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Model with One Spatial Variable for Design of a Technical Device

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Model with one spatial cylindrical variable for optimization of a technical device is presented. Device is in the form of a multilayer cylindrical sample with a pulse source operating at cryogenic temperatures. The results of numerical experiments are shown.

Key words and phrases: heat equation, partial differential equation, finite-difference scheme, mathematical modeling.

1. Introduction

The main goal of this work is to implement a model of thermal processes for a given object (Fig. 1) for systematic studies in order to find optimal materials and composition of the technical device. The object subject to the investigation is a cryogenic cell pulse (in the millisecond range) feeding the working gases into the working space of charged ion source [1].

Thermal processes occur at periodic passage electric current through conductive layer. Full period of the process is requested to be 10 ms: the heating period is 1 ms (current on, the object surface should be heated up to 60 K), for 9 ms it is cooling (current off, the surface should be cooled down to 4.2 K $\sim$ 10 K).

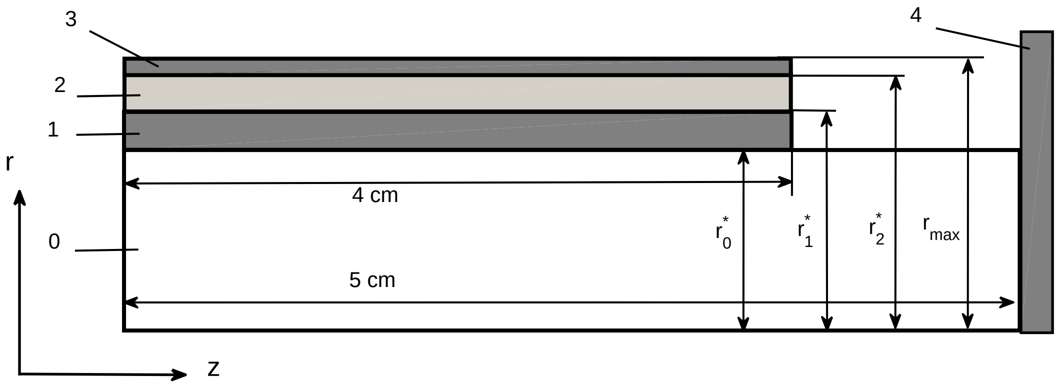

Figure 1. Schematic view of the object slice. The slice of the object: 0 − cooler ($r_0^* = 0.12$), 1 − electrical insulator ($r_1^* = 0.125$), 2 − heat source (conductive layer, $r_2^* = 0.13$), 3 − external insulator ($r_{\max} = 0.1301$), 4 − liquid helium temperature terminal with $T = 4.2$ K

Due to the cylindrical symmetry of the object, the heat conductivity inside it can be simulated by a model with two spatial cylindrical variables r and z (instead of three) and time variable. This model was presented and discussed in [2] and [3]. Possibility of using model with one spatial cylindrical variable r was proposed and discussed in [3], it is shown that thermal flow in the z direction can be neglected for beginning part of thermal process.

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2. Main Equations and Boundary Conditions

The thermal processes in the object can be described by the following system of parabolic partial differential equations with temperature depended coefficients [4]:

$$\rho_m C_{Vm}(T)\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_m(T)\frac{\partial T}{\partial r}\right) + X_m(T), \tag{1}$$

where $r \in [0, r_{\max}]$ and $t \in [0, t_{\max} = 10\text{ ms}]$. For $m = 0, 1$ and 3 the function $X_m(T) \equiv 0$ (there is no source).

The object involves different materials in construction with different densities and thermal coefficients; thus the index m is introduced for each material ($m = 0$ — cooler (cooper), $m = 1$ — electrical insulator, $m = 2$ — heat source (graphite), $m = 3$ — external insulator).

Coefficients C_{Vm} and λ_m are specific heat capacity and thermal conductivity, respectively for each material m (see Fig. 2 and Fig. 3). It is well known that thermal conductivity and specific heat for many materials could vary up to order of magnitude as temperature varies from $T = 4.2$ K to $T = 60$ K. For the chosen materials the corresponding data is obtained from the [5], and then these dependencies were approximated by the least-squares method using the linear and nonlinear analytical functions.

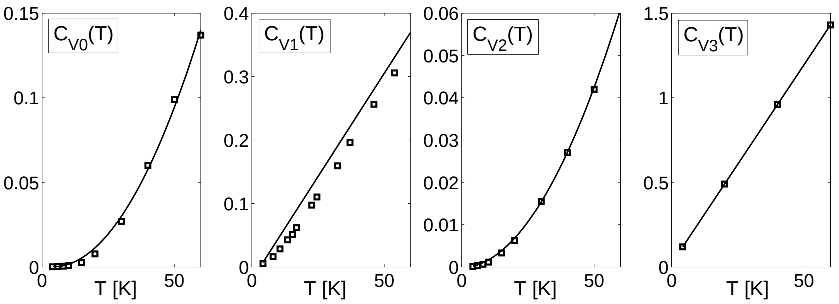

Figure 2. The heat capacities for different materials, $C_V(T)$

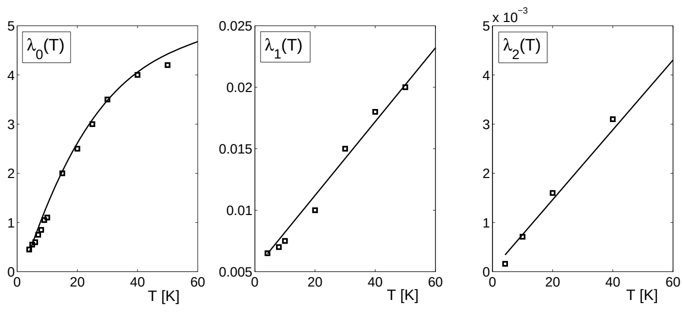

Figure 3. The thermal conductivities for different materials, $\lambda(T)$

Source dependence was approximated by function $X_2(T) = \chi(T)I^2/S$, where $\chi(T) = 1.8/\sqrt{T}$ and constant $I^2/S = 10^6$. In this formula I means electric current in the graphite slice along z direction and S is graphite cross section in the direction orthogonal to the current propagation [2].

The initial condition is given below

$$T(r, t = 0) = T_0, \tag{2}$$

where $T_0 \equiv 4.2$ (liquid helium temperature) and the boundary ones are taken as

$$\left.\frac{\partial T}{\partial r}\right|_{r=0 \text{ and } r_{\max}} = 0. \tag{3}$$

The functions C_V, λ and $X_i = X(T_i)$ have discontinuities of first kind at the following points: r_0^*, r_1^* and r_2^* in the interval $[0..r_{\max}]$. Conjugation condition between materials considered to be ideal:

$$\lambda_m(T)\left.\frac{\partial T}{\partial r}\right|_{r=r_m^*-0} = \lambda_{m+1}(T)\left.\frac{\partial T}{\partial r}\right|_{r=r_m^*+0} \tag{4}$$

where $m = 0, 1, 2$ and r_m^* is point of the border between the materials m and $m+1$ (discontinuity points).

3. Numerical Algorithm

For numerical solution the initial-boundary-value problem (1)–(3), the equation (1) was approximated by the following explicit finite difference scheme [6]:

$$\rho\, C_{V\,i}\frac{\widehat{T}_i - T_i}{h_t} = \frac{1}{r_i}\Lambda_i\left[r_i\lambda_i T_i\right] + X_i. \tag{5}$$

The spatial finite difference operator is:

$$\Lambda_i\left[r_i\lambda_i T_i\right] = \frac{1}{h_r^2}\left[r_{i+\frac{1}{2}}\lambda_{i+\frac{1}{2}}\left(T_{i+1} - T_i\right) - r_{i-\frac{1}{2}}\lambda_{i-\frac{1}{2}}\left(T_i - T_{i-1}\right)\right], \tag{6}$$

where

$$T_i = T(r_i, t), \quad C_{V\,i} = C_V(T_i), \quad \lambda_i = \lambda(T_i), \quad X_i = X(T_i),$$
$$r_{i\pm\frac{1}{2}} = r_i \pm \frac{h_r}{2}, \quad \lambda_{i\pm\frac{1}{2}} = \lambda\left(\frac{T_i + T_{i\pm 1}}{2}\right).$$

The boundary conditions have been approximated by the following set of formulas:

$$\begin{cases} \widehat{T}_0 = \dfrac{1}{3}\left(4\widehat{T}_1 - \widehat{T}_2\right), \\ \widehat{T}_n = \dfrac{1}{3}\left(4\widehat{T}_{n-1} - \widehat{T}_{n-2}\right), \end{cases} \tag{7}$$

for $r = 0$ and $r = r_{\max}$, respectively.

The conjugation conditions are approximated using one-sided derivatives:

$$\lambda_m(\widehat{T}_{i^*})\frac{3\widehat{T}_{i^*} - 4\widehat{T}_{i^*-1} + \widehat{T}_{i^*-2}}{2h_r} = \lambda_{m+1}(\widehat{T}_{i^*})\frac{-3\widehat{T}_{i^*} + 4\widehat{T}_{i^*+1} - \widehat{T}_{i^*+2}}{2h_r}, \tag{8}$$

where i^* is corresponding index of discontinuity point.

Numerical calculations to solve the problem (1)–(3) are carried out on a uniform grid of variables r and t with constant spatial and time steps h_r and h_t:

$$\begin{cases} h_r = \dfrac{r_{\max}}{n}, \\ h_t < \dfrac{h_r^2}{2}\min\left|\dfrac{\rho C_V(T)}{\max|\lambda(T)|}\right|, \end{cases} \tag{9}$$

here n is the number of partitions, and respectively, $r_i = i \cdot h_r$, for $i = 0, \ldots, n$.

The scheme (5)–(8) is tree points explicit difference scheme and it approximates the initial-boundary-value problem (1)–(3) with the accuracy $O(|h_t| + h_r^2)$ [4].

4. Results

Set of numerical simulations have been performed using spacial and time steps $h_r = 10^{-5}$ cm and $h_t = 10^{-11}$ s, for which the schema (5) converges. For the fourth material (external insulator) thermal conductivity was taken as constant $\lambda_4 = 10^{-3}$. Other thermal coefficient dependencies for each materials are given on Fig. 2 and Fig. 3. There are $T(r)$ distributions at different time moments on the Fig. 4.

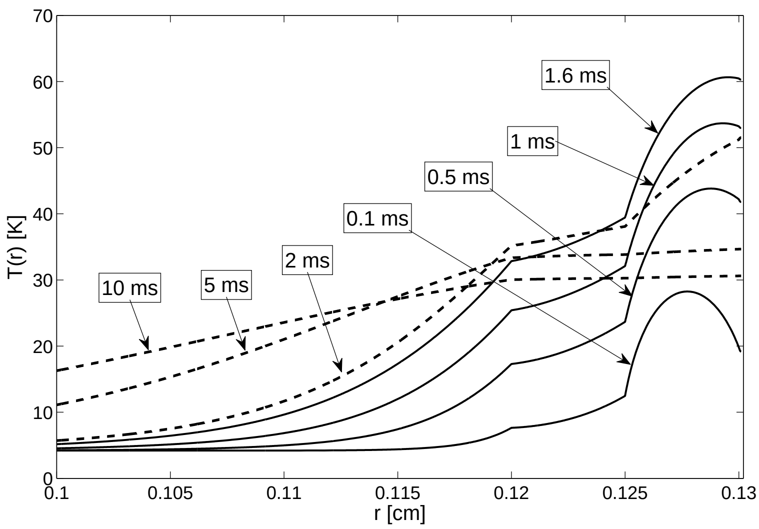

Figure 4. $T(r)$ distribution at different time moments. The solid lines correspond to heating. The dashed lines correspond to cooling

5. Conclusion and Discussion

In this paper the model with one spatial cylindrical variable for optimization of the object design is presented. This model allows modeling the beginning part of the thermal process (about 3 ms); thus it can significantly reduce the study of choosing the optimal design of the object in terms of practical needs and make a lot of preliminary studies.

For example, the results show that the given construction of the object using given materials does not satisfy the requirements (requirements for the thermal process are described in the "Introduction"). Firstly, the object surface (right point in Fig. 4) could not been heated up to 60 K in 1 ms. The heating time takes 1.6 ms. To speed up heating source should be increased. Secondly, for time ∼5–10 ms the results for this model will vary slightly disagree with reality, however one can be sure that the object surface could not been cooled down to ∼ 4.2–10 K in the remaining time. The electrical insulator between cooler and source does not allow to cool the object down for the requested time. It is needed to "play" with geometrical shape of the insulator or change the insulator material.

It is necessary to do a systematic studies in order to find optimal construction and materials of the object. One of the way of such study can be formulation and solution of the multiparameter optimal control problem [4] for the model with one spatial variable proposed in this paper.

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Модель с одной пространственной переменной для проектирования технического устройства

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В работе предложена модель с одной пространственной цилиндрической переменной для проектирования технического устройства. Устройство работает при криогенных температурах и имеет импульсный источник, его можно представить как многослойный цилиндрический образец. Представлены результаты вычислительных экспериментов.

Ключевые слова: уравнение теплопроводности, уравнения в частных производных, разностная схема, математическое моделирование.