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Numerical Stability of an Integral Equation Applied for High-Temperature Plasma Diagnostics

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The steady numerical solution using Tikhonov functional with a stabilizer of the second order was obtained for the integral equation of the first kind, which occurs for deconvolution of thin-target bremsstrahlung spectra to determine electron energy distributions function (EEDF)

Key words and phrases: integral equation, regularization, high-temperature plasma.

1. Introduction

One of the most common method of high-temperature plasma diagnostics is to study the spectrum of bremsstrahlung in X-ray wavelengths range. The experimentally registered shapes of the spectrum reflects the energy distribution function of the electron population (EEDF) over the photon energy. Whereas this is qualitatively obvious, the quantitative relation between the spectrum and the distribution is far from simple. In this paper we consider the possibility of restoring of the EEDF when it deviates from the Maxwellian, which is important for the investigation of strongly non-equilibrium high-temperature plasma [1].

In [2] under some assumptions and with some values of the parameters for concrete action of the experiment, we consider the integral equation relating the unknown distribution function of plasma electrons in the energy f and bremsstrahlung spector N

$$\int\limits_a^b H(E,\varepsilon)f(\varepsilon)d\varepsilon = N(\varepsilon), \quad \varepsilon \in [a,b], \tag{1}$$

where the kernel H - known continuous function. We write this equation in the form

$$Hf = N. \tag{2}$$

In [2] an approximate solution of this integral equation in the case, when $\|N - N^\delta\| = \delta$, it was proposed to look like extreme Tikhonov functional

$$M^\alpha[u] = \|Hu - N^\delta\|^2 + \alpha\|u'\|^2$$

with the stabilizer of the first order $\alpha\|u'\|^2$. Extreme can be obtained as the solution of the Euler equation

$$H^*Hu - \alpha u'' = H^*N^\delta,$$
$$u'(a) = 0, u'(b) = 0.$$

The solution of this problem as an approximate solution (1) has a defect (including those with numerical implementation) ¡¡deformation¿¿ of the solution to the zero value of the derivative at the points a, b. To avoid this ¡¡effect¿¿, in this paper, we propose to construct an approximate solution of the Euler equations for the Tikhonov functional with a stabilizer of the second order.

2. Situation of the problem

We construct an approximate solution of equation (2) as the extreme Tikhonov functional with a stabilizer of the second order

$$M^{\alpha}[u] = \|Hu - N^{\delta}\|^2 + \alpha\|u''\|^2, \tag{3}$$

Satisfies the boundary conditions

$$\begin{aligned} &u''(a) = 0, u''(b) = 0,\\ &u'''(a) = 0, u'''(b) = 0. \end{aligned}$$

In this case extremal of the functional (3) is the solution of the problem

$$\begin{aligned} &H^*Hu + \alpha u^{(4)} = H^*N^{\delta},\\ &u''(a) = 0, u''(b) = 0,\\ &u'''(a) = 0, u'''(b) = 0. \end{aligned} \tag{4}$$

The regularization parameter can be found from the condition of the discrepancy

$$\|Hu_{\alpha} - N^{\delta}\|^2 = \delta^2.$$

The convergence of the approximate solutions to the exact [3].

3. Discrization of the problem

To obtain the numerical solution of (4) we introduce a uniform grid on the interval $[a, b]$

$$\varepsilon_j = a + (j-1)h,\ j = 1, ..., n,\ h = (b-a)/(n-1).$$

The integral operator in equation (2) with kernel H goes into the matrix H_{ij}

$$H_{ij} = H(E_i, \varepsilon_j)hq_j, \quad q_j = 0.5 \text{ for } j = 1, n, \quad q_j = 1 \text{ for } j = 2, ..., n-1.$$

The solution of the problem (3) converts into the desired vector

$$y_j = u(\varepsilon_j).$$

The term $\alpha u^{(4)}$ with the boundary conditions of (4) becomes the product of a $\alpha Ty/h^4$ matrix T with rows

$$\begin{pmatrix}
1 & -2 & 1 & 0 & 0 & 0 & . & . & . & . & . & .\\
-2 & 5 & -4 & 1 & 0 & 0 & . & . & . & . & . & .\\
1 & -4 & 6 & -4 & 1 & 0 & . & . & . & . & . & .\\
0 & 1 & -4 & 6 & -4 & 1 & . & . & . & . & . & .\\
. & . & . & . & . & . & . & . & . & . & . & .\\
. & . & . & . & . & 0 & 1 & -4 & 6 & -4 & 1 & 0\\
. & . & . & . & . & 0 & 0 & 1 & -4 & 6 & -4 & 1\\
. & . & . & . & . & 0 & 0 & 0 & 1 & -4 & 5 & -2\\
. & . & . & . & . & 0 & 0 & 0 & 0 & 1 & -2 & 1
\end{pmatrix}$$

Thus, the problem (4) becomes a linear system

$$H^*Hy + \frac{\alpha}{h^4}Ty = H^*N^{\delta}. \tag{5}$$

The solution of this system is regarded as a grid solution of the problem (4).

4. Conclusion

Solution of the linear system (5) for values of the regularization parameter chosen by the discrepancy, gives a satisfactory approximate numerical solution of (1). This solution can be used for EEDF evolution study to provide detailed information on fundamental physical processes occurring in non-equilibrium high-temperature plasma experiments.

References

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Об устойчивом численном решении одного интегрального уравнения применимого в диагностике высокотемпературной плазмы

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Для интегрального уравнения первого рода, возникающего при операции восстановления функции распределения по энергии электронов (ФРЭЭ) по спектру тормозного излучения, получено устойчивое численное решение с использованием функционала Тихонова со стабилизатором второго порядка.

Ключевые слова: интегральное уравнение, регуляризация, высокотемпературная плазма.