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A Logical Analysis of the Two Widely Spread Misinterpretations of the Well Established Results of the Special Theory of Relativity

D. M. Davidović, L. Nadderd and S. Stanković

Institute of Nuclear Sciences Vinča,
12–13 Mike Petrovića Alasa, Belgrade, 11001, Serbia

By a rigorous logical analysis we show that the Ehrenfest paradox related to the relativistically rotating disc is not a paradox of the Special Theory of Relativity as it is usually treated in the literature because, as we proved rigorously, it does not follow from the principles of the Special Theory of Relativity but — contrary to this — contradicts them. We also analyse the logical status of the widely spread statement "moving clocks go slow" within the framework of the Special Theory of Relativity. Using the methods of the symbolic logic we show that this statement with correct interpretation of the meaning of the notion that the clock goes slow is, strictly speaking, at least misleading. We discuss the obtained results.

Key words and phrases: Ehrenfest paradox, special theory of relativity, clock paradox.

1. Introduction

Many problems in interpretation of the results of modern physics, which are discussed in the philosophy of physics, and which are particularly connected with modern discoveries, came out to be only terminological, independent of the success or inadequacy of physics proper [1,2]. Sometimes monstrous forms and dimensions are acquired by speculations related to the obscurity of terminology. Even simple problems turn out to be practically unsolvable and often obtain a mystical quasi religious flavour due to the ignorance or neglect of the logical methods.

In the present work, in the next Section, using rigorous methods of modern logic we prove that the Ehrenfest paradox related to the relativistically rotating disc is not the paradox of the Special Theory of Relativity (STR) because it, as we show, contradicts the principles of the STR.

In Section 2 we show that the clarification of the meaning of the sentence "moving clock goes slow" may lead to better elucidation of the proper results of the STR related to the rates of moving clocks.

In the last Section we discuss the obtained results.

2. Ehrenfest Paradox and the STR

Born [3] defined the "relativistic rigid-body" as such a body whose every infinitesimal element regarding Lorentz contraction behaves in the same way as would behave an analogous element being at rest and free in the inertial frame, which moves with the same velocity as the considered element, immersed in a continuum. Ehrenfest [4] immediately noticed that this definition even in simplest cases is self-contradictory. Namely for a rotating cylinder one would have (R is the radius of the cylinder when stationary, R' in rotation):

"(a) The circumference of the cylinder must show a contraction relative to the rest state, $2\pi R' < 2\pi R$ since each element of the circumference moves in its own direction with instantaneous speed ΩR.

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(*b*) If one considers an element of a radius, its instantaneous velocity is perpendicular to its length; thus, an element of the radius cannot show a contraction with respect to the rest state. Therefore $R = R'$ ..."

In this way (*a*) claiming that $R' < R$, and (*b*) claiming that $R' = R$, contradict each other.

The Ehrenfest paper marks the beginning of the problem which later became well known as the Ehrenfest paradox. There is a widely spread confusion that (*a*) and (*b*) follow from the first principles of the STR. Namely, analyzing geometry on relativistically rotating disc in his 1916 fundamental paper on general relativity [5] and a couple of years later in his popular book [6] Einstein used the following two assumptions:

(*i*) A measuring-rod laid down along the periphery of the rotating disc is Lorentz contracted because it is approximately in an inertial motion with a velocity along its axis.

(*ii*) Such a rod along radial direction is not contracted as it is approximately in inertial motion, this time with velocity perpendicular to its axis.

When the lengths of these rods compared to the radius of the disc become infinitesimally small, these relations become exact.

Einstein assumptions (*i*) and (*ii*) are very broadly perceived as almost identical to (*a*) and (*b*). This fact is somewhat surprising because Eddington [7], long ago, clearly stated the root of the Ehrenfest paradox and the logical error contained in the Born model:

"Consider a wheel revolving rapidly. Each portion of the circumference is moving in the direction of its length, and might be expected to undergo the Fitzgerald contraction due to its velocity; each portion of a radius is moving transversely and would therefore have no longitudinal contraction. It looks as though the rim of the wheel should contract and the spokes remain the same length, when the wheel is set revolving. The conclusion is absurd, for a revolving wheel has no tendency to buckle - which would be the only way of reconciling these conditions. *The point which the argument has overlooked is that the result here appealed to apply to unconstrained bodies ...*" (Emphasized by us.)

To show rigorously by purely logical means that (*a*) and (*b*) are not logical consequences of (*i*) and (*ii*), although at first shallow sight they may seem equivalent, and that opposite claims contain invalid argumentation, let us consider the problem in a slightly broader context.

What is the dividing line between valid and invalid argumentation and conclusions? There are two main approaches to this question [8,9]. In the first approach which may be called the rule-based or syntactic one, we count a complicated argument as valid if it can be broken down into simple steps which we immediately recognize as valid. If any of these steps is invalid the whole argument is also invalid. Checking an argument by this approach requires very detailed, step by step analysis. However, for our purpose this approach is inadequate because (*a*) and (*b*) usually are not derived from anywhere but simply stated by fiat as being consequences of the STR. On the other hand (*i*) and (*ii*) have a well founded logical status in the framework of STR. They are well established approximations valid exactly in the limit.

There is another approach to check validity of the argument and to see whether some statements may be considered as logical consequences of given premises. This is the semantic approach. In this approach a given statement is counted as a logical consequence of a set of premises if every interpretation on a model, which makes the premises true, makes the consequence true to. In the opposite case the argument leading to such conclusion must be considered as invalid. This means that only one counter-example or in other words an interpretation on one model which turns the premises into true sentences and the conclusion or consequence into a false sentence shows that the corresponding argumentation is false.

To find convenient models for such an approach we must find the case where we can see easily and unequivocally how the disc behaves when it comes into rotation. To this end, we may construct discs from measuring-rods arranged in such a way that we know in advance their behavior during rotation. In this way, without any further investigation, we can simply read of the behavior of the so constructed disc as a whole, from the known behavior of measuring-rods - the constitutive elements of such a disc.

Now, we have only two situations in which we know the behavior of measuring-rods in rotation, and these correspond to Einstein assumptions (i) and (ii).

To put in action the assumption (ii) we construct a "star" disc. This consists from rods arranged all along the radial direction, fixed in the centre, like playing cards in a hand or a fan. Keeping in mind that measurement of geometric relations of moving bodies in the framework of the STR is by definition, the measurement of their simultaneous traces, we may obviously conclude that the "star" disc does not change its shape during rotation; both radius and circumference and their ratio remain the same when measured from the stationary frame.

To use the assumption (i) we construct the "ring" disc. This one consists from short rods arranged circularly and say elastically connected so that when their ends touch each other there is no force between them and when their ends separate an attractive force arises.

Let us now return to our original problem. We consider (i) and (ii) as evident consequences of the STR. On the "star" model, as well as on every our model, (i) and (ii) are fulfilled together because they do not depend in whatever way from the behavior of discs during rotation but exclusively from the behavior of free rods in inertial motion, and only from this. However on the star model (b) is fulfilled and (a) is not. For the "ring" model we have the opposite situation - (a) is fulfilled while (b) is not. (Of course, again, both (i) and (ii) are fulfilled.) This shows that (a) and (b) are not logical consequences of (i) and (ii) and these two follow from the first principles of the STR. This terminates the proof that (a) and (b) do not follow from the first principles of the STR because they contradict (i) and (ii), and due to this they are in contradiction with the STR.

This result should put an end to the futile efforts to solve the Ehrenfest paradox in the framework of the STR, as if it were an inherent contradiction of this theory. These efforts have a goal which is absolutely unachievable because this paradox does not fit to the framework in which its solution is hopelessly seek for, and due to this are from the outset a lost case.

3. A Logical Analysis of the Statement "Moving Clocks Go Slow"

In our analysis by a clock (stopwatch) we mean an oscillator which is characterized by its period T and its reading, which roughly speaking counts the number of periodic processes done by the oscillator and which we denote by n. To make the essence of our logical argument as transparent and clear as possible we will first consider the following completely classical case. Suppose that we have three stopwatches, denoted by 0, 1 and 2, which are all located at the same place in a stationary closed box so that we can not directly see how they work. Let the time-measuring mechanisms of the stopwatches 0 and 1 start both to run at an accidental instant. Let at another accidental instant the stopwatch 1 stops and at the same instant the stopwatch 2 starts to run and finally, at the third accidental instant, both stopwatches 0 and 2 stop running. If all the stopwatches run correctly i.e. with a same rate, we will, after opening the box, find that their readings satisfy the obvious equality: $n_0 = n_1 + n_2$.

In the case that this equality is not satisfied we would conclude purely on the logical grounds that at least one of them has a different period. Since, we assumed that during the described process the box was closed, we were not able to compare directly the periods of the clocks.

If in a repeated series of such experiments we would always have $n_0 > n_1 + n_2$ we obviously could, without whatever further measurements, conclude that either:

$$T_0 < T_1 \quad \text{or} \quad T_0 < T_2 \quad \text{or both.} \tag{1}$$

Now, with only slight changes of the above arrangement, which from the logical point of view are irrelevant and almost trivial, we can apply the same reasoning and obtain the same results for moving clocks in the framework of the STR. To this end, let now our stopwatches 0, 1 and 2 are not closed in a box but are collinearly moving with corresponding constant speeds v_0, $v_1$and v_2. Let in a chosen inertial frame these speeds satisfy the inequalities:

$$0 < v_0 < v_1 < c, \quad -c < v_2 < v_0 \,. \tag{2}$$

With such a choice of speeds and the adequate initial positions each stopwatch will encounter two other stopwatches once and only once. So, each stopwatch will have two encounters with the other two remaining stopwatches. Let it's time mechanism starts to run at the moment of the first encounter and stops to run at the moment of the second encounter. At the end of the experiment the stopwatches record the corresponding time readings, which are numbers which we denote by: n_0, n_1 and n_2. It is an experimentally confirmed fact, completely in accord with the STR, that these time readings of the stopwatches satisfy the following relation:

$$n_0 > n_1 + n_2 \,. \tag{3}$$

In classical physics it was assumed *a priory* that the equality must hold. As stopwatches are classical systems in a sense that they have their characteristics independently of whatever observations and measurements, contrary to the quantum mechanical systems which get at least some characteristics only in the act of measurements, all stopwatches will have well defined periods independently of the fact whether we can measure and directly compare them or not. Due to this by the same reasoning and intuition the inequality (3) would imply the same conclusion as in (1), when all clocks were in a box at the same place.

Namely, the logical situation is the same. In both cases we have the same inequality. Also, in both cases we were unable to compare or perceive the periods T_i directly. Only the reasons for this impossibility were different. In the first case this was the fact that the clocks during the experiment were closed in the box and so inaccessible for direct observation, while in the second case the direct observation and comparison was impossible due to the movement of clocks. Also, the supposed outcome of the first experiment is somewhat artificial, especially constructed as a model for corresponding logical conclusion, while the result of the second experiment is in fact in such circumstances always present in nature. But for our logical conclusion which is the same in both cases, the mentioned differences are obviously irrelevant. So, using logical symbols, we can claim that the statement:

$$(T_1 > T_0) \vee (T_2 > T_0), \tag{4}$$

is true.

The relation (4) is absolute and invariant in a sense that it is not dependent on whatever convention. Also, it is theory independent. Various theories can not deny the physical fact described by this relation but can only supply corresponding, possibly different, explanations.

We will now use the obtained result to elucidate the logical status of the statement "moving clocks go slow".

Consider the inertial reference frame in which the stopwatch 1 is stationary and in which the standard synchronization is applied. In this reference frame the stopwatch 0 is moving. It is a very well established fact that in the standard synchronization time reading of the moving clock while it moves between the two fixed stationary points is always smaller than the difference of corresponding readings of the clocks fixed in these points. Now, this undeniable fact is used in almost all existing texts for the conclusion that "moving clock goes slow" (see e.g. [10, 11]). However, in any rational interpretation of the meaning "go slow" this would imply that the period of the moving clock is greater then the period of clocks which are stationary in the considered frame. In our case this would mean that $T_0 > T_1$. Applying the same procedure to the clock 2 one would get in the same way $T_0 > T_2$.

This can be written as:

$$(T_0 > T_1) \wedge (T_0 > T_2). \tag{5}$$

Applying the logical law $\neg(p \vee q) \iff (\neg p \wedge \neg q)$, simple calculations give:

$$\neg\Big[(T_1 > T_0) \vee (T_2 > T_0)\Big] \iff \Big[(T_0 > T_1) \wedge (T_0 > T_2)\Big]. \tag{6}$$

This relation shows that the statement "moving clock goes slow" is in contradiction with the well established result of the STR because the relation (5) contradicts the relation (4), which may be considered as an experimental fact. Our results simply show that the fact that the time reading of moving clock which is always smaller than the difference of corresponding readings of two fixed clocks, and this fact is symmetric and incontestable, does not give the information about relative relation between periods of moving and stationary clocks. This result is intuitively clear because in Lorentz transformations only time readings, $n - s$ in our notations, are present but not corresponding $T - s$.

The above analysis shows that the statement "moving clock goes slow" is misleading and if properly understood — contradicts the unequivocally established results of the STR, (which are reflected in (3) and (4)), and which were confirmed experimentally. In this way this statement may be regarded as an example of the misuse of language contradicting the laws of logic, and as such may be an obstacle in correct understanding and interpreting of the STR.

4. Discussion

We would like now to compare the results obtained for relativistically rotating disc and for moving clocks.

Born model for the "relativistic rigid-body", which is reflected in the Ehrenfest paradox directly contradicts STR and is self-contradictory.

The statement "moving clocks go slow" if "goes slow" is understood in a most rational way as "has a greater period", also contradicts the results of the STR.

It may be considered as correct if and only if we identify "go slow" with the fact that the reading of the moving clock while it moves between two fixed stationary points is always smaller than the corresponding difference of time readings of clocks in these points. However, understood in this way it obscures the real conclusion of the STR about moving clocks.

But where does this famous conclusion that "moving clock goes slower" comes from? It comes from the fact that always when the time interval which a moving clock shows in its movement from one point in an inertial frame to the other point, is compared with the difference of two local times in these points. From Lorentz transformations follow that the last number must always be greater. But this relation is symmetric and leads to a confusing statements of the type "from the point of view of the observer", or still worse "one observer sees so" and the other differently. In fact, they only have in corresponding inertial frames, which may be thought as physical laboratories, to perform analogous measurements with the same protocols and it would be in contradiction with the principle of relativity if they would obtain different results. In this sense the experiments with moving mesons only confirm the principle of relativity. The experiments which transported the clocks around the Earth, according to correct interpretation supplied, did not prove that moving clocks go slow, but that in a set of at least three clocks moving with different velocities not all of them can have equal rates.

To recapitulate: We have shown that the conclusion "moving clocks go slow" is not a consequence of the Special Theory of Relativity, contrary to the almost unanimously held belief, still less — its important consequence. We have shown that it is a consequence of the misinterpretation of the results of Lorentz transformations and the STR itself, and also of the misuse of the ordinary language. We have further shown that in order to speak cogently about the different rates of clocks, within the framework of the STR, one must have at least three clocks moving with different speeds and without acceleration. In such a situation, unequivocal conclusion of the STR is that at least one of these clocks has a different rate than the others but one does not have whatever physical mean to find which one, whether "moving" or this "at rest". The order among their rates may be established by convention and only by convention, choosing by a completely free choice whichever of them as “stationary”.

Let us mention at the very end that we intentionally avoided all technicalities and formulas, and retained only physical and logical arguments, in order to help the reader to see the physical situation in considered problems aright, the very essence of which is simple, but which is greatly obscured with a heavy ballast of many misunderstandings, both old and modern, but mainly the latter. In these, very often, sophisticated mathematical apparatus or ambiguity and uncertainty of terminology thoroughly obscure underlining physical reality.

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Логический анализ двух широко распространённых ошибочных интерпретаций известных результатов специальной теории относительности

Д. М. Давидович, Л. Наддерд, С. Станкович

Институт ядерных наук «Винча»
Сербия, 11001, Белград, ул. Мика Петровича, 12–13

Путём строгого логического анализа мы показываем, что парадокс Эренфеста, относящийся к релятивистски вращающемуся диску, не является парадоксом специальной теории относительности, как это обычно трактуется в литературе. Мы строго показываем, что он не следует из постулатов специальной теории относительности, — напротив, он противоречит им. Мы также анализируем логическое содержание широко распространённого утверждения о том, что, согласно специальной теории относительности, «движущиеся часы идут медленнее». Используя методы символической логики, мы показываем, что это утверждение, если правильно интерпретировать его содержание, по крайней мере, вводит в заблуждение.

Ключевые слова: парадокс Эренфеста, специальная теория относительности, парадокс часов.