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Chiral 8-Spinor Model with Pseudo-Vector Interaction

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In the paper the structure of the chiral 8-spinor field model is discussed, the interaction with the electromagnetic, Yang–Mills and gravitational fields being included.

Key words and phrases: spinor, topological charge, solitons.

1. Introduction

The Skyrme's idea to describe baryons as topological solitons [1] proved to be fruitful in nuclear physics for modeling the internal structure of hadrons [2, 3] and light nuclei [4, 5]. In the Skyrme Model the topological charge $Q = \deg(S^3 \to S^3)$ is interpreted as the baryon number B and serves as the generator of the homotopy group $\pi_3(S^3) = \mathbb{Z}$. The similar idea to describe leptons as topological solitons was announced by L. D. Faddeev [6]. In the Faddeev Model the Hopf invariant Q_H is interpreted as the lepton number L and serves as the generator of the homotopy group $\pi_3(S^2) = \mathbb{Z}$. The unification of these two approaches was suggested in [7], hadrons and leptons being considered as two possible phases of the effective 8-spinor field model.

The basic idea was to take into account the existence of the special 8-spinors identity discovered by the Italian geometer Brioschi [8]:

$$j_\mu j^\mu - \tilde{j}_\mu \tilde{j}^\mu = s^2 + p^2 + \mathbf{v}^2 + \mathbf{a}^2, \tag{1}$$

where the following quadratic spinor quantities are introduced:

$$s = \bar{\Psi}\Psi, \quad p = \imath\bar{\Psi}\gamma_5\Psi, \quad \mathbf{v} = \bar{\Psi}\lambda\Psi,$$
$$\mathbf{a} = \imath\bar{\Psi}\gamma_5\lambda\Psi, \quad j_\mu = \bar{\Psi}\gamma_\mu\Psi, \quad \tilde{j}_\mu = \bar{\Psi}\gamma_\mu\gamma_5\Psi,$$

with $\bar{\Psi} = \Psi^+\gamma_0$ and λ standing for Pauli matrices in the flavor (isotopic) space. Here the diagonal (Weyl) representation for $\gamma_5 = \gamma_5^+$ is used and γ_μ, $\mu = 0, 1, 2, 3$, designate the unitary Dirac matrices acting on Minkowsky spinor indices.

If one defines 8-spinors as columns:

$$\Psi = \mathrm{col}(\psi_1, \psi_2), \quad \psi_i = \mathrm{col}(\varphi_i, \chi_i), \quad i = 1, 2,$$

with φ_i and χ_i being 2-spinors, then one easily finds that the following identity holds

$$2j_\mu j^\mu = s^2 + p^2 + \mathbf{v}^2 + \mathbf{a}^2 + \Delta^2, \tag{2}$$

showing the time-like character of the 4-vector j_μ, where the denotation is introduced:

$$\Delta^2 = 8\big[(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2) - |\varphi_1^+\varphi_2|^2 + (\chi_1^+\chi_1)(\chi_2^+\chi_2) - |\chi_1^+\chi_2|^2\big] \geqslant 0.$$

The structure of the identity (2) leads to the natural conclusion that Higgs potential V in the effective spinor field model can be represented as the function of $j_\mu j^\mu$:

$$V = \frac{\sigma^2}{8}(j_\mu j^\mu - \varkappa_0^2)^2, \tag{3}$$

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with σ and $\varkappa_0$ being some constant parameters. If one searchs for localized soliton-like configurations in the model, one finds the natural boundary condition at space infinity:

$$\lim_{|\mathbf{r}|\to\infty} j_\mu j^\mu = \varkappa_0^2. \tag{4}$$

As follows from the identity (2), the condition (4) determines the fixed (vacuum) point on the surface S^8. Using (4) and the well-known property of homotopic groups of spheres: $\pi_3(S^n) = 0$ for $n \geqslant 4$, one concludes that the two phases with nontrivial topological charges may exist in the model (3). The first one corresponds to the choice $\pi_3(S^3) = \mathbb{Z}$ (Skyrme Model) and the second one coresponds to the choice $\pi_3(S^2) = \mathbb{Z}$ (Faddeev Model).

For example, if the vacuum state Ψ_0 defines $s(\Psi_0) \neq 0$, then the configurations characterized by the chiral invariant $s^2 + \mathbf{a}^2$ determining sphere S^3 as the field manifold are possible, that corresponds to Skyrme Model phase. On the contrary, if only $v_3(\Psi_0) \neq 0$, then the $SO(3)$ invariant $\mathbf{v}^2$ determines the S^2 field manifold, that corresponds to Faddeev Model phase.

2. The effective nonlinear 8-spinor field model

In view of these topological arguments, using the analogy with Skyrme (or Faddeev) Model, we suggested in [7] the following Lagrangian density for the effective 8-spinor field model:

$$\mathcal{L}_{\text{spin}} = \frac{1}{2\lambda^2}\overline{\partial_\mu\Psi}\gamma^\nu j_\nu \partial^\mu\Psi + \frac{\epsilon^2}{4} f_{\mu\nu}f^{\mu\nu} - V, \tag{5}$$

where $f_{\mu\nu}$ stands for the antisymmetric tensor of Faddeev–Skyrme type:

$$f_{\mu\nu} = (\bar{\Psi}\gamma^\alpha\partial_{[\mu}\Psi)(\overline{\partial_{\nu]}\Psi}\gamma_\alpha\Psi), \tag{6}$$

with λ and ϵ being constant parameters of the model. It should be stressed that the first term in (5) generalizes the σ-model term in Skyrme Model and includes the projector $P = \gamma^0\gamma^\nu j_\nu$ on the positive energy states. The second term in (5) gives the generalization of Skyrme (or Faddeev) term.

Now we intend to generalize the model by including the interaction of the spinor field with the electromagnetic, Yang–Mills and gravitational fields. To this end we consider the following structure for the extended covariant derivative:

$$D_\mu\Psi = \partial_\mu\Psi - \imath e_0\Gamma_e A_\mu\Psi + (A_\mu^L + A_\mu^R - \Gamma_\mu)\Psi, \tag{7}$$

where A_μ, A_μ^L, A_μ^R stand for the vector-potentials of the electromagnetic and left and right Yang–Mills fields respectively. Γ_μ in (7) denotes the spinor connection with the gravitational field, and Γ_e in (7) stands for the electric charge operator, e_0 being the corresponding coupling constant. The standard form of Γ_e, A_μ^L and A_μ^R reads as follows:

$$\Gamma_e = \frac{1}{2}(\lambda_3 - 1); \quad A_\mu^{L,R} = P_{L,R}\frac{e_{1L,R}}{2\imath}A_\mu^{aL,R}\lambda^a; \quad P_{L,R} = \frac{1}{2}(1 \pm \gamma_5), \tag{8}$$

with e_{1L}, e_{1R} being the corresponding coupling constants.

However, the Yang–Mills interaction in (7) can be simplified if one accepts that for the leptonic sector it should vanish. Taking into account that in the leptonic sector the pseudo-vector part $\mathbf{a} = \imath\bar{\Psi}\gamma_5\lambda\Psi = 0$ one gets the following constraint:

$$e_{1L}A_\mu^{aL} + e_{1R}A_\mu^{aR} = 0. \tag{9}$$

In view of (9) one finds

$$A_\mu^L + A_\mu^R = \gamma_5 e_{1L} \frac{\lambda^a}{2\imath} A_\mu^{aL} \equiv g_0 \gamma_5 \frac{\lambda^a}{2\imath} A_\mu^a \equiv g_0 \gamma_5 \mathbb{A}_\mu, \tag{10}$$

where the new pseudo-vector field $\mathbb{A}_\mu$ and the new coupling constant g_0 were introduced.

Now we take into account the invariance of the theory under the space reflection:

$$x_i \to -x_i, \quad \Psi \to \gamma_0 \Psi, \quad \mathbb{A}_0 \to -\mathbb{A}_0, \quad \mathbb{A}_i \to \mathbb{A}_i, \quad i = 1, 2, 3. \tag{11}$$

The condition (11) stems the following structure of the Yang–Mills Lagrangian density:

$$\begin{aligned}\mathcal{L}_{\text{YM}} = \frac{1}{32\pi g_0^2} \text{Sp}\left(F_{\mu\nu}^L F^{\mu\nu L} + F_{\mu\nu}^R F^{\mu\nu R}\right) = \\ = \frac{1}{8\pi} \text{Sp}\big\{(\partial_\mu \mathbb{A}_\nu - \partial_\nu \mathbb{A}_\mu)(\partial^\mu \mathbb{A}^\nu - \partial^\nu \mathbb{A}^\mu) + g_0^2 [\mathbb{A}_\mu, \mathbb{A}_\nu][\mathbb{A}^\mu, \mathbb{A}^\nu]\big\},\end{aligned} \tag{12}$$

where the strengths of the Yang–Mills fields were defined as

$$F_{\mu\nu}^{L,R} = \partial_\mu A_\nu^{L,R} - \partial_\nu A_\mu^{L,R} + [A_\mu^{L,R}, A_\nu^{L,R}].$$

Thus the Lagrangian density of the model in question reads

$$\mathcal{L} = \mathcal{L}_{\text{spin}} + \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{em}} + \mathcal{L}_{\text{g}}, \tag{13}$$

where ∂_μ in $\mathcal{L}_{\text{spin}}$ is changed to D_μ and the gravitational part corresponds to the Einstein theory:

$$\mathcal{L}_{\text{g}} = \frac{1}{2\varkappa} R, \quad \varkappa = \frac{8\pi G}{c^4},$$

R being the scalar curvature and G — the Newton gravitational constant.

The electromagnetic part of the Lagrangian density was investigated in [7] and it corresponds to the Mie generalized electrodynamics:

$$\mathcal{L}_{\text{em}} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} [1 + G(I)] - \frac{1}{8\pi} H(I), \tag{14}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $G(I)$, $H(I)$ are some functions of the Mie invariant $I = A_\mu A^\mu$. As was shown in [7], the model (14) admits the existence of static solitons with fixed electric charge and positive energy.

The final generalization concerns the Higgs-like potential (3), where the scalar multiplier σ^2 is supposed to be taken in the form [9]:

$$\sigma^2 = \frac{M^2}{8\lambda^2 \varkappa_0^2}, \quad M^2 = \frac{c^6}{\hbar^6 G^2} I_1^4 I_2^{-3}, \tag{15}$$

with I_1, I_2 being Riemannian invariants:

$$I_1 = \frac{1}{48} R_{\mu\nu\sigma\lambda} R^{\mu\nu\sigma\lambda}; \quad I_2 = -\frac{1}{432} R_{\mu\nu\sigma\lambda;\tau} R^{\mu\nu\sigma\lambda;\tau}. \tag{16}$$

The form (15) of the Higgs-like potential permits one to satisfy the quantum-mechanics correspondence principle, since the asymptotic behavior of the spinor field Ψ at large distances from the soliton center is described by the Klein–Gordon equation with the Schwarzschild mass M of the soliton as the mass term.

3. Conclusion

Using Brioschi 8-spinor identity in the form (2) and introducing the Higgs-like potential (3), we considered, following some ideas of Mie [10], the effective nonlinear 8-spinor model that includes, via the mechanism of spontaneous symmetry breaking, Skyrme and Faddeev models as particular cases. Using the space-reflection invariance of the model we simplify the Yang–Mills part of the Lagrangian density by considering the single pseudo-vector field instead of the pair of vector fields: left and right ones. Finally we generalize the Higgs-like potential V, the latter being multiplied by the special Riemannian invariant to satisfy the correspondence with quantum mechanics.

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Киральная 8-спинорная модель с псевдовекторным взаимодействием

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В работе обсуждается структура 8-спинорной полевой модели, включающей взаимодействие с электромагнитным полем, полем Янга–Миллса и гравитацией.

Ключевые слова: спинор, топологический заряд, солитоны.