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# Light-Diode Holoellipsometer with Binary Modulation of Polarization Employing Light Scattering from Uniaxial Bi-Dimension Crystal

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A holoellipsometer with binary modulation of polarization employing almost normal light scattering by the sample representing itself an optically uniaxial crystal is presented. The device is actual for the tomometric tools which are widely used in nanotechnologies and medicine. The main equations of the holoellipsometry employing normal light scattering method are obtained.

**Key words and phrases:** ellipsometry, light scattering, polarization, uniaxial bi-dimensional crystals.

## 1. Introduction

Efficient techniques of physical-chemical processes monitoring in various bodies' surface and near-surface regions are called for by high-end technologies, medicine, ecology, catalysis chemistry, electrochemistry, and in particular, atoms layers precipitation physics [1–3].

The problem of monitoring quality and efficiency is solved by its appropriate hardware and software. Here, an ever-increasing role is gained by non-destructive optical diagnostics techniques of applied-physics parameters of semiconductor and dielectric materials, films, coatings and interfaces, being formed in multi-stage technological processes [4].

Optical techniques – are first of all, ellipsometry techniques, such as direct, remote, non-contact and non-destructive techniques, quite fitting to various technological lines and allowing their efficient automatization [5].

Monitoring of anisotropic, *in medias res* nanometric layers, actual for surface systems, can be efficiently implemented by way of polarization registration of scattered light [6] from optically uniaxial crystal systems under the use of holoellipsometry and its implementation techniques [7].

## 2. Optical diagram and operation of light diode holoellipsometer with binary modulation of light polarization employing almost normal light scattering by an uniaxial bi-dimensional crystal

Optical diagram of light diode holoellipsometer of the almost normal light scattering by uniaxial bi-dimensional crystal sample with the use of light flux binary modulation is given in Fig. 1. Here LD — light diode; $\mathrm{BS}_i$ — beam splitter; $\mathrm{Ph}_o$ – photodetector at input; BMP — binary modulator of polarization [8]; $\mathrm{M}_{1,2}$ — plane mirrors in the incident to sample S and reflected from it light fluxes; ACh — reflected flow absorption chamber; C — collimator; $C_p$ — compensator; PBS — polarization beam splitter; $L_{1,2}$ — lenses; $\mathrm{Ph}_{1,2}$ — photodetectors; DPU – electric signals data processing unit; PC — computer; D — display.

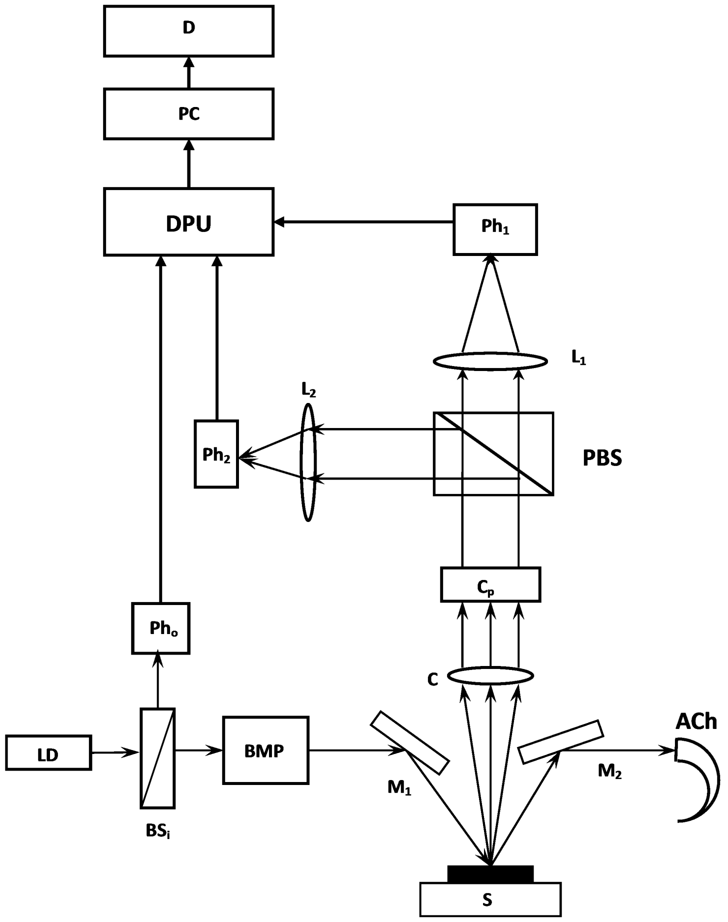

**Figure 1. Optical diagram of light diode holoellipsometer of the almost normal light scattering by uniaxial bi-dimensional crystal**

Some auxiliary but necessary units are not shown in Fig. 1 not to overload it. These are: source of light diode start and power supply, photodetectors power supply, electronic systems of registration, enforcement, synchronous detecting and digitization of electric signals from photodetectors, incorporated to DPU; control lines of complex blocks and units. They are standard; meet the requirements of stability, sensitivity and measurements accuracy.

Under operation of the device (Fig. 1), light flux from light diode LD comes to beam splitter BS, which splits light flux illuminating it into two beams. Beam reflected by the beam splitter is registered by photodetector Ph$_o$ as reference one, in order to control intensity $I_i(t)$ of light diode radiation, and working beam transmitted by BS illuminates mirror M $_1$ after passing through binary modulator of polarization BMP. The mirror sends light flux to sample S almost normally at small angle of incidence $\vartheta_i \approx 5^\circ$. Sample S reflects and partially scatters the light flux illuminating it. The reflected part is sent by mirror M$_2$ to absorption chamber ACh and is absorbed by it. A part of scattered light comes from sample S almost normally to its surface within the cone angle $\Delta\vartheta \approx 2.5^\circ$ (which is smaller than the angle of incidence) to collimator C. The latter forms a collimated flow from the scattered light directing it through compensator CP to polarization beam splitter PBS, which splits the light flux in two beams with linear $p$- and $s$-polarizations. Lenses L$_{1,2}$ focus these light beams onto photodetectors Ph$_{1,2}$. Electric signals from the detectors come to data processing unit DPU, which digitizes them and sends to computer PC, processing the obtained data and automating the device operation. Final results can be viewed on display D.

Control of the light-diode radiation intensity $I_i\,(t)$ is performed with the aid of optron pair consisting of beam splitter $\mathrm{BS}_i$ and photodetector $\mathrm{Ph}_o$, which allows to normalize signal $I_k\,(t)$ coming from photodetector $\mathrm{Ph}_k$ ($k$=1, 2) by dividing $k$-th signal on signal $I_i\,(t)$ and obtain analogue $I_k\,(t)$ and normalized $i_k\,(t)$ signals:

$$i_k\,(t) = [I_k\,(t)/I_i\,(t)]\,. \tag{1}$$

Normalized values (1) facilitate mathematical algorithms stability in data processing. *In situ* measurements require: first, simultaneity of $I_i\,(t)$ and $I_k\,(t)$ signal registration; second, appropriate electric signals processing operational speed for all detecting, amplification, synchronous detecting, signals digitizing, and inputting into PC operations. Relatively weak link in this chain of operations providing *in situ* measurement arrangements is first of all radiation intensity stability and electric signal duration due to the photodetectors response time limitation. Measurement data contain information about sample S as well as about other optical elements the light beams interact on their ways from light diode LD to the photodetectors.

As is known complex components $\tilde{S}_{uv}$ of scattering Jones matrix $\hat{S}$ carry information about sample $S$ [9–11]:

$$\tilde{S}_{uv} = S_{uv} \cdot \exp \cdot (i\varphi_{Suv})\,, \tag{2}$$

where $S_{uv}$ and $\varphi_{Suv}$ – module and phase of component $\tilde{S}_{uv}$ of Jones matrix $\hat{S}$; each index in their pair $(uv)$ correlates with index $(p)$ or $(s)$ of the light flux or with index $(\xi)$ or $(\eta)$ polarizations of extraordinary $(\xi)$ and ordinary $(\eta)$ light waves in the crystal.

Contribution of other optical elements is made by complex polarization instrument functions $\tilde{A}_{p,s}$ [5]:

$$\tilde{A}_{p,s} = A_{p,s} \cdot \exp \cdot (i\delta_{Ap,s})\,, \tag{3}$$

where $A_{p,s}$ and $\delta_{Ap,s}$ – module and phase of complex function $\tilde{A}_{p,s}$.

Instrument function $\tilde{A}_{p,s}$ contains factors and amplitudes $E_{i(p,s)}$ of the electric vector $p$ and $s$-components at the device input, and volt-Watt sensitivities $D_k$ of photodetectors $\mathrm{Ph}_k$, etc. The functions $\tilde{A}_{p,s}$ in signals data set $I_k\,(t)$ can be excluded by means of supplementary measurements by arranging other reference (standard) pattern $S_r$ put on the way of light flux instead of the sample S. Data set $I_{kr}\,(t)$ obtained as a result of measurements with so called etalon $S_e$ is registered prior performing measurements with sample S and used when processing data set $I_k\,(t)$.

## 3. Specific characters of scattered light polarization registration

The essential moment in the creation of scattered light when sample S is irradiated by weak intensity light flux is related to spontaneous discharge of particles excited states in the scattering medium [12]. Partial light wave, scattered by a certain $j$-th particle, departs it and lives absolutely independently from other scattered light waves. Partial wave of scattered light according to the Maxwell electrodynamics propagates in the bi-dimensional crystal like it happens in an optical Fabry-Perot resonator. Any other $k$-th component of the scattered light flux generated by a particle in $k$-th point of the medium produces a similar interference picture. Due to temporal incoherence of partial $j$-th and $k$-th waves in scattered light flux these pictures overlap and intensities are summarized. Interference picture from incoherent light sources was first studied by O. Fresnel [13].

Any partial wave of the scattered light emitted from a $j$-point can be presented as a sum of mutually normal vectors $E'_{\xi j}$ and $E'_{\eta j}$ both of which are perpendicular to the scattered light wave vector $k'$. Preservation of temporal coherence for partial waves emitted by $j$-th particle in the scattering medium is insured by two circumstances in case such waves propagate in an optical uniaxial medium.

The first of them is obliged to the fact that thin layer of sample S is Fabry-Perot resonator, for example, for normally scattered partial extraordinary and ordinary light waves propagating in anisotropic medium of sample S.

The second moment is obliged to the interference of the electric vector projections of the scattered waves $E'_{\xi j}$ and $E'_{\eta j}$ $\left(E'_{\xi j} \perp E'_{\eta j}\right)$ onto a certain direction in space. For example, such selected directions can be linear $p$- and $s$-polarizations set by the device polarization elements. The light wave illuminating sample S and initiating scattering is set by complex amplitudes of vectors $\tilde{E}\left(z_j\right)_{1(\xi,\eta)}$ at depth $z$ from the sample surface where point $j$ in medium l:

$$\tilde{E}(z_j)_{1(\xi,\eta)} = \tilde{E}_{0(\xi,\eta)} \left\{ \frac{\tilde{\tau}_{01} \cdot \exp(i\varphi_{1zj}) \left[1 + \tilde{\rho}_{12} \cdot \exp(2i\varphi_{1(d-zj)})\right]}{1 + \tilde{\rho}_{01}\tilde{\rho}_{12} \cdot \exp(2i\varphi_{1d})} \right\}_{(\xi,\eta)}, \tag{4}$$

here indexes 0, 1, 2 are related to the environment, sample layer and substrate, respectively; $\tau_{\alpha\beta(\xi,\eta)}$ and $\rho_{\alpha\beta(\xi,\eta)}$ are amplitude Fresnel coefficients for transmitted and reflected light with linear ($\xi$, $\eta$)-polarizations at interface ($\alpha\beta$) of media $\alpha$ and $\beta$; $\varphi_{1z(\xi,\eta)}$ is phase shift for normally scatted light wave with linear ($\xi$, $\eta$)-polarization at output from the sample acquired due to its run from depth $z$ in medium 1:

$$\varphi_{1z(\xi,\eta)e} = \left(2\pi\nu' n'_{(\xi,\eta)} z \middle/ c\right) \tag{5}$$

$\varphi_{1d(\xi,\eta)}$ — phase shift for wave when propagating in layer of depth d.

Partial wave $\tilde{E}'\left(z_j\right)_{1i(\xi,\eta)}$ of the scattered light created at point $j$ has the form:

$$\tilde{E}'\left(z_j\right)_{1i(\xi,\eta)} = \tilde{E}\left(z_j\right)_{1(\xi,\eta)} \tilde{S}_{(\xi\xi,\eta\eta)}\left(z_j\right). \tag{6}$$

At the surface ($e$) of sample S with thickness $d$ partial wave $\tilde{E}'\left(z_j\right)_{(\xi,\eta)e}$ of the scattered light created at point $j$ located at depth $z$ from the sample surface has the following form:

$$\begin{aligned} &\tilde{E}'\left(z_j\right)_{(\xi,\eta)e} = \\ &\qquad = \tilde{E}'\left(z_j\right)_{1i(\xi,\eta)} \left\{ \frac{\tilde{\tau}_{10} \cdot \exp\left(-i\varphi_{1zj}\right) \left[1 + \tilde{\rho}_{12} \cdot \exp\left(2i\varphi_{1(d-zj)}\right)\right]}{\left[1 + \tilde{\rho}_{01}\tilde{\rho}_{12} \cdot \exp\left(2i\varphi_{1d}\right)\right]} \right\}_{(\xi,\eta)}. \end{aligned} \tag{7}$$

Or in more expanded form with the account of formulae (4)–(6):

$$\tilde{E}'\left(z_j\right)_{(\xi,\eta)e} = \tilde{E}_{0(\xi,\eta)} \left\{ \left(\tilde{\tau}_{10}\tilde{\tau}_{01}\tilde{S}\left(z_j\right)\right) \frac{\left[1 + \tilde{\rho}_{12} \cdot \exp\left(2i\varphi_{1(d-zj)}\right)\right]^2}{\left[1 + \tilde{\rho}_{01}\tilde{\rho}_{12} \cdot \exp\left(2i\varphi_{1d}\right)\right]^2} \right\}_{(\xi,\eta)}. \tag{8}$$

Due to the fact that sample layer S acts on both the incident light wave and the scattered wave created by the first one in the same way as Fabry-Perot resonator does, electric vectors $\tilde{E}'\left(z\right)_{(\xi)e}$ and $\tilde{E}'\left(z\right)_{(\eta)e}$ of the normally scattered light field straight at output from the layer can be linked to vectors $\tilde{E}_{0(\xi)}$ and $\tilde{E}_{0(\eta)}$ of the incident wave, respectively, via the efficient components $\tilde{S}_{\text{eff}}\left(z_j\right)_{(\xi,\eta)}$ of Jones matrix $\tilde{S}_{\text{eff}}$ for the light scatted along the normal:

$$\tilde{S}_{\text{eff}}\left(z_j\right)_{(\xi,\eta)} = \tilde{S}\left(z_j\right)_{(\xi,\eta)} \tilde{t}_{\text{eff}}\left(z_j\right)_{(\xi,\eta)}, \tag{9}$$

here

$$\tilde{t}_{\text{eff}}\left(z_j\right)_{(\xi,\eta)} = \left(\tilde{\tau}_{10}\tilde{\tau}_{01}\right)\left[1+\tilde{\rho}_{12}\cdot\exp\left(2i\varphi_{1(d-zj)}\right)\right]^2 \Big/ \left[1+\tilde{\rho}_{01}\tilde{\rho}_{12}\cdot\exp\left(2i\varphi_{1d}\right)\right]^2 \tag{10}$$

is the efficient complex amplitude coefficient of the output light flux with linear ($\xi$, $\eta$)-polarization under the conditions the light flux comes out directly from an optically anisotropic medium.

## 4. Basic equations of light diode holoellipsometry of almost normal light scattering by optically uniaxial sample

Let us derive a formula for modules $S_{uv}$ and phase difference $\Delta_s$ of Jones matrix $\hat{S}$ components $\tilde{S}_{uv}$ for almost normal light scattering [9–11] when light scattering holoellipsometer is used (Fig. 1). Let us note that non-diagonal components $\tilde{S}_{\xi\eta}$ and $\tilde{S}_{\eta\xi}$ of light scattering Jones matrix $\hat{S} \sim \sin\Phi_s$ (here $\Phi_s$ is scattering plane azimuth) and in case of the almost normal light scattering when $\Phi_s = 0$ components $\tilde{S}_{\xi\eta}$ and $\tilde{S}_{\eta\xi}$ of matrix $\hat{S}$ are equal to zero as well [11]. And by virtue of temporal incoherence of scattered light flux partial $j$-th components escaping from $j$-points located at depths $z_j$ intensities of these partial waves are simply summed up. This summation $\Sigma_j$ of scattered light flux on the $j$-th particles set in the scattering volume, actual for normal light scattering, can be substituted by integral $\int f\left(z\right) dAz$ on the depth of occurrence $z$ of $j$-th particle scattering light within the limits of area A of sample S region being exposed.

In fact, at sample surface component $\tilde{S}_{\xi\xi}$ and $\tilde{S}_{\eta\eta}$ matrices $\hat{S}$ are multiplied, according to (9), to efficient complex factor of output $\tilde{t}_{\text{eff}(\xi,\eta)e}\left(z\right)$ of scattered light flux with ($\xi$, $\eta$)-polarization under its output from optically anisotropic medium layer. In order not to complicate writing of formulae, let us understand components $\tilde{S}_{(\xi\xi,\eta\eta)}$ as efficient without fixing this moment by corresponding index and simplify the writing of components themselves:

$$\tilde{S}_{(\xi\xi,\eta\eta)} \equiv \tilde{S}_{(\xi,\eta)}. \tag{11}$$

Let optical axis $\xi$ of uniaxial crystal sample S form angle $\alpha$ with linear $p$-polarization of light flux incident on sample S (Fig.1) and $E_{i\xi}$, $E_{i\eta}$ are projections of electrical vector $E_i$ of light wave on linear $\xi$- and $\eta$-polarizations. Polarized light wave normally incident from vacuum onto an optically uniaxial crystal is divided into two waves with mutually transverse linear polarizations [14]. Under normal incidence light wave electric-flux density $D_\xi$ and electric field tension $E_\xi$ of the extraordinary wave oscillate along optical axis $\xi$, while for the ordinary wave vectors $D_\eta$ and $E_\eta$ oscillate transversely to axis $\xi$ and along direction $\eta$.

Complex amplitudes $\tilde{E}_{\text{pe}}$ and $\tilde{E}_{se}$ of these vectors of the output flow ($e$) of normally scattered light flux components with linear $\xi$- and $\eta$-polarizations are found by vectors and Jones matrices [5]:

$$\hat{E}_e = \hat{R}\left(\alpha\right)\cdot\hat{S}\cdot\hat{R}\left(-\alpha\right)\cdot\hat{A}\cdot\hat{E}_i, \tag{12}$$

$$\begin{pmatrix}\tilde{E}_p\\ \tilde{E}_s\end{pmatrix}_e = \begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}\tilde{S}_\xi & 0\\ 0 & \tilde{S}_\eta\end{pmatrix}\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}\tilde{A}_\xi & 0\\ 0 & \tilde{A}_\eta\end{pmatrix}\begin{pmatrix}\tilde{E}_p\\ \tilde{E}_s\end{pmatrix}_i. \tag{13}$$

Having done proper actions with (13) one can obtain:

$$\tilde{E}_{\text{pe}} = \left[\tilde{E}_{\text{pi}}\tilde{A}_p\left(\tilde{S}_\xi\cos^2\alpha + \tilde{S}_\eta\sin^2\alpha\right) + \tilde{E}_{\text{si}}\tilde{A}_s\left(\tilde{S}_\xi - \tilde{S}_\eta\right)\sin\alpha\cos\alpha\right]_e, \tag{14}$$

$$\tilde{E}_{se} = \left[\tilde{E}_{\text{pi}}\tilde{A}_p\left(\tilde{S}_\xi - \tilde{S}_\eta\right)\sin\alpha\cos\alpha + \tilde{E}_{\text{si}}\tilde{A}_s\left(\tilde{S}_\xi\sin^2\alpha + \tilde{S}_\eta\cos^2\alpha\right)\right]_e. \tag{15}$$

Transmitting light flux with linear $p$- or $s$-polarization (employing, for example, polarization binary modulator [8]), one can essentially simplify this formulae. In particular, in the case of $p$- polarization the simplification has the form:

$$\tilde{E}_{\text{pep}} = \tilde{E}_{\text{pi}}\left[\tilde{A}_p\left(\tilde{S}_\xi\cos^2\alpha + \tilde{S}_\eta\sin^2\alpha\right)\right]_e, \tag{16}$$

$$\tilde{E}_{\text{sep}} = \tilde{E}_{\text{pi}}\left[\tilde{A}_p\left(\tilde{S}_\xi - \tilde{S}_\eta\right)\right]_e\sin\alpha\cdot\cos\alpha. \tag{17}$$

In the case of only $s$-polarizations:

$$\tilde{E}_{\text{ses}} = \tilde{E}_{\text{si}}\left[\tilde{A}_s\left(\tilde{S}_\xi\sin^2\alpha + \tilde{S}_\eta\cos^2\alpha\right)\right]_e, \tag{18}$$

$$\tilde{E}_{\text{pes}} = \tilde{E}_{\text{si}}\left[\tilde{A}_s\left(\tilde{S}_\xi - \tilde{S}_\eta\right)\right]_e\sin\alpha\cdot\cos\alpha. \tag{19}$$

Correspondingly intensities $I_{(p,s)ep}$ and $I_{(p,s)es}$:

$$I_{\text{pep}} = I_{\text{pi}}\left[\left|\tilde{A}_p\right|^2\left(S_\xi^2\cos^4\alpha + S_\eta^2\sin^4\alpha + S_\xi S_\eta\sin\Delta\varphi_{\xi\eta}\left(\sin^2 2\alpha/2\right)\right)\right]_e, \tag{20}$$

$$I_{\text{ses}} = I_{\text{si}}\left[\left|\tilde{A}_s\right|^2\left(S_\xi^2\sin^4\alpha + S_\eta^2\cos^4\alpha + S_\xi S_\eta\sin\Delta\varphi_{\xi\eta}\left(\sin^2 2\alpha/2\right)\right)\right]_e, \tag{21}$$

$$I_{\text{sep}} = I_{\text{pi}}\left[\left|\hat{A}_p^*\right|^2\left|S_\xi^* - S_\eta^*\right|^2\right]_e\cdot\left(\sin^2 2\alpha\right)/2, \tag{22}$$

$$I_{\text{pes}} = I_{\text{si}}\left[\left|\tilde{A}_s\right|^2\left|\tilde{S}_\xi - \tilde{S}_\eta\right|^2\right]_e\cdot\left(\sin^2 2\alpha\right)/2, \tag{23}$$

where $\Delta\varphi_{\xi\eta}$ is phase difference $\varphi_{Suv}$ of Jones matrix $\hat{S}$ basic complex components of light scattering, and sinus character of its contribution to ratios (20) and (21) is due to compensator $C_p$ action onto the light flux.

According to (22) and (23) signals $I_{\text{sep}}$ and $I_{\text{pes}}$ are equal to zero when $\alpha = 0$, that enables to select necessary orientation of sample S optical axis $\xi$.

In case of so called etalon sample $S_e$ the measurements should be performed in the reflection mode when the reflection coefficients $R_{\xi e} = R_{\eta e} = 1$:

$$I_{(\text{pep})e} = \left|\tilde{E}_p\right|_e^2 = I_{\text{pi}}\left|\tilde{A}_p\right|_e^2, \tag{24}$$

$$I_{(\text{ses})e} = \left|\tilde{E}_s\right|_e^2 = I_{\text{si}}\left|\tilde{A}_s\right|_e^2, \tag{25}$$

here $I_{\text{pi}}$ and $I_{\text{si}}$ are input intensities of $p$- and $s$-polarized waves, respectively.

According to (1) we have normalized signals $i_{\text{pep}}$ and $i_{\text{ses}}$ from sample S:

$$i_{\text{pep}} = \left[I_{\text{pep}}/I_{\text{pi}}\right] =$$

$$= \left[ \left| \tilde{A}_p \right|^2 \times \left( S_\xi^2 \cos^4 \alpha + S_\eta^2 \sin^4 \alpha + S_\xi S_\eta \sin \Delta\varphi_{\xi\eta} \left( \sin^2 2\alpha/2 \right) \right) \right]_e, \quad (26)$$

$$i_{\text{ses}} = [I_{\text{ses}}/I_{\text{si}}] =$$
$$= \left[ \left| \tilde{A}_s \right|^2 \times \left( S_\xi^2 \sin^4 \alpha + S_\eta^2 \cos^4 \alpha + S_\xi S_\eta \sin \Delta\varphi_{\xi\eta} \left( \sin^2 2\alpha/2 \right) \right) \right]_e, \quad (27)$$

$$i_{\text{sep}} = [I_{\text{sep}}/I_{\text{pi}}] = \left[ \left| \tilde{A}_p \right|^2 \left| \tilde{S}_\xi - \tilde{S}_\eta \right|^2 \right]_e \times \left( \sin^2 2\alpha \right)/2, \quad (28)$$

$$i_{\text{pes}} = [I_{\text{pes}}/I_{\text{si}}] = \left[ \left| \tilde{A}_s \right|^2 \left| \tilde{S}_\xi - \tilde{S}_\eta \right|^2 \right]_e \times \left( \sin^2 2\alpha \right)/2. \quad (29)$$

And signals $i_{(\text{pep})e}$ and $i_{(\text{ses})e}$ received from etalon $S_e$:

$$i_{(\text{pep})e} = \left| \tilde{A}^*_{(p)} \right|_e^2, \quad (30)$$

$$i_{(\text{ses})e} = \left| \tilde{A}^*_{(s)} \right|_e^2. \quad (31)$$

Finally, one can obtain ratios for so called reduced signals $i_{\text{red(pep)}}$, $i_{\text{red(ses)}}$ and $i_{\text{red(sep)}} = i_{\text{red(pes)}}$ for sample S:

$$i_{\text{red(pep)}} = \left[ i_{\text{pep}}/i_{(\text{pep})e} \right] =$$
$$= \left[ \left( S_\xi^2 \cos^4 \alpha + S_\eta^2 \sin^4 \alpha + S_\xi S_\eta \sin \Delta\varphi_{\xi\eta} \left( \sin^2 2\alpha/2 \right) \right) \right]_e \quad (32)$$

$$i_{\text{red(ses)}} = \left[ i_{\text{ses}}/i_{(\text{ses})e} \right] =$$
$$= \left[ \left( S_\xi^2 \sin^4 \alpha + S_\eta^2 \cos^4 \alpha + S_\xi S_\eta \sin \Delta\varphi_{\xi\eta} \left( \sin^2 2\alpha/2 \right) \right) \right]_e \quad (33)$$

$$i_{\text{red(sep)(pes)}} =$$
$$= \left[ \left| \tilde{S}_\xi - \tilde{S}_\eta \right|^2 \right]_e \times \left( \sin^2 2\alpha \right)/2 = \left( S_\xi^2 + S_\eta^2 - 2S_\xi S_\eta \sin \Delta\varphi_{\xi\eta} \right) \left( \sin^2 2\alpha \right)/2 \quad (34)$$

Note also that complex components $\hat{S}_\xi$ and $\hat{S}_\eta$ of light scattering Jones matrix $\hat{S}$ in ratios (32)–(34) should be understood as efficient in full accordance with ratios (9) and (10).

Excluding the singular cases, when angle $\alpha$ is equal to 0, $(\pi/4)$ or $(\pi/2)$, and considering only non-singular cases for equation system (32)–(34), when angle $\alpha$ is equal, for instance, to $(\pi/3)$ or $(2\pi/3)$, we obtain a system of the three independent equations, which allows to find by means of the algebra operations parameters of light scattering holoellipsometry: modules $S_\xi$ and $S_\eta$, phase difference $\Delta\varphi_{\xi\eta}$ of the components $\tilde{S}_{(\xi,\eta)}$ of light scattering Jones matrix $\hat{S}$.

Strictly speaking the parameters of the true complex components $\tilde{S}_{(\xi,\eta)}$ scattering Jones matrix $\hat{S}$ are obtained by means of taking into account of their received effective parameters and efficient complex amplitude coefficient of the output light flux with linear $(\xi, \eta)$-polarization in accordance with the formula (10). With the aim to compute the latter the measurements of the complex reflection coefficients $R_\xi$ and $R_\eta$ should be performed in the reflection mode when the holoellipsometer replaces absorption chamber ACh (Fig.1) and $R_\xi$ and $R_\eta$ replaces modules $S_{(\xi,\eta)}$ into formulas (32)—(34). As the result of the solution of the holoellipsometry reverse task refraction indexes $n_e$ and $n_0$ as well as thickness $d$ can be computed.

The obtained result can be considered as physical basis of experimental determination of modules $S_{\text{eff}(\xi,\eta)}$ and phase difference $\Delta\varphi_{\xi\eta}$ of basic efficient complex components $\tilde{S}_{\text{eff}(\xi,\eta)}$ of the light scattering efficient Jones matrix $\hat{S}_{(\xi,\eta)}$ employing only one measuring polarization channel instead of two ones used in known noninterferometric holoellipsometers [7].

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# Светодиодный холоэллипсометр с бинарной модуляцией поляризации рассеяния света одноосным двумерным кристаллом

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Представлен холоэллипсометр почти нормального рассеяния света оптически одноосным кристаллом с использованием бинарной модуляции поляризации света. Он актуален для топометрических систем, широко используемых в нанотехнологиях и медицине. Получены основные уравнения холоэллипсометрии нормального рассеяния света.

**Ключевые слова:** холоэллипсометрия, рассеяние света, поляризация, одноосные кристаллы.