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Propagation of the Monochromatic Electromagnetic Waves in Irregular Waveguides. A Brief Introduction to an Analysis in the Case of Smooth or Statistic Irregularities

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Two cases are examined in the paper: propagation of waves in smoothly irregular and statistically irregular dielectric waveguides. The peculiarities of approximate solutions of vector electrodynamic problems in both cases are discussed. The offered methods are applicable for analysis of similar dielectric, magnetic, optic and meta materials structures in enough broad band of electromagnetic wavelengths.

Key words and phrases: Maxwell's equations, vector electrodynamics' problems, smoothly irregular dielectric waveguide, multilayer waveguide, Luneburg waveguide lens, boundary conditions, asymptotic method, quasi-waveguide modes, statistic waveguide irregularities, TE and TM modes, waveguide scattering, Green functions method.

1. Introduction

Many papers (see, for example, [1, 2] and quoted there references) are devoted to the analysis of propagation of a plane monochromatic light wave in planar multilayer regular dielectric few-mode waveguides. A number of methods [1–13] are used for an analysis of processes of transformation of quasi-waveguide modes, accompanying an exchange of energy between modes and between modes with surroundings.

The asymptotic method of a solution of Maxwell's equations [6] in our view is better than other approaches for the description of processes of an evolution of quasi-waveguide modes. The conservation in an obtained solution and boundary conditions of terms, proportional to gradient of a dielectric permeability, allows taking into account a vectorial character of propagation of a monochromatic electromagnetic field along smoothly-irregular three-dimensional (3D) sections of multilayer dielectric multimode waveguide.

Generalized Luneburg waveguide lens is a key functional element in such, for example integrated-optical processors as radio-frequency-spectrumanalyzer, working in real time scale [4]. A request to exactitude of calculation of parameters of similar elements of integrated structures in nanometer range will hardly increase in connection with existence of restrictions stipulated by diffraction effects.

The integrated-optical waveguide is one of basic elements of the integrated optics and waveguide optoelectronics [2, 4, 8, 10]. In most cases waveguide serves as the basis for creation of the various optical integrated circuits [4]. In this connection the important direction in technology is the development of methods of creation of a waveguide with a low level of losses of the intensity of the directed mode on scattering by 3D irregularities (boundaries roughness and heterogeneities of the waveguide layers) of the structure of a waveguide [14–36]. The light scattered in a waveguide can be registered as radiated substrate-cover (substrate-air) and substrate modes, and as the radiation, scattered in the plane of a waveguide.

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When neglecting polarizing effects the problem of waveguide 3D scattering is reduced to a solution of known two-dimensional (2D) wave equation. Then we assume absence of the cross-correlation relations between all types of irregularities (roughness of the boundaries and/or heterogeneity of the layers of waveguide).

2. Maxwell's Equations. Adiabatic Modes of Smoothly Irregular Waveguiding Structure. Boundary Conditions

Let's begin a consideration from the analysis of features of propagation of eigenmodes of an asymmetrical thin-film dielectric waveguide in the smoothly-irregular segment (right part of the Fig. 1).

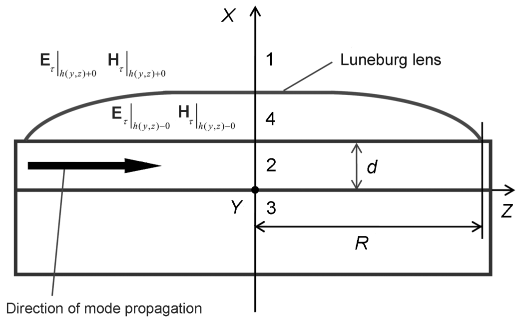

Figure 1. **The cross-section of a researched integrated structure constructed by layers 1-4. The regular three layer waveguide is created by layers 1, 2, and 3**

On the picture are designated: 1 is the framing medium or covering layer (air) with an index of refraction n_c; 2 is the waveguide layer (three-layer regular part of an integrated structure) with an index of refraction n_f; 3 is the substrate with an index of refraction n_s; 4 is the thin-film Luneburg's lens (irregular four-layer part of an integrated structure) with an index of refraction n_l; R is the radius the aperture of a thin-film lens; d is the thickness of a regular part of an integrated waveguide structure; $h\left(y,z\right)$ is the boundary of waveguide layer 4 and covering layer 1.

An example of such smooth irregularity is a thin-film generalized waveguide Luneburg lens. As is known, the generalized Luneburg waveguide lens is an important functional element in integrated-optic devices such as high-frequency spectrum analyzer operating on a real time scale. Requirements to the accuracy of calculation of such waveguide lens strongly increase on the passage in a range of sub wave sizes, which is related to the appearance of restrictions imposed by the diffraction effects [2].

Similar problems are encountered in numerous conjugating devices, which provide the connection of various elements, for example, in an integrated optical circuit. The efficiency of conjugation strongly depends on the matching of fields of the incident wave and the waveguide mode, both in front of and behind the conjugation element.

The Maxwell's equations for an electromagnetic field in a case of not absorbing inhomogeneous linear isotropic medium (in the absence of currents and electric charges) in a SI system receive the form:

$$\mathrm{rot}\tilde{\vec{H}} = \varepsilon\frac{\partial\tilde{\vec{E}}}{\partial t}, \quad \mathrm{rot}\tilde{\vec{E}} = -\mu\frac{\partial\tilde{\vec{H}}}{\partial t}, \tag{1}$$

where $\varepsilon = \varepsilon_r\varepsilon_0$ is the dielectric permeability of a medium (layer); $\mu = \mu_r\mu_0$ is the magnetic permeability of a layer; ε_r, μ_r are the relative dielectric and magnetic permeability accordingly (in a non magnetized medium it is necessary to take $\mu =$

1); ε_0 and μ_0 are the electrical and magnetic constants accordingly; $\omega\sqrt{\mu\varepsilon} = nk_0$, n is the index of refraction of a layer, $k_0 = 2\pi/\lambda_0$, $\omega = 2\pi\nu$, ν is the frequency of an electromagnetic field. $\vec{E}$, $\vec{H}$ are electric and magnetic field intensity vectors; a symbol tilde at vectors of fields reflects their complex character.

When deriving equations (1) one takes into account, that for a linear isotropic medium the following relations are valid:

$$\vec{D} = \varepsilon\vec{E}, \quad \vec{B} = \mu\vec{H},$$

where $\vec{D}$ is the electric induction vector, $\vec{B}$ is the magnetic induction vector.

In regular four-layer waveguide (see. Fig. 1) the thicknesses $h\,(y,z)$ of a second waveguide layer is constant also eigenmodes propagating along the plane yOz in the direction Oz, have well known kind [10].

In smoothly-irregular four-layer waveguide (right part of the 1) the thicknesses of the second waveguide layer is not constant. In this case the Fourier method of a separation of variables, used in regular case, is inapplicable.

In the paper [9] it is offered to use an asymptotic method of a solution of a set of equations (1) for a smoothly-irregular dielectric integrated waveguide structure. Within the framework of the approach given in paper [9] authors have kept two contributions, circumscribing a so-called adiabatic approximation [6].

Then for search a concrete solution the method of "partial" separation of variables (Kantorovich's method, known as a modified Galerkin's method [11] of "partial" separation of variables) was used.

This approach allows searching for a solution of the Maxwell's equations with the components of fields $U\,(x,y,z)$, dependent on three space variables, as the final series:

$$U\,(x,y,z) = \sum_{m=1}^{M} U_v^m\,(x;y,z)\,U_h^m\,(y,z)\,. \tag{2}$$

The sequence (2) contains factors $U_h^m\,(y,z)$ dependent on horizontal variables y and z, and factors $U_v^m\,(x;y,z)$ dependent functionally on a vertical variable x, and dependent parametric from horizontal variables.

In regular case: $M = 1$ and $U_v^1\,(x;y,z) = U_v^1\,(x)$. The necessary condition that is possible to neglect the following members with $m > 1$ is the condition $|U_{m+1}/U_m| \ll 1$.

3. Scattering of Electromagnetic Monochromatic Waves in a Statistically Irregular Waveguide

The scattering problem of a directed waveguide mode in a planar optical waveguide containing stochastic irregularities is solved with the help of the theory of perturbations [2, 18, 31, 32].

Generally for the description of an electromagnetic field $\vec{E}$ in an irregular waveguide the equation is used, which in the rectangular Cartesian coordinates has the following form:

$$\nabla^2\vec{E} + \nabla\left(\vec{E}\frac{\nabla\varepsilon}{\varepsilon}\right) + \omega^2\mu\varepsilon\vec{E} = 0, \tag{3}$$

where $\nabla^2 = \Delta$ is Laplacian. The equation (3) is found with the help of the Maxwell's equations (1).

Then one considers the case of propagation in a waveguide along z-axis of the main TE-mode (for TM-mode the analysis is carried out similarly). Then the full field in an irregular planar optical waveguide can be written as the sum of fields of an incident waveguide mode and the field of a scattered wave.

Then we can write equation (1) as the approximate three-dimensional equation. Keeping in the obtained equation only members of the first order of smallness in respect of $\vec{E}_s$ and $\Delta\varepsilon(\vec{r})$, we shall receive an approximate inhomogeneous wave equation, which can be considered as a homogeneous wave equation with perturbation as a source $\vec{E}_{0y}$ in the right part [31, 32]:

$$\nabla^2 \vec{E}_s(x,y,z) + \omega^2 \mu\varepsilon_0 \vec{E}_s(x,y,z) \approx -\omega^2 \mu\varepsilon_0 \Delta\varepsilon(x,y,z)\, \vec{E}_{0y}(x,z), \tag{4}$$

where $\vec{E}_{0y}$ is the solution of the homogeneous undisturbed equation circumscribing the propagation of the main TE-mode in a waveguide.

From the power point of view, the "*source*" in a right part of the equation (4) is the intensity of the mode, incident on an irregular area of waveguide and scattered in all enclosing space (3D-*scattering*).

The solution of the given inhomogeneous wave equation can be obtained as a convolution of some Green function $G(x,\ y,\ z;\ x',\ y',\ z')$ with the expression for the source $\Delta\varepsilon(x',y',z')$ [31, 32]:

$$\vec{E}_s(x,y,z) = \\ = -\omega^2\mu\varepsilon_0 \iiint \Delta\varepsilon(x',y',z')\, G(x,\ y,\ z;\ x',\ y',\ z')\, \vec{E}_{0y}(x',z')\, \mathrm{d}x'\mathrm{d}y'\mathrm{d}z'. \tag{5}$$

The analysis shows, that in this case it is impossible to neglect polarizing effects and the consideration of the problem of waveguide scattering of light on three-dimensional irregularities becomes hardly complicated, since the determination of analytical expression for a Green function represents here not at all trivial problem.

When neglecting the polarizing effects, originating during scattering, it is enough to require, that the relative variation of the dielectric permittivity on a distance of one wavelength was much less than unit. Then it is possible to use a simple wave equation:

$$\Delta\vec{E} + n^2 k_0^2 \vec{E} = 0, \tag{6}$$

which is fair for each Cartesian component of the vector of electrical field. For fundamental TE-mode, propagating along the axes z, under condition $\partial/\partial y = 0$, this formula accepts next form:

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} + n^2 k_0^2 E_y = 0, \tag{7}$$

where $n^2(x,z) = n_0^2(x,z) + \Delta n^2(x,z)$, $n_0^2(x,z)$ describes regular properties of an appropriate medium of waveguide (accepts values n_1, n_2 or n_3 accordingly), and the component $\Delta n^2(x,z)$ describes irregularities of the structure of a waveguide (both irregularity of the boundaries, and heterogeneity of a refraction index).

For application of the theory of perturbations the addendum $\Delta n^2(x,z)$ should not be necessarily the value of the small order. There is quite enough, that the area, within the limits of which this component differs from zero, was very narrow. The solution of the equation (7) with help of the approximate method of "*ideal modes*" [2] is then finding as the expansion of certain scattering field on the orthogonal set of the modes of the rectilinear optical waveguide.

Thus the solution for component of a scattered field in any point of space with coordinates x, z takes form:

$$E_y = \int q(\rho, L)\, E_y(\rho, z)\, \mathrm{d}\rho, \tag{8}$$

where q is the effective amplitude of scattering TE-modes, defined as factor of expansion of a field on all radiation modes; $\tilde{\rho}$ is the transversal component of the propagation constant $\tilde{\beta}$ of the radiation modes.

The factors of expansion are fined from the orthogonality relations with the help of the theory of perturbations. Both numerical methods of direct calculation and analytical methods of determination of an approximate value of the integral in expression (8) can be used, for example, method of the stationary phase or saddle point method. If the condition $\partial/\partial y = 0$ is executed, it is possible to express any distribution of the field of a waveguide as the superposition of orthogonal TE- and TM-modes of an ideal rectilinear waveguide.

4. Dispersion Relations and Results of Numerical Calculations for Three-Layer Regular Waveguide

For simulation the well known representation of dispersion relation in a trigonometrical kind for thee layer waveguide was used [37]:

$$\beta h = \operatorname{arctg}\,(\rho/\beta) + \operatorname{arctg}\,(\eta/\beta) + (m-1)\,\pi, \tag{9}$$

where $\beta = k_0\sqrt{n_2^2 - \gamma^2}$ is the propagation constant of directed TE-mode along an axes z (see Fig. 2); h is the thickness of a waveguide layer; γ is the factor of phase slowing down (effective waveguide refraction index); $\rho = k_0\sqrt{\gamma^2 - n_1^2}$ is the vertical component of the propagation constant of directed TE-mode for $x > 0$; $\eta = k_0\sqrt{\gamma^2 - n_3^2}$ is the vertical component of the propagation constant of directed TE-mode for $x < -h$; $m = 0, 1, 2, \ldots$.

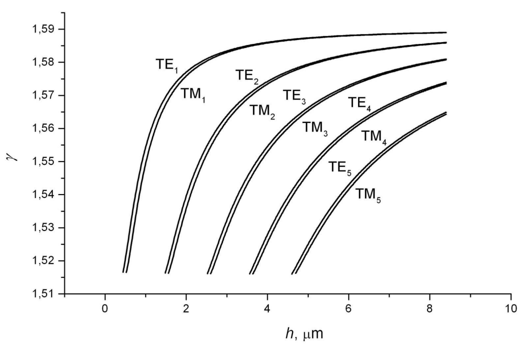

Figure 2. **Dispersing relation $\gamma = \gamma\,(h)$ for the first five TE and TM modes of three-layer polystyrene planar integrated-optical waveguide**

The dispersing relations $\gamma = \gamma\,(h)$ for the first five TE and TM modes of a regular three-layer polystyrene integrated-optical waveguide are represented in the Fig. 2.

The amplitude of the field in arbitrary units of the radiated substrate TE_5-mode for $\gamma = 2.099$ are represented as the example in the Fig. 3.

The field of a radiated substrate TE modes are set as follows [37]:

$$|\mathbf{E}_y\,(x;\gamma)| =$$

$$= \begin{cases} \left\{ \dfrac{4\omega_0\mu_0\rho_f^2\rho_s^2 P_0}{\pi|\beta|k_0^2\left(n_2^2-n_1^2\right)\left[\rho_s^2+\left[\rho_f^2-\rho_s^2\right]\sin^2(\varphi_c-\rho_f h)\right]} \right\}^{1/2} \exp\left[-\rho_c\,(x-h)\right], x > h, \\ \left\{ \dfrac{4\omega_0\mu_0\rho_s^2 P_0}{\pi|\beta|\left(n_2^2-n_1^2\right)\left[\rho_s^2+\left[\rho_f^2-\rho_s^2\right]\sin^2(\varphi_c-\rho_f h)\right]} \right\}^{1/2} \cos\left[\rho_f\,(x-h)+\varphi_c\right], 0 < x < h, \\ \left\{ \dfrac{4\omega_0\mu_0 P_0}{\pi|\beta|} \right\}^{1/2} \cos\left[\rho_s x + \varphi\right], x < 0, \end{cases} \tag{10}$$

where P_0 is the power, transportable of the wave in waveguide in a direction of an z axes through the width unit (on the y axes), in calculations we took $P_0 = 1W/m$.

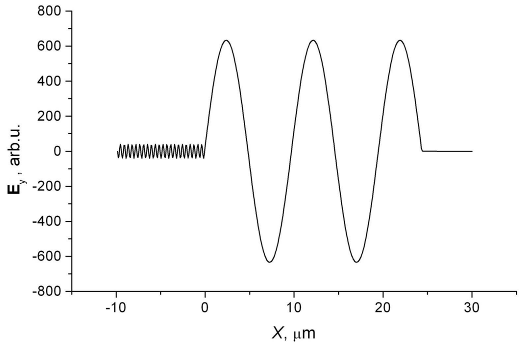

Figure 3. **The amplitude of the field of a radiated substrate** TE_5**-mode for** $\gamma = 2.099$ **of three-layer** Ta_2O_5 **planar integrated-optical waveguide**

5. Dispersion Relations for Smoothly-Irregular Waveguide

Dispersion relations for thin-film generalized waveguide Luneburg's lens, being the example of investigated by us smoothly-irregular integrated waveguide structures, were obtained [5] in an approximation, when the sloping tangential boundary conditions were replaced by their projections with a horizontal plane.

Taking into account of non horizontality of tangential boundary conditions, introduces to Southwell's relations a small correction on a parameter δ, determined as follows:

$$\delta = \max \left| \vec{\nabla}_{y,\,z} \beta \right| \left(k_0 \beta^2 \right)^{-1},$$

this is two-dimensional waveguide's analogy of magnitude $\left| \vec{\nabla} \varepsilon \big/ \varepsilon \right|$.

With the help of additional differentiations and with allowance for adiabatic behavior of field intensities, we received from the Maxwell's equations (1) expressions for field's components E_x, E_y, H_y, H_x and quasi-waveguide equations for field's components E_z, H_z [9]. Let's remark, that quasi-wave is the equation which is looking like a wave equation in the left part of equality, in which right part instead of zero there is an expression from unknown function, first order of smallness δ on a comparison with expression in the left part.

In each of homogeneous areas:

$$\begin{gathered} I_s = \{x \in (\, -\infty, -d\,]; \quad y, z \in (-\infty, +\infty)\}, \\ I_f = \{x \in (-d, 0); \quad y, z \in (-\infty, +\infty)\}, \\ I_l = \{x \in [0, h(y,z)]; \quad y, z \in (-\infty, +\infty)\}, \\ I_c = \{x \in [h(y,z), \, +\infty); \quad y, z \in (-\infty, +\infty)\}, \end{gathered} \tag{11}$$

obtained quasi-wave equations are simplified to wave equations:

$$\frac{\mathrm{d}^2 E_z}{\mathrm{d}x^2} + \chi^2 E_z = 0, \quad \frac{\mathrm{d}^2 H_z}{\mathrm{d}x^2} + \chi^2 H_z = 0, \tag{12}$$

where χ^2 is the transversal wave number equal in zero order of smallness δ approximation to magnitude $\chi_0^2 = k_0^2 \left(\varepsilon\mu - \beta^2 \right)$. The solutions of the equations (12) are well-known [1, 2, 10].

Further we determine horizontal boundary conditions on the plane $x = -d$ and on the plane $x = 0$. Then we determine "not horizontal" boundary conditions. From three components of a tangential field $\vec{E}_\tau$ only two are linearly independent. Therefore, it is quite enough to determine boundary conditions only for E_y^τ, E_z^τ. Similarly from three components of a magnetic field only two H_y^τ and H_z^τ also are linearly independent. Therefore, the boundary conditions need to be determined only for them.

All twelve relations for three boundaries form a homogeneous system of linear algebraic equations for amplitude coefficients $A_s, B_s, A_f^\pm, B_f^\pm, A_l^\pm, B_l^\pm, A_c, B_c$. This system has a nontrivial solution, if it degenerates, i.e. if the determinant of the given matrix $Matr$ is equal to zero [9]. The condition $\det(Matr) = 0$ is, as a matter of fact, the dispersion relations for hybrid quasi-waveguide modes in a smoothly-irregular section of a dielectric waveguide [9, 13].

6. Results of Numerical Calculations for Smoothly-Irregular Four-Layer Waveguide

The parameters of the structure considered are following: refractive index of substrate (SiO_2) $n_s = 1.470$, refractive index of regular waveguide film (glass of the mark Corning 7059) $n_f = 1.565$, refractive index of second waveguide layer (Ta_2O_5) with varying thickness $h(y, z)$ $n_l = 2.100$, refractive index of covering (air) $n_c = 1.000$.

The dispersion dependences of phase slowing down coefficient of TE_0 mode (effective refraction index) on waveguide layers thickness in four-layer integrated-optical structure including regular three-layer planar waveguide and thin-film generalized Luneburg waveguide lens are presented in [38]. Therefore in the given work we do not examine the dispersion dependences in detail.

On the vertical axis the values of β for TE_0 mode are indicated, and on the horizontal axis the thicknesses waveguide layers in relative units d/λ and h/λ are indicated accordingly.

In the Fig. 4 the relative thickness d of first (regular) waveguide layer from ≈ 0.4 to 3.0 are depicted, and the relative thickness $h(z)$ of second (irregular) waveguide layer from 3.0 to 3.8 are depicted accordingly.

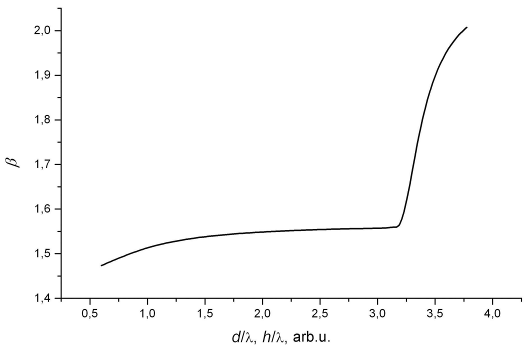

Figure 4. **The dispersion dependence of** TE_0 **mode in three-layer and four-layer parts of the integrated-optical structure presented in Fig. 1**

Hence the left part of the figure presents the dispersion dependence of regular three-layer planar waveguide and the right part is the dispersion dependence of four-layered smoothly irregular waveguide.

The part of the dispersion dependence from $h \approx 0.0\lambda$ to $h \approx 0.2\lambda$ (directly after 3.0 on the Fig. 4) presents some transition (unsteady) regime in the Luneburg lens.

Taking into account the longitudinal wave number correct transition ($\chi \to i\gamma, \gamma \to -i\chi$) along the same branch of two-valued function of taking square root of a complex variable during coefficient β transition from region $\beta < n_f$ to region $\beta > n_f$ is the indispensable condition for elaborating strict calculations there.

The amplitude of the field in arbitrary units of the guided TE_4-mode for $h \approx 0.5$ (see Fig. 1), as the example of calculated fields, are represented in the Fig. 5.

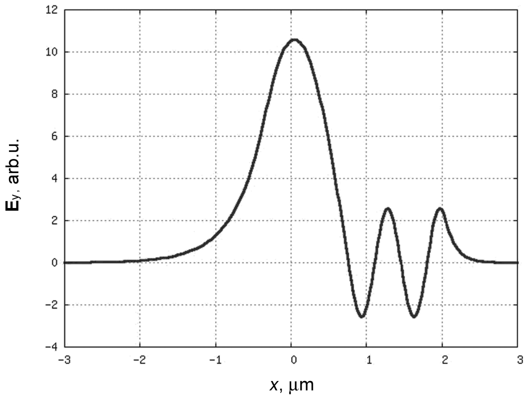

Figure 5. **The amplitude of the field of a guided TE_4-mode for $h \approx 0.5$**

7. Conclusion

We used the solution of the electrodynamic problem that takes into account the vector character of propagating fields, thus providing a more adequate (compared to the scalar case) description of real irregular and smoothly-irregular waveguide structures. Using this solution, we can analytically describe the fields of smoothly deforming modes of a dielectric waveguide, their interrelation, and the dispersion relations.

The offered method is applicable for analysis of similar dielectric, magnetic, and meta- materials' structures, including nonlinear in enough broad band of electromagnetic wavelengths, that is doubtless advantage.

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Распространение монохроматических электромагнитных волн в нерегулярных волноводах. Краткое введение в анализ для случая плавных и статистических нерегулярностей

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В статье рассмотрены два случая: распространение волн в плавно-нерегулярных и статистически нерегулярных диэлектрических волноводах. Обсуждены особенности приближённых решений векторных электродинамических задач в обоих случаях. Предлагаемые методы применимы для анализа подобных структур из диэлектрических, магнитных, оптических и мета материалов в достаточно широком диапазоне электромагнитных длин волн.

Ключевые слова: уравнения Максвелла, векторная электродинамическая проблема, плавно-нерегулярный диэлектрический волновод, многослойный волновод, волноводная линза Люнеберга, граничные условия, асимптотический метод, квази-волноводные моды, статистические волноводные нерегулярности, TE и TM моды, волноводное рассеяние, метод функций Грина.