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Nonlinear Model of the Earth Atmosphere with Regard to Rocket and Satellite Data

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The well-known model of the Earth standard atmosphere becomes unsatisfactory when altitudes are higher than 150 km. That is why in the paper a new model of the atmosphere is proposed. In this model strong electric fields in the Earth atmosphere are taken into account and described by means of the *SU(2)* Yang-Mills nonlinear generalization of the Maxwell equations. It is shown that the proposed nonlinear model is in a good balance with experimental data in the considered range of altitudes from zero to 1000 km.

Key words and phrases: Atmospheric regions, quasi-neutral plasma, nonlinear electric fields, Yang-Mills equations, nonlinear atmospheric model.

1. Introduction

The mathematical model of the standard atmosphere is based on the following equation [1] describing its equilibrium under the action of the Earth gravitation:

$$\frac{\mathrm{d}p}{\mathrm{d}r} + g\left(\frac{r_0}{r}\right)^2 \rho = 0, \tag{1}$$

where $p = p(r)$ is the atmospheric pressure at the distance r from the Earth center, r_0 is the Earth radius, $\rho = \rho(r)$ is the atmospheric density as a function of r for a given latitude and longitude, and g is the free fall acceleration at the Earth surface.

The pressure p is determined by the Clapeyron ideal gas law

$$p = \frac{R_0 T}{\mu}\rho, \tag{2}$$

where R_0 is the universal gas constant, T is the Kelvin temperature, and μ is the gas molar mass.

Numerical computations performed by means of equations (1) and (2) demonstrate that the standard atmosphere model can correctly describe only atmospheric regions with altitudes not exceeding 150 km. At higher altitudes which relate to the ionospheric region F there are considerable deviations of the standard atmosphere model from observational data derived from rockets and space satellites. That is why further we may consider a new atmospheric model. In this model, in contrast with the standard model, the strong electric fields in the Earth atmosphere are taken into account.

The necessity of the inclusion of strong electric fields in our atmospheric model can be justified by the following arguments:

1. A number of atmospheric phenomena can be explained by assuming strong electric fields in high-altitude regions of the Earth atmosphere. So one can attribute the phenomena of polar lights, magnetic storms, the conservation of the Earth negative electric charge, and some others [2].
2. Explorations of the Earth ionosphere show that it is a quasi-neutral plasma in which the number of positively charged particles is only approximately equal to that of negatively charged ones. That is why electric fields in this substantially ionized region could play a considerable role [3, 4].

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3. Measurements of electrostatic fields near the Earth surface show that both their values averaged over large periods of time and their instantaneous values can undergo substantial changes during polar lights. This fact confirms the supposition that in high-altitude atmospheric regions there can exist large potential differences and bulk electric charges [2].

2. Nonlinear Generalization of the Standard Atmosphere Model

Let us take into account the influence of the atmospheric electric field. Then equation (1) describing the atmosphere equilibrium can be generalized as follows:

$$\frac{\mathrm{d}p}{\mathrm{d}r} + g\left(\frac{r_0}{r}\right)^2 \rho - \Theta E = 0. \tag{3}$$

Here $E = E(r)$ is the strength of the electric field at the distance r from the Earth center and $\Theta = \Theta(r)$ is the charge density. In each atmospheric layer the charge density can be determined by the formula

$$\Theta = \chi\rho, \tag{4}$$

where χ is the coefficient of proportionality of the charge and mass densities.

This formula is a condition of stationarity of an atmospheric layer. Then not only the ingoing and outgoing gas masses are equal in its arbitrary volume, but the equality of the ingoing and outgoing charges also holds.

According to Coulomb's law, the electric field strength $E = E(r)$ can be determined by the following formula in the system of electrostatic units (esu):

$$E = \frac{q(r)}{r^2}, \quad q(r) = 4\pi \int\limits_0^r r^2 \Theta(r) \mathrm{d}r, \tag{5}$$

where $q(r)$ is the charge of the spherical region with the radius r.

In order to describe strong electric fields in the atmosphere, we suggest a new nonlinear model for them.

This model is based on the Yang-Mills equations with *SU(2)* symmetry which play an important role in the field theory of electroweak interactions [5]. In Refs [6–8] it is shown that the Yang-Mills equations give a reasonable nonlinear generalization of the Maxwell equations which can be applicable in the case of very powerful field sources. In these works a new class of spherically symmetric solutions to the Yang-Mills equations is found and investigated. In the stationary case they give that the electric field strength E at the distance r from the center of a charged spherically symmetric source can be determined by the formula [6–8]

$$E = \frac{q_{\mathrm{eff}}(r)}{r^2}, \tag{6}$$

where q_{eff} is the effective charge of the spherical region with radius r which includes not only electric charges of the field source but also charges of quanta of the Yang-Mills field.

The effective charge can be determined as follows [6–8]:

$$q_{\mathrm{eff}}(r) = K \sin\left(\frac{q(r)}{K}\right), \tag{7}$$

where $q(r)$ is the charge of the region with the radius r determined by the classical formula (5) and K is a constant.

The above formulas (6) and (7) were applied in Refs [6,8] to explain the phenomenon of a ball lightning and a relation of the constant K to its maximum diameter was found.

The choice of the value ~ 100 cm for the maximum diameter of the ball lightning derived from a large number of observations gives the following estimate for the constant K:

$$K \sim 10^7 \text{ coul} = 3 \cdot 10^{16} \text{ esu}. \tag{8}$$

When $|q| \ll K$, from (7) we have $q_{\text{eff}} = q$ and hence the proposed formula (6) coincides with the classical formula for the electric field strength.

Therefore, formulas (6) and (7) can differ from the classical formula only when the charge of a field source is very high.

Taking into account formulas (4), (6), and (7), let us represent equation (3) in an atmospheric layer $r_1 \leqslant r \leqslant r_2$, where r_1 and r_2 are its boundaries, in the form of the integro-differential equation

$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} + g\left(\frac{r_0}{r}\right)^2 \rho(r) - \frac{\chi \rho(r) K}{r^2} \sin\left(\frac{4\pi\chi}{K} \int\limits_{r_1}^{r} \rho(r) r^2 \mathrm{d}r\right) = 0. \tag{9}$$

It is worth noting that the following conditions correspond to the lower and upper boundaries of each atmospheric layer:

$$\delta(r_1) = 0, \quad \delta(r_2) = 2\pi, \tag{10}$$

where $\delta(r)$ is the argument of the sine in equation (9).

Then above the surface $r = r_1$ the electric force acting on ions is directed upwards and under the surface it is directed downwards. This separates a layer under consideration from the layer below it.

Analogously, when conditions (10) are fulfilled, under the surface $r = r_2$ the electric force acting on ions is directed downwards and above the surface it is directed upwards. This separates the considered layer from the layer above it.

To come to a nonlinear differential equation from (9), let us introduce the function

$$u(r) = \int\limits_{r_1}^{r} \rho(r) r^2 \mathrm{d}r. \tag{11}$$

From (11) we obtain the formulas

$$\rho(r) = \frac{u'(r)}{r^2}, \quad \rho'(r) = \frac{1}{r^2}\left(u''(r) - \frac{2u'(r)}{r}\right). \tag{12}$$

Let us also put

$$a = \frac{R_0 T}{\mu}. \tag{13}$$

The molar mass μ of a gas mix is defined as the weighted average of the molar masses of its different gases by the following classical formula:

$$\frac{1}{\mu} = \sum_i \frac{1}{\mu_i} \frac{M_i}{M}, \tag{14}$$

where μ_i is the molar mass of the i th gas, M_i is the mass of the i th gas and M is the mass of all the gas mix per unit volume.

Then using correlations (2), (12), and (13), we find

$$\frac{\mathrm{d}p}{\mathrm{d}r} = \frac{a(r)}{r^2}\left(u''(r) - \frac{2u'(r)}{r}\right) + \frac{a'(r)u'(r)}{r^2}. \tag{15}$$

Instead r, let us introduce the dimensionless variable x:

$$x = r/r_0. \tag{16}$$

As a result of the transition to the dimensionless variable x, formulas (12) and (15) acquire the form

Then from (11), (12), and (16) we find that equation (9) can be represented as

$$u''(x) - \frac{u'(x)}{x}\left[2 - \frac{x^2 a'(x) + g r_0}{x a(x)} + \frac{\chi K}{r_0 x a(x)} \sin\left(\frac{4\pi\chi}{K} u(x)\right)\right] = 0. \tag{17}$$

Using formulas (11) and (16), let us write down the following conditions for the function $u(x)$ at the lower boundary $r = r_1$ of the considered region:

$$\begin{aligned} &u(x_1) = 0, \quad x_1 = r_1/r_0, \\ &u'(x_1) = r_0^3 \rho(r_1) x_1^2. \end{aligned} \tag{18}$$

Thus, we have the differential equation (17) of the second order with conditions (18) for the function $u(x)$ which describes the Earth atmosphere for different values of latitude and longitude. The equation contains of two unknown parameters depending on the geographic coordinates, considered day and day time: the coefficient χ of the proportionality between the charge and mass densities and the constant K with the approximate estimate given in (8).

To solve the problem under examination a set of computer programs has been worked out. It includes the numerical solution of the nonlinear differential equation (17) by means of the Runge-Kutta method of the forth order and a program of nonlinear optimization to choose values of the model unknown parameters.

For numerical computations a number of cases have been chosen which correspond to different geographic coordinates, days and day times. Further we can demonstrate our results in the two cases given in Table 1. The results in other cases considered by us are similar to them.

Table 1

Dates, day times and geographic coordinates

Case	Date	Time (UTC), hrs	Latitude	Longitude
Case 1	01.06.2000	12.0	55°	45°
Case 2	01.05.2001	1.5	80°	150°

Further we may consider nonlinear models of different atmospheric regions.

3. Nonlinear Model of the Ionosphere

For each case under consideration a large series of computations by using our set of computer programs has been carried out for different values of the unknown parameters. These parameters are chosen so that the best balance with experimental data is achieved.

Experimental data necessary to solve the examined problem were obtained as a result of long standing measurements of characteristics of the Earth atmosphere by means of spacecrafts. They were the basis of the known empirical models MSIS-E-90 and IRI-2007 [9]. These models are used by us to compare them with results of our computations and to get experimental data necessary for us. Applying the empirical models, we have calculated values of the function $a(x)$ which enters into equation (17) and is determined by formula (13) describing its dependence on temperature and molar mass. Experimental values of the temperature inside the ionosphere are derived from the model IRI-2007. The concentrations of the gases O, O_2, N, N_2, H, He, Ar per unit volume derived from the model MSIS-E-90 are used to calculate molar masses of gas mixtures by means of formula (14).

Using computations carried out by us, we have determined values of the parameters K and χ for the atmospheric regions F and E and also found the altitude of the lower boundary of the region F. They are given in Table 2.

Table 2

Computed values of the nonlinear model parameters

Case	χ, esu		K, esu	Altitude of the lower boundary of the ionospheric region F, km
	E	F		
Case 1	17.8	750	$2.8 \cdot 10^{16}$	124
Case 2	26.6	800	$3.1 \cdot 10^{16}$	140

As it is seen from this table, the obtained values of K correspond to the estimate (8) and the found lower boundaries of the region F conform to experimental data [3, 4].

These results are also represented in Figures 1 and 2.

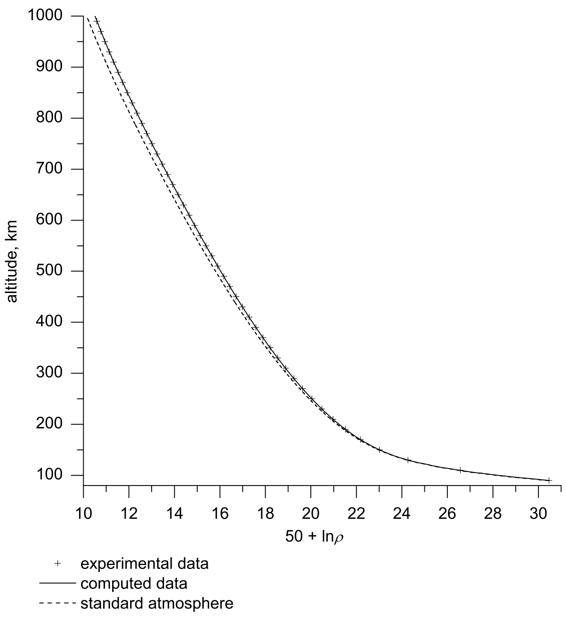

Figure 1. Plots of the density versus altitude in the ionosphere for different models in the case 1

The obtained distributions of the density in the ionosphere for different models in the considered cases 1 and 2 are given in Tables 3 and 4.

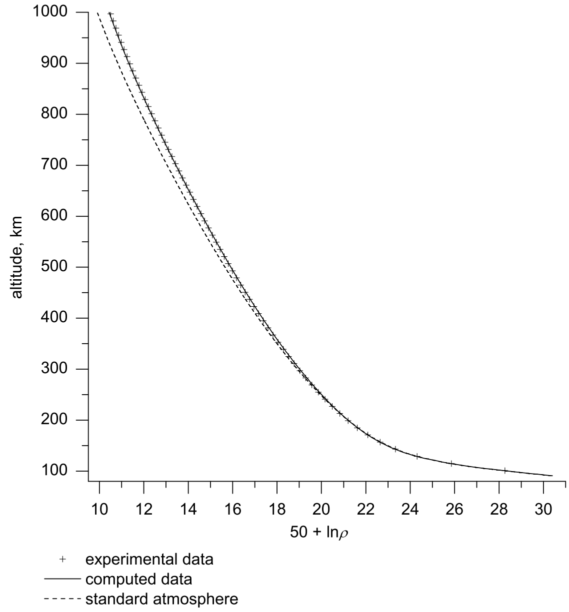

Figure 2. Plots of the density versus altitude in the ionosphere for different models in the case 2

Table 3

Distribution of the density (g/cm^3) in the ionosphere for different models in the case 1

Altitude, km	Experimental data	Data of the proposed nonlinear model	Data of the standard atmosphere model
100.0	$3.735 \cdot 10^{-10}$	$3.566 \cdot 10^{-10}$	$3.719 \cdot 10^{-10}$
120.0	$1.798 \cdot 10^{-11}$	$1.756 \cdot 10^{-11}$	$1.828 \cdot 10^{-11}$
160.0	$1.220 \cdot 10^{-12}$	$1.225 \cdot 10^{-12}$	$1.194 \cdot 10^{-12}$
200.0	$3.232 \cdot 10^{-13}$	$3.241 \cdot 10^{-13}$	$3.018 \cdot 10^{-13}$
240.0	$1.215 \cdot 10^{-13}$	$1.213 \cdot 10^{-13}$	$1.096 \cdot 10^{-13}$
280.0	$5.343 \cdot 10^{-14}$	$5.330 \cdot 10^{-14}$	$4.703 \cdot 10^{-14}$
320.0	$2.583 \cdot 10^{-14}$	$2.573 \cdot 10^{-14}$	$2.225 \cdot 10^{-14}$
360.0	$1.330 \cdot 10^{-14}$	$1.326 \cdot 10^{-14}$	$1.125 \cdot 10^{-14}$
400.0	$7.177 \cdot 10^{-15}$	$7.170 \cdot 10^{-15}$	$5.981 \cdot 10^{-15}$
450.0	$3.497 \cdot 10^{-15}$	$3.495 \cdot 10^{-15}$	$2.857 \cdot 10^{-15}$
500.0	$1.776 \cdot 10^{-15}$	$1.777 \cdot 10^{-15}$	$1.424 \cdot 10^{-15}$
550.0	$9.304 \cdot 10^{-16}$	$9.312 \cdot 10^{-16}$	$7.330 \cdot 10^{-16}$
600.0	$4.990 \cdot 10^{-16}$	$4.999 \cdot 10^{-16}$	$3.865 \cdot 10^{-16}$
650.0	$2.730 \cdot 10^{-16}$	$2.736 \cdot 10^{-16}$	$2.080 \cdot 10^{-16}$
700.0	$1.520 \cdot 10^{-16}$	$1.524 \cdot 10^{-16}$	$1.140 \cdot 10^{-16}$
750.0	$8.608 \cdot 10^{-17}$	$8.641 \cdot 10^{-17}$	$6.364 \cdot 10^{-17}$
800.0	$4.965 \cdot 10^{-17}$	$4.988 \cdot 10^{-17}$	$3.621 \cdot 10^{-17}$
850.0	$2.924 \cdot 10^{-17}$	$2.939 \cdot 10^{-17}$	$2.105 \cdot 10^{-17}$
900.0	$1.764 \cdot 10^{-17}$	$1.774 \cdot 10^{-17}$	$1.256 \cdot 10^{-17}$
1000.0	$7.032 \cdot 10^{-18}$	$7.079 \cdot 10^{-18}$	$4.914 \cdot 10^{-18}$

Table 4
Distribution of the density (g/cm^3) in the ionosphere for different models in the case 2

Altitude, km	Experimental data	Data of the proposed nonlinear model	Data of the standard atmosphere model
100.0	$4.482 \cdot 10^{-10}$	$4.278 \cdot 10^{-10}$	$4.582 \cdot 10^{-10}$
120.0	$1.774 \cdot 10^{-11}$	$1.743 \cdot 10^{-11}$	$1.856 \cdot 10^{-11}$
160.0	$1.154 \cdot 10^{-12}$	$1.147 \cdot 10^{-12}$	$1.213 \cdot 10^{-12}$
200.0	$3.034 \cdot 10^{-13}$	$3.104 \cdot 10^{-13}$	$3.142 \cdot 10^{-13}$
240.0	$1.128 \cdot 10^{-13}$	$1.173 \cdot 10^{-13}$	$1.137 \cdot 10^{-13}$
280.0	$4.890 \cdot 10^{-14}$	$5.127 \cdot 10^{-14}$	$4.769 \cdot 10^{-14}$
320.0	$2.329 \cdot 10^{-14}$	$2.442 \cdot 10^{-14}$	$2.187 \cdot 10^{-14}$
360.0	$1.183 \cdot 10^{-14}$	$1.237 \cdot 10^{-14}$	$1.069 \cdot 10^{-14}$
400.0	$6.324 \cdot 10^{-15}$	$6.576 \cdot 10^{-15}$	$5.495 \cdot 10^{-15}$
450.0	$3.053 \cdot 10^{-15}$	$3.149 \cdot 10^{-15}$	$2.529 \cdot 10^{-15}$
500.0	$1.545 \cdot 10^{-15}$	$1.579 \cdot 10^{-15}$	$1.221 \cdot 10^{-15}$
550.0	$8.101 \cdot 10^{-16}$	$8.201 \cdot 10^{-16}$	$6.118 \cdot 10^{-16}$
600.0	$4.363 \cdot 10^{-16}$	$4.378 \cdot 10^{-16}$	$3.155 \cdot 10^{-16}$
650.0	$2.403 \cdot 10^{-16}$	$2.390 \cdot 10^{-16}$	$1.667 \cdot 10^{-16}$
700.0	$1.350 \cdot 10^{-16}$	$1.332 \cdot 10^{-16}$	$9.001 \cdot 10^{-17}$
750.0	$7.731 \cdot 10^{-17}$	$7.568 \cdot 10^{-17}$	$4.967 \cdot 10^{-17}$
800.0	$4.518 \cdot 10^{-17}$	$4.392 \cdot 10^{-17}$	$2.805 \cdot 10^{-17}$
850.0	$2.702 \cdot 10^{-17}$	$2.610 \cdot 10^{-17}$	$1.626 \cdot 10^{-17}$
900.0	$1.659 \cdot 10^{-17}$	$1.594 \cdot 10^{-17}$	$9.716 \cdot 10^{-18}$
1000.0	$6.909 \cdot 10^{-18}$	$6.574 \cdot 10^{-18}$	$3.867 \cdot 10^{-18}$

4. Nonlinear Model of the Troposphere, Stratosphere, and Mesosphere

The troposphere is the lowest layer of the atmosphere. Its upper boundary ranges from 8 to 20 km. The altitude of the boundary depends on the geographic coordinates and seasons. As altitude increases, temperature in this region diminishes. The stratosphere is the atmospheric layer ranging from 11 to 55 km. In this region the temperature increases with the increase of altitude. The mesosphere ranges from 55 to 90 km. In this layer temperature diminishes with the increase of altitude.

The troposphere, stratosphere, and mesosphere end above with the narrow domains tropopause, stratopause, and mesopause, respectively, in which temperatures are almost independent of altitudes.

Using the above facts and data of the model MSIS-E-90, we have found boundaries of the regions under consideration which are given in Table 5.

Table 5
Altitude of the upper boundary of the troposphere, stratosphere, and mesosphere in the chosen cases

Case	Altitude of the upper boundary, km		
	troposphere	stratosphere	mesosphere
Case 1	17	47	89
Case 2	12	48	91

In each case by means of the set of programs worked out by us numerical computations have been carried out. In them for the parameter K we used its value obtained above for the ionosphere, since it should not depend on altitude. These computations allowed us to find values of the unknown parameter χ for each layer under consideration which are given in Table 6.

Table 6

Values of the model parameters χ of the troposphere, stratosphere, and mesosphere in the chosen cases

Case	χ, esu		
	troposphere	stratosphere	mesosphere
Case 1	$3.673 \cdot 10^{-5}$	$3.536 \cdot 10^{-4}$	$2.466 \cdot 10^{-2}$
Case 2	$4.436 \cdot 10^{-5}$	$2.002 \cdot 10^{-4}$	$3.243 \cdot 10^{-2}$

The distribution of the atmospheric density (g/cm^3) with altitude derived from the suggested nonlinear model and its experimental values are given in the plots 3 and 4. It should be noted that in the troposphere, stratosphere, and mesosphere the deviations of the data given by the standard model from experimental ones are not substantial. That is why the data derived from the standard model for these regions are not shown.

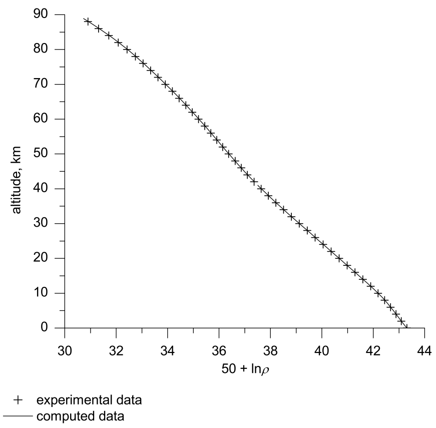

Figure 3. Plot of the density versus altitude in the troposphere, stratosphere, and mesosphere in the case 1

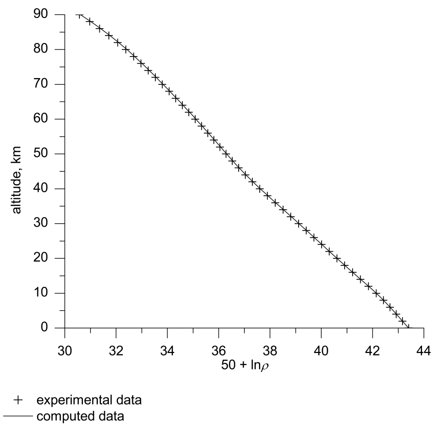

Figure 4. Plot of the density versus altitude in the troposphere, stratosphere, and mesosphere in the case 2

5. Conclusion

It is shown that the standard atmosphere model is not satisfactory at the altitudes higher than 150 km. That is why in the paper a new model of the Earth atmosphere is suggested. In it strong electric fields are taken into account and described within the framework of the Yang-Mills theory with *SU(2)* symmetry which gives a nonlinear generalization of the Maxwell equations. Applying the new model we have come to a nonlinear differential equation of the second order describing distributions of mass density in atmospheric layers.

To determine its unknown parameters we have used data derived from rockets and satellites. After finding numerical solutions of the considered nonlinear differential equation in different cases, we have obtained distributions of mass density in the ionospheric layers E and F and then in the tropospheric, stratospheric, and mesospheric layers. The comparison of the computed values of mass density with experimental data shows that they are closed to each other at the considered altitudes ranging from zero up to 1000 km.

The balance of the suggested nonlinear model with experimental data demonstrates that it can be applicable to describe the Earth atmosphere.

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Нелинейная модель атмосферы Земли с учётом ракетных и спутниковых данных

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Хорошо известная модель стандартной атмосферы Земли становится неудовлетворительной на высотах, превышающих 150 км от поверхности Земли. Поэтому в статье предлагается новая модель атмосферы. В ней учитываются сильные электрические поля в атмосфере Земли, которые описываются посредством нелинейного *SU(2)* обобщения Янга–Миллса уравнений Максвелла. Показывается, что предложенная нелинейная модель находится в хорошем согласии с экспериментальными данными в рассматриваемой области высот от нуля до 1000 км.

Ключевые слова: атмосферные области, квазинейтральная плазма, нелинейные электрические поля, уравнения Янга–Миллса, нелинейная модель атмосферы.