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Application of Method of Conjugate Equations to Research of Loss Stability of Shell Under the Action of Moving Loads

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Actual problem of the theory of stability is creation of strict and effective methods of research of the loss of stability of movement of systems with distributed parameters, in particular for continuous environments. This problem has a huge impact on both theoretical and applied uses.

In the article is presented the research of the loss of stability of the equations which describe mathematical models of soil massifs and the bases. These models reflect the character of the soil under load, are based on the laws of structural mechanics and the theory of elasticity.

The adjoin method offered by Kudinov A.N. was applied to research of the loss of stability. The main advantage of the adjoin method is that for use to the problems of dynamic stability studies in various fields of science and technology, if their equations can be reduced to the equations of second order, don't demand the introduction of Lyapunov functions. The developed algorithm allows to find positions of balance of the decision and to check up stability of system by adjoin method. The research of stability of the solution of linear systems by using Lyapunov's method of first approximation is presented in the article.

Key words and phrases: stability, dynamic criterion, Lyapunov's first method, rocks, soil.

1. Introduction

Dynamic processes are described by nonlinear autonomous systems of ordinary differential equations, systems of partial differential equations, integral and integral–differential equations. The system must have a dynamic and structural stability.

The results of the fundamental works of Nikolaja Pavlovicha Epygina, Viktora Alekcandpovicha Plica, Nikolaja Nikolaevicha Kpacovckogo, Aleksandra Aleksandrovicha Andronova, Anatolija Arkad'evicha Pepvozvanckogo, Marka Aponovicha Yzerman, Vladimira Vasil'evicha Bolotina and Arnol'da Sergeevicha Vol'mira research of complex dynamic systems described by nonlinear systems of differential equations, derived from the introduction and analysis of Lyapunov functions – is qualitative methods.

The proof of theorems, which represent a solution to the problem of Yzerman in the sense of establishing the necessary conditions for stability systems:

- the dissemination of necessary and sufficient conditions for the stability of nonlinear dynamical systems on the systems described by partial differential equations. The main idea is to conversion the partial differential equations into the frequency by means of the Fourier transform followed by the application of the theorem Planshepelya;
- the dissemination of necessary and sufficient conditions for the stability of nonlinear dynamical systems to the systems described integral and integral-differential equations. The main idea is to conversion the original integral or integral-differential equation to ordinary derivatives with the use of a linear differential operator.
- the validation of necessary and sufficient conditions for the structural stability of autonomous nonlinear dynamical systems. The main idea is the study dependence of eigenvalues of conjugate system from control parameters of the main system and that point of the changing sign of eigenvalues and conjugate system – is the point of transition of system from stable steady in unstable steady states.

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The various rocks (soil) are used for building of engineering constructions as basis. Therefore, the models of basis which reflect the character of work of soil under load, are built on laws of structural mechanics and the theory of elasticity. The equations which describe the model of basis are called as mathematical models of soil massifs and the basis.

2. The Adjoin Method

V. Z. Vlasov [1] proposed the theory of technical analysis of constructions on elastic basis, which have a distribution and persistence at a formula of jet pressure:

$$q = -2c\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} + m_0\frac{\mathrm{d}^2 w}{\mathrm{d}t^2} + kw, \tag{1}$$

where c, k, m_0 — shift, compression, inertance-based (Fig. 1).

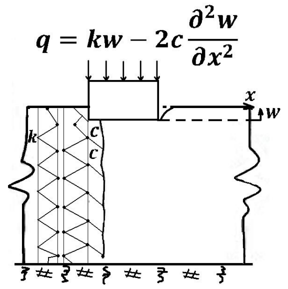

Figure 1. Constructions on Elastic Basis

The expressions for deflection [2] $w(x,t) = \varphi(t)\sin(nx)$, where n is a wave number.

Applying the method of Bubnova–Galerkina, we will receive the following differential equation of second order:

$$\frac{\partial^2\varphi}{\partial t^2} + \frac{\varphi}{m_0}\left(k + 2cn^2 - 4\frac{q}{\pi}\right) = 0. \tag{2}$$

Further this equation we reduce to the system

$$\begin{cases} \dot\varphi(t) = x(t), \\ \dot x(t) = \ddot\varphi(t) = -\dfrac{\varphi}{m_0}\left(k + 2cn^2 - 4\dfrac{q}{\pi}\right), \end{cases} \tag{3}$$

Then we write Hamiltonian:

$$H(p_1, p_2, \varphi(t), x(t)) = p_1(t)x(t) + p_2(t)\left[-\frac{\varphi}{m_0}\left(k + 2cn^2 - 4\frac{q}{\pi}\right)\right],$$

and compute the conjugate system that is linear relatively unknown functions $p_1(t)$ and $p_2(t)$

$$\begin{cases} \dot{p_1}(t) = p_2(t)\dfrac{1}{m_0}\left(k + 2cn^2 - 4\dfrac{q}{\pi}\right), \\ \dot{p_2}(t) = -p_1(t). \end{cases}$$

Calculate Jacobian the main system

$$J(\varphi(t), y(t)) = \begin{vmatrix} 0 & 1 \\ -\dfrac{1}{m_0}\left(k + 2cn^2 - \dfrac{4q}{\pi}\right) & 0 \end{vmatrix} = \frac{1}{m_0}\left(k + 2cn^2 - \frac{4q}{\pi}\right).$$

Find Jacobian the conjugated system

$$J(p_1(t), p_2(t)) = \begin{vmatrix} 0 & \dfrac{1}{m_0}\left(k + 2cn^2 - \dfrac{4q}{\pi}\right) \\ -1 & 0 \end{vmatrix} = \frac{1}{m_0}\left(k + 2cn^2 - \frac{4q}{\pi}\right).$$

Accordingly for the main system and conjugate system, the Jacobians are nonzero at $\dfrac{1}{m_0}\left(k + 2cn^2 - \dfrac{4q}{\pi}\right) \neq 0$, position of equilibrium exist at $q < \dfrac{k\pi}{4} + \dfrac{c\pi n^2}{2}$.

Let us write the linear equation of second order for the conjugate function $p_2(t)$. Since $\dot{p_2}(t) = -p_1(t)$, then $\ddot{p_2}(t) = -\dot{p_1}(t)$, substituting $\dot{p_1}(t)$ we get

$$\ddot{p_2}(t) = -p_2(t)\frac{1}{m_0}\left(k + 2cn^2 - \frac{4q}{\pi}\right)$$

or

$$\ddot{p_2}(t) + p_2(t)\frac{1}{m_0}\left(k + 2cn^2 - \frac{4q}{\pi}\right) = 0.$$

From the equations we can immediately see that the stability criterion of Hurwitz for the conjugate system is fulfilled, if $k + 2cn^2 - \dfrac{4q}{\pi} > 0$ or $q < \dfrac{k\pi}{4} + \dfrac{c\pi n^2}{2}$ at any moment. Hence the solution of the conjugate system will be stable and there will be the stability of nonperturbed state.

3. Lyapunov's Method on First Approximation

Also we have researched the stability of the system of differential equations (3) by Lyapunov's method on first approximation [3]. For this purpose the matrix of coefficients has been written:

$$A = \begin{pmatrix} 0 & 1 \\ -\dfrac{1}{m_0}\left(k + 2cn^2 - \dfrac{4q}{\pi}\right) & 0 \end{pmatrix},$$

The characteristic equation has form: $\det(A - kE) - 0$, where A — matrix of coefficients, E is the identity matrix.

$$\begin{pmatrix} 0 & 1 \\ -\dfrac{1}{m_0}\left(k + 2cn^2 - \dfrac{4q}{\pi}\right) & 0 \end{pmatrix} - k\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 0.$$

$$\det(A - kE) = \begin{vmatrix} -k & 1 \\ -\dfrac{1}{m_0}\left(k + 2cn^2 - \dfrac{4q}{\pi}\right) & -k \end{vmatrix} = k^2 + \frac{1}{m_0}\left(k + 2cn^2 - \frac{4q}{\pi}\right) = 0. \quad (4)$$

Solving the equations, we obtain that eigenvalues are equal:

$$k = \pm\sqrt{\frac{1}{m_0}\left(\frac{4q}{\pi} - k - 2cn^2\right)}.$$

Nonperturbed motion of the initial non-linear system is stable, as $Re(k) = 0$ and $q < \dfrac{k\pi}{4} + \dfrac{c\pi n^2}{2}$.

References

1. *Vlasov V. Z., Leontiev U. N.* Beams, Plates, and Shells on Elastic Foundation. — Jerusalem: Israel Program for Scientific Translations, 1966.
2. *Кудинов А. Н., Катулев А. Н., Кузнецов А. Ю.* Исследование устойчивости автономных нелинейных динамических систем // Материалы XVI Международного симпозиума «Динамические и технологические проблемы механики конструкций и сплошных сред» им. А. Г. Горшкова. — 2010. — С. 110–112. [Kudinov A. N., Katulev A. N., Kuznecov A. J. The Research of Stability of Independent Non-Linear Dynamic Systems // Materials of XVI International symposium "Dynamic and Technological Problems of Mechanic Constructions and Continuums" in the name of A. G. Gorshkov. — 2010. — Pp. 110–112. — (in russian).]
3. *Кудинов А. Н., Чусова Е. В.* Исследование динамической устойчивости цилиндрических оболочек // Материалы XVI Международного симпозиума «Динамические и технологические проблемы механики конструкций и сплошных сред» им. А. Г. Горшкова. — 2010. — Т. 2. — С. 121–134. [Kudinov A. N., Katulev A. N., Kuznecov A. J. The Research of Stability of Independent Non-Linear Dynamic Systems // Materials of XVII International symposium "Dynamic and Technological Problems of Mechanics of Constructions and Continuums" in the name of A. G. Gorshkov. — 2010. — Vol. 2. — Pp. 121–134. — (in russian).]

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Применение метода сопряжённых уравнений к исследованию процесса потери устойчивости оболочек при действии подвижных нагрузок

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Актуальной проблемой теории устойчивости является создание строгих и эффективных методов исследования процесса потери устойчивости движения систем с распределёнными параметрами, в особенности сплошных сред. Эта проблема имеет огромное теоретическое и прикладное значение.

В статье проведено исследование процесса потери устойчивости уравнений, которые описывают математические модели грунтовых массивов и оснований. Эти модели отражают характер работы грунтов под нагрузкой, строятся на законах строительной механики и теории упругости.

Для исследования процесса потери устойчивости был применён метод сопряжённых уравнений, предложенный Кудиновым А. Н. Данный метод даёт возможность его применения к задачам исследования динамической устойчивости в самых разных областях науки и техники, уравнения которых сводятся к уравнению второго порядка, при этом для исследования процесса потери устойчивости нет необходимости построения функции Ляпунова. В результате исследования были найдены положения равновесия и проверено условие, будет ли иметь место устойчивость невозмущённого состояния. Также было проведено исследование на основе метода Ляпунова по первому приближению, в результате получены условия устойчивости оболочек при действии подвижных нагрузок.

Ключевые слова: устойчивость, динамический критерий, первый метод Ляпунова, грунт.