Физика

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Lagrangian Density of Lepton and Baryon Phases in Nonlinear 8-Spinor Model

V. I. Molotkov

Department of Theoretical Physics and Mechanics
Peoples' Friendship University of Russia
6, Miklukho-Maklaya str., Moscow, Russia, 117198

The Skyrme's idea (1954) for describing baryons as topological solitons was based on the identification of baryon number B with the topological charge of the degree type $B = \deg(S^3 \to S^3)$. It serves as the generator of the third homotopy group $\pi_3(S^3) = \mathbb{Z}$. The similar idea to describe leptons as topological solitons was announced by Faddeev (1972). He identified the lepton number L with the Hopf invariant Q_H.

The 8-spinor field is suggested to unify Skyrme and Faddeev models describing baryons and leptons as topological solitons. The special 8-spinor Brioschi identity is used to include leptons and baryons as two possible phases of the effective spinor field model, with Higgs potential depending on the $j^\mu j_\mu$ being added to the Lagrangian.

To this end the generalization of the Mie electrodynamics within the scope of the effective 8-spinor field model is suggested. For this field model the quadratic spinor quantities entering the Brioschi identity are constructed. Then the symmetry groups, which generate S^2- and S^3-submanifolds in general S^8 biquadratic spinor manifold, are found. For unifying these phases, common vacuum state should conserve only one component in both lepton and baryon cases.

In the present paper we try to construct Lagrange density for homotopy groups $\pi_3(S^2)$ and $\pi_3(S^3)$, which describe lepton and baryon phases.

Key words and phrases: 8-spinor, topological charge, solitons, homotopy groups, Brioschi identity, Skyrme–Faddeev model.

1. Introduction

In the Skyrme model the particles-solitons possess the topological charge of the degree type $B = \deg(S^3 \to S^3)$, which is interpreted as the baryon number B. In the Faddeev model the particles are endowed with the topological invariant of the Hopf type, which is interpreted as the Lepton number. This invariant serves as the generator of the third homotopy group $\pi_3(S^2) = \mathbb{Z}$. Prof. Yu. P. Rybakov suggested [1] to unify these two approaches describing baryons and leptons as two possible phases of the effective 8-spinor field model. For this goal, the special 8-spinor Brioschi identity is used [2]:

$$j_\mu j^\mu - \tilde{j}_\mu \tilde{j}^\mu = s^2 + p^2 + v^2 + a^2. \tag{1}$$

In this identity the following quadratic spinor quantities are introduced:

$$\begin{aligned} s &= \overline{\psi}\psi, \\ p &= i\overline{\psi}\gamma_5\psi, \\ v &= \overline{\psi}\lambda\psi, \\ a &= i\overline{\psi}\gamma_5\lambda\psi, \\ j_\mu &= \overline{\psi}\gamma_\mu\psi, \\ \tilde{j}_\mu &= \overline{\psi}\gamma_\mu\gamma_5\psi, \end{aligned}$$

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with $\overline{\psi} = \psi^{+}\gamma_0$ and matrices $\lambda = \sigma_i \otimes I_4$ standing for the Pauli matrices in the isotopic space. Here and below we use the Weyl representation for Dirac matrices γ_μ, $\mu = 0$, 1, 2, 3, which may be written in the block form using the Pauli matrices σ_1, σ_2, σ_3:

$$\gamma_0 = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}, \quad \gamma_k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \quad k = 1, 2, 3.$$

Hence the matrix γ_5, which is the product of the four gamma matrices, is written as follows:

$$\gamma_5 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}.$$

We use the denotation I_n for the unit matrix of size n, $n \in \mathbb{N}$.

By analogy with [3, 4] we consider the 8-spinor space, in which 8-spinor is defined as column:

$$\psi = col(\varphi, \chi, \xi, \theta), \tag{2}$$

with $\varphi = col(\varphi_1, \varphi_2)$, $\chi = col(\chi_1, \chi_2)$, $\xi = col(\xi_1, \xi_2)$, $\theta = col(\theta_1, \theta_2)$ being 2-spinors.

In this 8-spinor space S^2- and S^3- submanifolds, which can describe the lepton and baryon sectors, were introduced [5]. The symmetry group $SU(2)$ was proposed to describe leptons:

$$\begin{pmatrix} \varphi \\ \chi \end{pmatrix} = U_L \begin{pmatrix} \xi \\ \theta \end{pmatrix} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \begin{pmatrix} \xi \\ \theta \end{pmatrix}, \tag{3}$$

Simple calculations show that we have the homotopy group $\pi_3(S^2)$, which generates the topological Hopf-like charge $s^2 + a_1^2 + a_3^2 \neq 0$:

$$\begin{aligned} s &= 2(\xi^{+}\theta + \theta^{+}\xi), \\ a_1 &= 2(\theta\theta^{+} - \xi\xi^{+}), \\ a_3 &= -2i(\theta^{+}\xi - \xi^{+}\theta), \\ s^2 + a_1^2 + a_3^2 &= 4\left(\theta^{+}\theta + \xi^{+}\xi\right)^2. \end{aligned}$$

The symmetry group $SU(2)$ was proposed for the description of baryons:

$$\begin{pmatrix} \varphi \\ \chi \end{pmatrix} = U_B \begin{pmatrix} \xi^* \\ \theta^* \end{pmatrix} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \begin{pmatrix} \xi^* \\ \theta^* \end{pmatrix}, \tag{4}$$

This S^3-manifold is generated by the structure $p^2 + s^2 + a_1^2 + a_2^2$:

$$\begin{aligned} s &= 2(\xi^{+}\theta + \theta^{+}\xi), \\ p &= 2i(\xi^{+}\theta - \theta^{+}\xi), \\ a_1 &= \Big(\left(\theta^{+}\right)^2 + \theta^2 - \xi^2 - \left(\xi^{+}\right)^2\Big), \\ a_2 &= i\Big(\left(\theta^{+}\right)^2 - \theta^2 + \xi^2 - \left(\xi^{+}\right)^2\Big), \\ s^2 + p^2 + a_1^2 + a_2^2 &= 4\Big[\left(\theta^{+}\theta + \xi^{+}\xi\right)^2 - \left(\xi^{+}\theta - \theta^{+}\xi\right)^2\Big]. \end{aligned}$$

To add to this, common vacuum state was found [6], which conserves only one component $s = 2(\xi^+\theta + \theta^+\xi)$ in both the lepton and the baryon phases:

$$\psi_V = \begin{pmatrix} iC \\ iC \\ C \\ C \end{pmatrix}, \tag{5}$$

where $\theta = \xi = C = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$, $C_1, C_2 \in \mathbb{C}$.

2. Current in 8-spinor model

We can write down the components of current $j_\mu = \overline{\psi}\gamma_\mu\psi = \psi^+\gamma_0\gamma_\mu\psi$ entering the Brioschi identity (1):

$$\begin{gathered} j_0 = \psi^+\gamma_0\gamma_0\psi = \varphi^+\varphi + \chi^+\chi + \xi^+\xi + \theta^+\theta, \\ j_1 = \psi^+\gamma_0\gamma_1\psi = -\varphi^+\varphi + \chi^+\chi - \xi^+\xi + \theta^+\theta, \\ j_2 = \psi^+\gamma_0\gamma_2\psi = i\left[\varphi_1^+\varphi_2 - \varphi_2^+\varphi_1 + \chi_2^+\chi_1 - \chi_1^+\chi_2 + \xi_1^+\xi_2 - \xi_2^+\xi_1 + \theta_2^+\theta_1 - \theta_1^+\theta_2\right], \\ j_3 = \psi^+\gamma_0\gamma_3\psi = -\varphi_1^+\varphi_1 + \varphi_2^+\varphi_2 + \chi_1^+\chi_1 - \chi_2^+\chi_2 - \xi_1^+\xi_1 + \xi_2^+\xi_2 + \theta_1^+\theta_1 - \theta_2^+\theta_2. \end{gathered}$$

According to (3) and (4), only one component of the current converses in lepton and baryon phases:

$$j_0 = \psi_L^+\gamma_0\gamma_0\psi_L = \psi_B^+\gamma_0\gamma_0\psi_B = 2\left(\xi^+\xi + \theta^+\theta\right), \quad j_1 = j_2 = j_3 = 0. \tag{6}$$

The same situation is in the vacuum state:

$$j_0 = \psi_V^+\gamma_0\gamma_0\psi_V = 4\left(\psi_1^*\psi_1 + \psi_2^*\psi_2\right), \quad j_1 = j_2 = j_3 = 0. \tag{7}$$

3. Lagrangian density

By analogy with [1], let us choose the following Lagrangian density for the 8-spinor field model:

$$\mathcal{L} = \frac{1}{2\lambda^2}\overline{D_\mu\psi}\gamma^\alpha j_\alpha D^\mu\psi + \frac{\epsilon^2}{4}f_{\mu\nu}f^{\mu\nu} - V\left(j_\mu j^\mu\right) - \frac{1}{16\pi}F_{\mu\nu}^2 \tag{8}$$

with $D^\mu = \partial_\mu\psi - ie_0A_\mu\Gamma_e\psi$ — gauge covariant derivative,$V\left(j_\mu j^\mu\right) = \frac{\sigma^2}{8}\left(j_\mu j^\mu - \kappa^2\right)^2$ — Higgs potential and $f_{\mu\nu}$ stands for the antisymmetric tensor of the Faddeev–Skyrme type:

$$f_{\mu\nu} = \left(\overline{\psi}\gamma^\alpha D_{[\mu}\psi\right)\left(\overline{D_{\nu]}\psi}\gamma_\alpha\psi\right) \tag{9}$$

with λ and ϵ being constant parameters of the model. We use the denotation $\mathcal{L}_i$ for terms in this Lagrangian density. Let us write the first σ-model term $\mathcal{L}_1$, which is a product of the covariant derivatives:

$$\begin{aligned} \mathcal{L}_1 &= \overline{D_\mu\psi}\gamma^\alpha j_\alpha D^\mu\psi = D_\mu\psi^+\gamma_0\gamma^\alpha j_\alpha D^\mu\psi = \\ &= \left(\partial_\mu\psi^+ + ie_0A_\mu\Gamma_e\psi^+\right)\gamma_0\gamma^\alpha\left(\psi^+\gamma_0\gamma_\alpha\psi\right)\left(\partial^\mu\psi - ie_0A^\mu\Gamma_e\psi\right) = \\ &= \partial_\mu\psi^+\gamma_0\gamma^\alpha\left(\psi^+\gamma_0\gamma_\alpha\psi\right)\partial^\mu\psi - \partial_\mu\psi^+\gamma_0\gamma^\alpha\left(\psi^+\gamma_0\gamma_\alpha\psi\right)ie_0A^\mu\Gamma_e\psi + \\ &\quad + ie_0A_\mu\Gamma_e\psi^+\gamma_0\gamma^\alpha\left(\psi^+\gamma_0\gamma_\alpha\psi\right)\partial^\mu\psi - \left|ie_0A_\mu\Gamma_e\right|^2\left(\psi^+\gamma_0\gamma_\alpha\psi\right)^2. \end{aligned} \tag{10}$$

Our model (8) supposes that at space infinity $|r| \to \infty$ the following boundary conditions hold:

$$\psi \to \psi_V, \quad j_\mu j^\mu \to \kappa^2, \quad A_\mu \to 0. \tag{11}$$

To satisfy this asymptotic behaviour of localized configurations charge generator Γ_e should satisfy the following conditions:

$$\Gamma_e \psi_V = 0, \quad \Gamma_e^2 = \Gamma_e. \tag{12}$$

Using (5) operator Γ_e has the following form:

$$\Gamma_e = I_4 \otimes \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{13}$$

We can calculate the spinor term $\overline{D_\mu \psi} \gamma^\alpha j_\alpha D^\mu \psi$ for two phases and the vacuum. For leptons and baryons it has the following form:

$$\mathcal{L}_1 = \overline{D_\mu \psi} \gamma^\alpha j_\alpha D^\mu \psi = D_\mu \psi^+ \gamma^0 \gamma^\alpha j_\alpha D^\mu \psi = \\ = 4 \left(\xi^+ \xi + \theta^+ \theta \right) \left[D_\mu \xi^+ D^\mu \xi + D_\mu \theta^+ D^\mu \theta \right]. \tag{14}$$

Similarly, for the vacuum state one gets:

$$\mathcal{L}_{V1} = \overline{D_\mu \psi_V} \gamma^\alpha j_\alpha D^\mu \psi_V = 16 \left(C_1^* C_1 + C_2^* C_2 \right) \left[D_\mu C_1^* D^\mu C_1 + D_\mu C_2^* D^\mu C_2 \right], \tag{15}$$

where C_1 and C_2 take complex values, which describe the vacuum state in (5). Since C_1 and C_2 are constant and $\Gamma_e \psi_V = 0$ (12), the covariant derivative is identically zero in the vacuum state $D^\mu \psi_V = 0$. As a result the σ-model term of the Lagrangian density (15) is also identically zero:

$$\mathcal{L}_{V1} \equiv 0. \tag{16}$$

4. Conclusion

Finally we give the summary of the results obtained:
– The effective 8-spinor field model is suggested. For this field model the components of current quantities are constructed.
– For suggested lepton and baryon sectors σ-model term of Lagrange function is found.

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Лагранжиан лептонного и барионного секторов в нелинейной восьмиспинорной модели

В. И. Молотков

Кафедра теоретической физики и механики
Российский университет дружбы народов
ул. Миклухо-Маклая, д. 6, Москва, Россия, 117198

Идея Скирма (1954) состоит в том, что барион интерпретируется как частица-солитон, которая имеет топологический заряд $B = \deg(S^3 \to S^3)$. Этот заряд B служит генератором гомотопической группы $\pi_3(S^3) = \mathbb{Z}$. Аналогичная идея для описания лептонов используется в модели Фаддеева (1972). В ней в роли лептонного числа L используется инвариант типа Хопфа Q_H.

Для объединения двух подходов, описывающих лептоны и барионы как топологические заряды, предлагается использовать 8-спинорное поле. Использование специального 8-спинорного тождества Бриоски позволяет рассматривать лептоны и барионы как секторы в общей спинорной модели с потенциалом Хиггса, зависящего от $j^\mu j_\mu$, входящего в лагранжиан. С этой целью рассматривается обобщение электродинамики Ми в рамках эффективной 8-спинорной полевой модели. Кроме того были обнаружены группы симметрий, образующие S^2 и S^3 подмногообразия в общем биквадратном спинорном S^8-многообразии. Для объединения двух секторов было построено общее вакуумное состояние, сохраняющее лишь одну компоненту в каждом секторе.

В настоящей работе предлагается попытка выписать функцию Лагранжа для гомотопических групп $\pi_3(S^2)$ и $\pi_3(S^3)$, которые описывают барионы и лептоны.

Ключевые слова: 8-спинор, топологический заряд, солитоны, гомотопические группы, тождество Бриоски, модель Скирма-Фаддеева.

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