gibbs-Einstein and Blokhintsev, in contrary to Boltzmann's assembly of particles, we enter the ensemble [7], containing an infinite set of complete system copies under certain external conditions. This choice allows us to apply the obtained relations to a single microparticle as well. *TfST*is based on Gibbs'-Einstein's canonical initial distribution in the space of macroparameters at the module of distribution associating by the effective temperature T_{ef}. It takes the form [8]

$$\rho(\varepsilon) = \exp\left\{\frac{F_{\text{ef}} - \varepsilon}{k_B T_{\text{ef}}}\right\} \Omega(\varepsilon).$$

Expanding the structural function $\Omega(\varepsilon)$into a series up to the second order, it is possible to find fluctuations of any system macroparameters, including effective temperature as well. In the most simple case at $\Omega(\varepsilon) = 1$, from the given distribution, the expression for effective entropy follows in the form

$$S_{\rm ef} = -k_B \int {\rm d}\varepsilon \rho(\varepsilon) \ln \tilde{\rho}(\varepsilon) = \frac{\langle \varepsilon \rangle - F_{\rm ef}}{T_{\rm ef}}.$$

Here effective free energy

$$F_{\rm ef} = -k_B T_{\rm ef} \ln \coth \frac{\hbar\omega}{2k_B T}$$

and $\tilde{\rho}(\varepsilon)$, in contrary to $\rho(\varepsilon)$, is a dimensionless density of probability. Formulae for fluctuations of energy and temperature of a system are similar to Einstein's formulae, but they additionally show that fluctuations of frequency $\Delta\omega = \langle \omega \rangle$ and energy $\Delta\varepsilon = \langle \varepsilon \rangle$ always take place including the ground state.

Some advantage of *TfST* over *QSM* ground on the different choice of thermobath models. Studying the entropy in the frame of *TfST* we can take the quantum oscillator in the TCCS as a macrosystem in the thermal equilibrium. Its effective entropy $S_{\rm ef}$ has the form

$$S_{\rm ef} = k_B \left\{ 1 + \ln \coth \frac{\hbar\omega}{2k_B T} \right\}$$

in which the contribution of the energy fluctuations in the ground state is included. But according to *QSM,* we have the essentially other expression

$$S = k_B \left\{ \frac{\hbar\omega}{k_B T} \left(\exp \frac{\hbar\omega}{k_B T} - 1 \right)^{-1} - \ln \left(1 - \exp \left(-\frac{\hbar\omega}{k_B T} \right) \right) \right\}.$$

We note the entropies $S_{\rm ef}$ and S are different. Particulary, at $T \to 0 : S_{\rm ef} \to k_B \neq 0$ while $S \to 0$ but at $T \gg \hbar\omega/2k_B$ we have $S_{\rm ef} \approx S$. The condition of applicability of thermodynamics $\Delta T_{\rm ef} = T_{\rm ef}$ at $T \to 0$ in *TfST*, in contrary to *QSM*, are satisfied.

The legitimacy of our results can be also proved with the help of the *TfD* calculations. We define the effective entropy on the ground of the wave function $\psi(q, T)$ obtained by us

$$S_{\rm ef} = -k_B \left\{ \int {\rm d}q\, \rho(q) \ln \tilde{\rho}(q) + \int {\rm d}p\, \rho(p) \ln \tilde{\rho}(p) \right\},$$

where $\tilde{\rho}(q)$ and $\tilde{\rho}(p)$ are dimentionless densities of probability. This expression might be rearranged through the "effective quantum of action"

$$S_{\rm ef} = k_B\{1 + \ln(\hbar_{\rm ef}/\hbar)\} = k_B\{1 + \ln W\},$$

where W — is a number of microstates in the given macrostate.

As a result, Fenyes' idea [9] about an opportunity of the Nature description simultaneously at micro- and macrolevels by the means of two alternative ways proves to be true. We have demonstrated the language of *TfST* allows using the generalized diffusion equation for density $\rho_N(q, T_{\rm ef})$, at the effective coefficient of diffusion

$$D_{\rm ef} = \frac{k_B}{m\omega} T_{\rm ef},$$

while the language of *TfD* allows using the Shroedinger equation for wave function $\psi(q, T)$. Moreover the both ways of the description give us the equivalent results, and they are applicable even up to the ground state of oscillator equal in rights (at $T \to 0$).

We believe that using *TfST* principles and the main *TfT* ideas together can be propagated on other systems (relic radiation in the early Universe, quark-gluon plasma, quantum computers, nanostructures etc.). Then we can obtain their adequate description at any temperatures and avoid some difficulties which appear in the frame of *QSM* for any system at

$$\langle \varepsilon \rangle = k_B T_Q \geqslant k_B T.$$

We hope that efficiency of the suggested theory at the description of nearly perfect fluids [10] can be confirmed in the near future both at the analysis of quark - gluon plasma and the further studying of He-4 superfluidity and Bose-Einstein condensates.

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Два подхода к квантовому обобщению термодинамики

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Предлагаются два подхода к описанию системы, находящей под одновременным квантовым и тепловым стохастическим воздействием, каждый из которых является термополевым аналогом классической термодинамики Клаузиуса и статистической термодинамики Эйнштейна соответственно. Мы называем их *термополевой термодинамикой* и *термополевой статистической термодинамикой*. Исходя из с термополевого вакуума устанавливается соответствие между основными понятиями квантовой механики и термодинамики. Предлагается рассматривать термополевой вакуум как эффективный термостат и основное состояние как состояние теплового равновесия. На этом основании вводятся такие понятия как эффективная температурная и эффективная энтропия и открывается возможность описания системы, находящейся под одновременным квантовым и тепловым воздействием.

Ключевые слова: эффективная температура, эффективная энтропия, комбинированное квантовое и тепловое воздействие, стохастическое воздействие, термополевой вакуум.