Bulletin of PFUR Series *Mathematics. Information Sciences. Physics.* No 3. 2009. Pp. 99–105

UDC 535.42:681.786.23

Noether Theorem and Variation of Charges of the Universe

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We present new approach to explanation of possible cosmological evolution of fundamental charges related to spinor particles. This approach is based on the use of modified version of massless Dirac equation with analytic dependence of wave-function on extra complex time-like parameter. We identify real and imaginary parts of complex time with absolute cosmological time and temperature, respectively, and study canonical structure of the theory. It is established, that gauge and energy-momentum Noether charges are temperature-dependent, and, moreover, that these quantities are related through their temperature dependence. It is argued, that total charges of the Universe must vary in accordance with temperature evolution in framework of realistic cosmological scenario.

Key words and phrases: Noether theorem, fundamental charges, cosmological evolution.

1. Introduction

Variations of fundamental charges is old and famous activity [1–3]. Usually it is based on the hypothesis, that fundamental charges change their values during the cosmological evolution [4, 5]. In part, this approach is supported by the Dirac's laws of huge numbers [6]. Actually, these laws are relations between fundamental charges, related to micro-world, and cosmological quantities, which describe the total Universe. For example, the Universe time of life is one of the global characteristics, which is included into this mysterious numerical "game". If the corresponding dimensional analysis will be supported by the fundamental theory, then one will be able to state that the elementary charge changes due to the global cosmological scenario [7, 8].

The most well out-worked approach to explanation of possible cosmological evolution of fundamental charges and constants is given in framework of multidimensional and superstring theories [9, 10]. In these theories, one deals with fundamental multidimensional system considered over the appropriate cosmological ansatz. All four-dimensional quantities, including fundamental constants, are calculated according to compactification scheme taken. They depend on the cosmological time (the time passed after the Big Bang) together with decreasing extra dimensions [11–15].

Our approach to possible variations of fundamental constants is based on the use of generalized quantum theory with extra complex parameter of evolution, developed in [16], and its relativistic spinor realization, presented in [17]. In this paper we establish canonical structure of this theory, and calculate its Noether charges. We show, that these charges do not depend on the extra time (which is the real part of complex parameter of evolution) explicitly, but are the functions on the temperature of the system (which is defined by the imaginary part of this parameter). We identify extra time and temperature with the corresponding cosmological characteristics, and consider given scenario with fixed global temperature function. Finally, Noether charges become non-explicit functions on the cosmological time, and we obtain consistent theoretical scheme for variations of different quantities which can be calculated using the Noether charges. Such quantities characterize the total Universe starting with fundamental canonical micro-scheme. Thus, this approach can be useful for providing

Received 4th June, 2009.
Authors are really grateful to prof. B. Ishkhanov, Yu. Rybakov, P. Pronin, G. Costa and M. Matone for fruitful discussions while this investigation was carried out.

a theoretical base for the Dirac laws of huge numbers and variation of fundamental constants after the appropriate out-working.

2. Modified Dynamics of Spinor Field

General principles of quantum theory [18] lead to the Schrodinger's equation

$$i\hbar\,|\Psi\rangle_{,\,\tau} = \hat{\mathcal{H}}\,|\Psi\rangle \tag{1}$$

to the state vector Ψ. Here τ means the evolutionary parameter, whereas $\hat{\mathcal{H}}$ is the operator of Hamilton, which is independent on $|\Psi\rangle$. For fundamental systems this operator is stationary one, i.e. it is τ-independent; thus, $\hat{\mathcal{H}}_{,\,\tau} = 0$.

Structure of the theory preserves its form if one admits a complex nature of the parameter of evolution, and postulates the holomorphic conditions

$$|\Psi\rangle_{,\,\tau^*} = 0 \tag{2}$$

and $\hat{\mathcal{H}}_{,\,\tau^*} = 0$, and also the restriction $[\mathcal{H},\, \mathcal{H}^+] = 0$ on the Hamiltonian of the theory. In [16] it was shown, that the theory with non-Hermitian Hamiltonian ($\mathcal{H}^+ \neq \mathcal{H}$) with positive norm of the states ($\langle\Psi|\Psi\rangle > 0$) possesses arrow of time in its dynamics. Namely, the dynamics is non-reversible for isothermal and other significant temperature regimes of evolutions. Such thermodynamical conclusions are based on decomposition

$$\tau = t - \frac{i\hbar}{2}\beta \tag{3}$$

of the complex parameter τ to the "usual" time t and inverse absolute temperature β. We identify these absolute time and temperature with cosmic time — the time of life of the Universe, and CMB temperature — the temperature of Cosmic Microwave Background. Thus, we introduce these "macro-dynamical" characteristics into the "micro-dynamics" of the spinor field. The resulting theoretical scheme is consistent from the both cosmological and particle physics points of view; it realizes unity of these two fundamental branches of fundamental theory.

Our variant of modified dynamics of spinor field is defined by the Hamiltonian

$$\hat{\mathcal{H}} = -c\left(\gamma^4\right)^{-1}\gamma^\mu \hat{p}_\mu, \tag{4}$$

where γ^μ is the corresponding conventional Dirac matrix, $\hat{p}_\mu = i\hbar\partial_\mu$ is the momentum operator, and $\mu = 0$, ..., 3. The matrix γ^4 can be taken in two forms: as -1 and also as γ^5. In the both cases $\hat{\mathcal{H}}^2 = \hat{p}^2 c^2$, where $\hat{p}^2 = g^{\mu\nu}\hat{p}_\mu\hat{p}_\nu$, i.e. the operator of Hamilton satisfies a relativistic mass relation. This fact allows us to introduce the mass operator $\hat{m} = \hat{\mathcal{H}}/c$; in [17] it was shown, that it is Hermitian one. Then, in this work it was also established, that spectrum of the theory contains states with non-positive norm $\langle\Psi|\Psi\rangle$. This means absence of arrow of time in the dynamics of the spinor system under consideration. Below we study canonical structure of this relativistic reversible theory.

3. Five-Dimensional Noether Analysis

Using Eqs. (4) and (3), it is possible to rewrite the dynamical relations (1)–(2) of the theory in the following five-dimensional form:

$$\gamma^M \Psi_{,\,M} = 0, \quad \Psi_{,\,\beta} = -\frac{i\hbar c}{2}\Psi_{,\,4}. \tag{5}$$

Here $M = 0, ..., 4$, and we have introduced the 5th coordinate $x^4 = ct$. Note, that $x^4 \neq x^0$: these two temporary variables are kinematically independent. Thus, the theory is based on the double time formalism with x^0 and x^4 as Minkowskian and Hamilton's times, respectively. In this theoretical scheme, the time x^4 "counts" four-dimensional space-times with the coordinates x^μ, and the "process of counting" is defined by choice of the observer. In this simple model, the observer is chosen if the temperature function $\beta = \beta(t)$ is fixed. For example, one can consider isothermal evolution with $\beta = \mathrm{const}$, or the thermodynamic regime with other restriction on the macroscopic observables.

The first relation in (5) is the conventional five-dimensional massless Dirac equation in the case of $\gamma^4 = \gamma^5$. Actually, in this case the anti-commutator algebra $\{\gamma^M, \gamma^N\} = 2g^{MN}$ takes place, where $g^{MN} = \mathrm{diag}(1, -1, -1, -1, -1)$ is five-dimensional Minkowsky metrics. However, for the both realizations of γ^4, this equation can be derived from the action

$$\mathcal{S}_5 = \int \mathrm{d}^5 x \mathcal{L}_5 \tag{6}$$

with the Lagrangian

$$\mathcal{L}_5 = \frac{i}{2}\left(\bar{\Psi}\gamma^M\Psi_{,\,M} - \bar{\Psi}_{,\,M}\gamma^M\Psi\right). \tag{7}$$

Below we preserve a generality and consider both realizations of the model. Our goal is to study integrals of motion in this theory in respect to the cosmic time t. It is clear, that they are given by the Noether's procedure of constructing of conserving charges, which correspond to different symmetries of the action (6) [19, 20]. These charges include:

1. The gauge charge $\mathcal{Q}$ defined by the relation

$$\mathcal{Q} = \int \mathrm{d}^4 x\, \mathcal{J}^4, \tag{8}$$

where $\mathrm{d}^4 x = \mathrm{d}x^0 \mathrm{d}x^1 \mathrm{d}x^2 \mathrm{d}x^3$, and $\mathcal{J}^4$ is the 4-thcomponent of the five-dimensional current

$$\mathcal{J}^M = \bar{\Psi}\gamma^M\Psi. \tag{9}$$

2. The translational vector $\mathcal{P}_M$,

$$\mathcal{P}_M = \int \mathrm{d}^4 x\, \mathcal{T}^4_M, \tag{10}$$

where

$$\mathcal{T}^N_M = \frac{i}{2}\left(\bar{\Psi}\gamma^N\Psi_{,\,M} - \bar{\Psi}_{,\,M}\gamma^N\Psi\right). \tag{11}$$

The charges $\mathcal{Q}$ and $\mathcal{P}_M$ correspond symmetries in respect to the gauge and five-dimensional space-time shift transformations, respectively. The statement is that these charges do not depend on the Hamilton's time t (or x^4) in the explicit form, i.e. $\mathcal{Q}_{,\,t} = \mathcal{P}_{M,\,t} = 0$, they are functions on the temperature parameter β only. However, if one takes into account the temperature function $\beta = \beta(t)$, which defines the observer in the theory, for the total t-dependence of these charges one will has a non-trivial result.

Note, that temperature dependence of Noether charges is a remarkable property of this theory. Actually, this establishes a relation between conservative mechanics with its absolute integrals of motion, and thermodynamics of equilibrium states with the same quantities as temperature functions [21, 22].

4. Temperature Relations Between Noether Charges

Performing straightforward calculations (based on the use of the second relation from Eq. (5)), for the β-dependence of the gauge current one obtains: $\mathcal{J}^M{}_{,\beta} = -\hbar c\,\mathcal{T}_4^M$. From this it follows, that

$$\mathcal{Q}_{,\beta} = -\hbar c\,P_4, \tag{12}$$

i.e. the relation between gauge invariance and x^4-shift (which corresponds to the formal energy conservation law) in the theory. This fact is highly non-waited and non-trivial, it follows from analytical structure of the system. This leads to infinite chain of Noether currents in the system, i.e. any theory of our type possesses reach symmetry structure.

Namely, let us consider some continuous symmetry and the corresponding Noether charge $\mathcal{Q}_{(0)} = \mathcal{Q}_{(0)}(\beta)$. Let us define the charge $\mathcal{Q}_{(n)}$ by the relation

$$\mathcal{Q}_{(n)} = (-\hbar c)^{-n}\,\frac{\mathrm{d}^n\,\mathcal{Q}_{(0)}}{\mathrm{d}\beta^n}, \tag{13}$$

which generalizes Eq. (12) for the case of arbitrary integer n. It is easy to see, that such quantities are t-independent: actually, from the starting conservation law $\mathcal{Q}_{(0),\,4} = 0$ and Eq. (13) it follows the analogous relation

$$\mathcal{Q}_{(n),\,4} = 0 \tag{14}$$

for arbitrary $n > 1$. Of course, the total t-dependence of these charges is non-trivial; it is given by the relation $\mathrm{d}\mathcal{Q}/\mathrm{d}t = \dot{\beta}\,\mathcal{Q}_{,\beta}$, where $\dot{\beta} = \mathrm{d}\beta/\mathrm{d}t$. In the theory under consideration, Eq. (12) gives the concrete realization of the scheme presented.

5. Charge Conservation and Temperature Regime

It is easy to see, that the conservation law

$$\mathcal{Q} = \mathrm{const} \tag{15}$$

for the charge $\mathcal{Q} = \mathcal{Q}(t,\,\beta)$ is, in fact, the temperature regime $\beta = \beta(t)$ which guarantees such conservation. Moreover, for the Noether charges, this regime is equivalent to the isothermal one, because $\mathcal{Q} = \mathcal{Q}(\beta)$ in this case. This statement takes place for the all on-shell states which yield the Noether theorem conditions. For the states with non-vanishing bounding term the Noether charge conservation law leads to non-trivial temperature evolution.

Actually, let us consider the gauge charge (8) again, and impose the restriction (15) on the evolution. Taking into account the relation $d\mathcal{J}^4/dt = \mathcal{J}^4_{,t} + \dot{\beta}\mathcal{J}^4_{,\beta}$ and the Noether's null-divergent relation $\mathcal{J}^M_{,M} = 0$, from Eq. (15) one obtains, that

$$\dot{\beta} = c\,\frac{\int \mathrm{d}^4x\,\mathcal{J}^\mu{}_{,\mu}}{\int \mathrm{d}^4x\,\mathcal{J}^4{}_{,\beta}} = \frac{\Sigma_4}{\hbar\,\mathcal{S}_4}. \tag{16}$$

Here

$$\Sigma_4 = \oint \mathrm{d}\sigma_\mu\,\mathcal{J}^\mu \tag{17}$$

is the surface term (which arises after the Gauss theorem application), whereas

$$\mathcal{S}_4 = \int \mathrm{d}^4x\,\frac{i}{2}\,\left(\bar{\Psi}\gamma^\mu\Psi_{,\mu} - \bar{\Psi}_{,\mu}\gamma^\mu\Psi\right) \tag{18}$$

means the standard kinetic part of the four-dimensional action for the spinor field (it is generated by the second relation from Eq. (5)).

Note, that the temperature regime (16) does not coincide with the isothermal one in the general case. For the cosmological applications it is natural to start with some singularity with non-zero surface term Σ_4 at initial time $t = t_0$. Then, for the evolution with $\Sigma_4 \to 0$ at $t \to +\infty$, one obtains a consistent quantum model with dynamical realization of the isothermal state.

6. Temporal Variations of Electric Charge and Energy-Momentum of the Universe

In the theory under consideration, the electric charge Q and the energy-momentum vector P_μ are defined through the conventional three-dimensional integrals

$$Q = \int \mathrm{d}^3x\, J^0, \qquad P_\mu = \int \mathrm{d}^3x\, \mathcal{T}^0_\mu, \tag{19}$$

where $\mathcal{T}^0_\mu$ is given by Eq. (11). These quantities are not integrals of motion now: they are functions on x^0 and x^4 (we understood the temperature regime $\beta = \beta(t)$ fixed here). Of course, the theory possesses the states which cannot be find in the standard Dirac's theory. However, if one restricts a consideration by subspace of solution with

$$Q = Q(x^0,\, x^4) = \mathrm{const}, \tag{20}$$

then one deals with dynamical scheme with the familiar one-time formalism. We would like to stress that the restriction (20) is not necessary for the adequate interpretation of the theory: we consider not closed system of the infinite size with not trivial thermodynamical behavior and relate all its surprising properties with the thermostat action. Of course, it is possible to identify this formal thermostat with the physical vacuum.

Performing the calculations, it possible to show that for any quantity Θ, is related to the null-divergent relation $\Theta^M_{,M} = 0$ and defined as $\Theta = \int \mathrm{d}^3x\, \Theta^0$, the following identity takes place:

$$\Theta_{,\,x^0} = -\Xi_{,\,x^4}, \tag{21}$$

where $\Xi = \int \mathrm{d}^3x\, \Theta^4$. Here one can identify Θ with Q and P_μ — in the any case motivated by the null-curvature relation this identity will be hold. Eq. (21) means, that the quantity Θ is not a motion integral; the reason of this fact is determined by the five-dimensional structure of the theory. Actually, for the five-dimensional theory all Noether charges are defined as some four-dimensional integrals, which have all the conservational properties. It is interested to note, that in the theory under consideration it arises the covariant four-dimensional force. This reads:

$$F_\mu \equiv \frac{\mathrm{d}P_\mu}{\mathrm{d}x^0} = P_{\mu,\,x^0} + \frac{\mathrm{d}x^4}{\mathrm{d}x^0}\left(P_{\mu,\,x^4} + \frac{\dot\beta}{c}\,P_{\mu,\,\beta}\right), \tag{22}$$

where $P_{\mu,\,x^0} = -\left(\int \mathrm{d}^3x \mathcal{T}^4_\mu\right)_{,\,x^4}$ in accordance with Eq. (21), whereas

$$P_{\mu,\,x^4} = -\frac{i}{2}\int \mathrm{d}^3x\left[\Psi^+_{,\nu}\gamma^{\nu+}\left(\gamma^{4+}\right)^{-1}\Psi_{,\,\nu} + \Psi^+\left(\gamma^4\right)^{-1}\gamma^\nu\Psi_{,\,\mu\nu} - \mathrm{c.c.}\right],$$

$$P_{\mu,\,\beta} = \frac{\hbar c}{4}\int \mathrm{d}^3x\left[\Psi^+_{,\nu}\gamma^{\nu+}\left(\gamma^{4+}\right)^{-1}\Psi_{,\,\nu} - \Psi^+\left(\gamma^4\right)^{-1}\gamma^\nu\Psi_{,\,\mu\nu} + \mathrm{c.c.}\right];$$

these terms are nontrivial ones in the general case. Here we have supposed the temperature regime $\beta = \beta(x^4)$ fixed, as well as the two-time resolution function $x^4 = x^4(x^0)$. This force can be identified with the force of the universal vacuum (or thermostat) resistance. We will not give explicit form of all terms in Eq. (22), they can be calculated by the reader using the dynamical equations (5) of the theory.

7. Conclusion

Let us summarize starting points and results of this paper. The system considered is the Dirac's theory with Poincare-invariant extra complex parameter of evolution. The real part of this time parameter is identified with cosmological time, i.e. with time passed after the Big Bang. Its imaginary part is understood as inverse proportional to the cosmic (CMB) temperature. We have studied a canonical structure of this quantum system and established non-trivial temperature dependence of its Noether charges. For the given cosmological scenario CMB temperature is the function on the cosmological time, so the Noether charges must vary in accordance to the Universe evolution. It is argued, that fundamental constants can be related to Noether charges using the procedure of averaging over ensemble of fermion particles. Finally, this leads to consistent theoretical scheme of the cosmological variations of fundamental constants.

We would like to stress the relation between gauge and space-time shift symmetries established in this paper. Analytical structure of the theory studied guaranties such non-trivial unification of different Noether symmetries into the single structure. We hope to continue investigation of this remarkable property of the theory in forthcoming publications.

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УДК 535.42:681.786.23

Теорема Нётер и вариации зарядов во Вселенной

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Представлен новый подход к объяснению возможной космологической эволюции фундаментальных зарядов, определяемых спинорными частицами. Этот подход основан на использовании модифицированной версии безмассового уравнения Дирака с аналитической зависимостью волновой функции от дополнительного комплексного временного параметра. Мы отождествили реальную и мнимую части комплексного времени соответственно с абсолютными космологическим временем и температурой и исследовали каноническую структуру теории. Установлено, что калибровочный заряд и вектор энергии-импульса в теории являются температурно-зависящими и, более того, связанными друг с другом. Предложена аргументация в пользу утверждения об изменении полных зарядов Вселенной в соответствии с температурной эволюцией в рамках реалистического космологического сценария.

Ключевые слова: теорема Нётер, фундаментальные заряды, космологическая эволюция.