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Time Characteristics of Queuing System with Renovation and Reservice

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This article is devoted to time characteristics of queuing system with recurrent input flow, one server, exponential service time distribution and infinite queue. The mechanism of renovation with reservice (repeated service) is introduced. It means that a packet at the moment of the end of its service with some probability may just leave the system or with complementary probability will drop all other packets in the system and return for service.

Assuming that we know the steady-state probability distribution of number of packets (calculated with help of embedded by the moments of arrival Markov chain) the main emphasis of the article will be on system time characteristics such as steady-state distributions of time in system for serviced or dropped packets, average time characteristics — mean service time, mean waiting time for a dropped, serviced and an arbitrary packet.

Key words and phrases: renovation, reservice, general input flow, time characteristics, serviced packet, dropped packet.

1. Introduction

Mathematical methods of queuing theory allow to create stochastic models of data network protocols and make possible to find analytical solutions of multiple tasks such as network quality service and performance characteristics, time and probabilistic characteristics of network nodes, estimate losses and data links download.

The one-server queuing system with general input flow, infinite buffer and exponential service time is considered. The general renovation with reservice is introduced. According to this mechanism a packet under service at the moment of the end of service with some probability may drop all other packets from a buffer or with complementary probability just leave the system.

The first introduction of systems with renovation was in [1] and application of such systems was shown in [2], but the general renovation (without repeated service) was introduced in [3] and we will use some notation and style of discussion from [3–5]. The simple example of such system was discussed in [6], when input flow was Poisson.

Some application of queuing system with general renovation is shown in [7–9]. Some results obtained for queuing systems with general renovation may be used in such area of research as P2P networks (for example, [10]).

The article is structured as follows: in the next section system notation is presented, then some formulae for steady-state distribution and service and loss probabilities are given. The section after that presents time characteristics of served and dropped (loss) packets. The last section is conclusion where some future problems will be presented.

2. Notation

1. $A(x)$ — arrival distribution function.
2. μ, $\mu > 0$ — the parameter of exponential service distribution function.
3. $\alpha(\cdot)$ — Laplase-Stieltjes transformation (LST) of $A(x)$ function.
4. p — the probability that a served packet will leave the queuing system without dropping other packets.

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5. $q = 1 - p$ — the probability of renovation with reservice — the packet at the moment of the end of the service will drop all other packets in the system and return to the buffer (and due to buffer emptiness will be immediately served again).
6. p_k^-, $k \geqslant 0$, — the steady-state probability (obtained by an embedded Markov chain) that a packet at the moment of arrival will find k, $k \geqslant 0$, packets in the system.
7. $p^{(\text{loss})}$ — the steady-state probability that arriving packet will be dropped (will not be served).
8. $p^{(\text{serv})}$ — the steady-state probability that arriving packet will be served.
9. $W^{(\text{serv})}(x)$ and $\omega^{(\text{serv})}(s)$ — the probability that a time of packet being on the server will be less than x (service time distribution function of served packet) and Laplase-Stieltjes transformation (LST) of this probability.
10. $W^{(\text{serv})}_{(\text{wait})}(x)$ and $\omega^{(\text{serv})}_{(\text{wait})}(s)$ — the probability that a time of a packet waiting for service will be less than x and this packet will not be dropped (waiting time distribution function of served packet) and Laplase-Stieltjes transformation (LST) of this probability.
11. $W^{(\text{loss})}(x)$ and $\omega^{(\text{loss})}(s)$ — the probability that a dropped packet will be in the queue for a time less than x (waiting time distribution function of dropped packet) and LST of this probability.
12. $W_{(\text{wait})}(x)$ and $\omega_{(\text{wait})}(s)$ — the probability that an arbitrary packet will stay in the queue for a time more than x (waiting time distribution function of an arbitrary packet) and Laplase-Stieltjes transformation (LST) of this probability.
13. $w^{(\text{loss})}$, $w^{(\text{serv})}_{(\text{wait})}$, $w_{(\text{wait})}$ — mean waiting times for dropped, served and arbitrary packets.

3. Embedded Markov Chain

3.1. Steady-State Probability Distribution of Packets Number

Let τ_n be the moment of n-th packet arrival in our system, $\nu(\tau_n - 0)$ — the number of packets in the queuing system at the moment $(\tau_n - 0)$ (just before arrival), $n \geqslant 1$. Random variables $\nu(\tau_n - 0)$, $n \geqslant 1$, construct embedded by arrival moments Markov chain which we will consider to be ergodic. The state space of embedded Markov chain is $\mathcal{X} = \{0, 1, \ldots\}$. Let's suppose [11] that steady-state probability distribution of packets number $p_0^-, p_1^- \ldots$ exists (where $p_k^- = P\{\nu(\tau_n - 0)\}$ and probabilities p_k^-, $k \geqslant 0$ are independent of n) and (the method of formulae derivation for probabilities p_k^-, $k \geqslant 0$ is similar to [3]) may be represented by geometric form [11, 12]:

$$p_k^- = g^{k-1} p_1^-, \quad k \geqslant 1, \tag{1}$$

where g is the unique solution $(0 < g < 1)$ of the equation

$$g = \alpha(\mu - gp\mu)$$

and

$$p_1^- = p_0^- \frac{q - gp}{p} \frac{q - p\alpha(\mu - \mu q) + p\alpha(\mu)}{p\alpha(\mu - \mu q) - q\alpha(\mu - \mu p) - (p - q)\alpha(\mu)}, \tag{2}$$

$$p_0^- = \frac{1 - g}{1 - g + \frac{q - gp}{p} \frac{q - p\alpha(\mu - \mu q) + p\alpha(\mu)}{p\alpha(\mu - \mu q) - q\alpha(\mu - \mu p) - (p - q)\alpha(\mu)}}. \tag{3}$$

3.2. Loss and Service Probability

The probability that an arbitrary arriving packet will be dropped (with the help of (1))

$$p^{(\mathrm{loss})} = \sum_{k=1}^{\infty} p_k^- \sum_{l=1}^{k} p^{l-1} q = p_1^- \frac{q}{(1-g)(1-gp)}. \tag{4}$$

The probability that an arbitrary arriving packet will be served may found by equation

$$p^{(\mathrm{serv})} = p_0^- + p_1^- \frac{p}{1-gp} = 1 - p^{(\mathrm{loss})}. \tag{5}$$

4. Time Characteristics of Served and Dropped (Loss) Packets

At first let's consider the waiting time of dropped packet.

4.1. Dropped Packet Waiting Time Distribution

As it was denoted before $W^{(\mathrm{loss})}(x)$ is the probability that a packet will be in the queue for a time less than x and will be dropped. This probability depends on a number of other packets in the system at the moment of arrival of considered packet. Let's denote the probability that the arriving packet will meet k, $k \geqslant 0$, other packets in the system and will be dropped for a time less than x as $W_k^{(\mathrm{loss})}(x)$. Then

$$W^{(\mathrm{loss})}(x) = \frac{1}{p^{(\mathrm{loss})}} \sum_{k=0}^{\infty} p_k^- W_k^{(\mathrm{loss})}(x),$$

where $p^{(\mathrm{loss})}$ is defined by (4), probabilities p_k^-, $k \geqslant 0$ are defined by (1)–(3).

It is obvious that a $W_0^{(\mathrm{loss})}(x) = 0$. For $k \geqslant 1$ the probability $W_k^{(\mathrm{loss})}(x)$ is defined by

$$W_k^{(\mathrm{loss})}(x) = \sum_{l=1}^{k} q p^{l-1} B^{*(l-1)}(x), \quad k \geqslant 1,$$

where $B(x)$ — in-service time distribution function (exponential one in our case), $B^{*(l-1)}(x)$ — $(l-1)$-th convolution of service times, $l \geqslant 1$, $B^{*0}(x) \equiv B(x)$, the probability qp^{l-1} means that $(l-1)$ packet will be served and leave the system and the l-th packet will repeat the service. The whole probability $\sum\limits_{l=1}^{k} qp^{l-1} B^{*(l-1)}(x), \quad k \geqslant 1,$ means that from k packets at least $(l-1)$ will be served without dropping other packets and the l-th packet will drop other packers from the queue.

In LST term we can write:

$$\omega_k^{(\mathrm{loss})}(s) = \int_0^{\infty} e^{-sx} \mathrm{d}W_k^{(\mathrm{loss})}(x) = \sum_{l=1}^{k} qp^{l-1}\beta^l(s) = \frac{q\beta(s)\left(1-(p\beta(s))^k\right)}{1-p\beta(s)}, \tag{6}$$

where $\beta(s)$ is LST of $B(x)$ and $\beta(s) = \frac{\mu}{\mu+s}$.

The LST $\omega^{(\text{loss})}(s)$ of $W^{(\text{loss})}(x)$ is

$$\omega^{(\text{loss})}(s) = \frac{1}{p^{(\text{loss})}} \sum_{k=1}^{\infty} p_k^- \omega_k^{(\text{loss})}(s). \tag{7}$$

Substituting (6) in (7) and using $\beta(s) = \dfrac{\mu}{\mu + s}$:

$$\omega^{(\text{loss})}(s) = \frac{1}{p^{(\text{loss})}} \sum_{k=1}^{\infty} p_k^- \frac{q\beta(s)\left(1 - (p\beta(s))^k\right)}{1 - p\beta(s)} =$$

$$= \frac{p_1^-}{p^{(\text{loss})}} \frac{q}{(1-g)(1-gp)} \frac{\mu(1-gp)}{(s + \mu(1-gp))}. \tag{8}$$

Having applied (4) to (8) finally we get

$$\omega^{(\text{loss})}(s) = \frac{\mu(1-gp)}{s + \mu(1-gp)}. \tag{9}$$

With the help of inverse transformation of LST (9) we get the probability that a packet before being dropped will be in the queue for a time less than x.

$$W^{(\text{loss})}(x) = \begin{cases} 1 - e^{-\mu(1-gp)x}, & x > 0, \\ 0, & x \leqslant 0. \end{cases}$$

Hence, time spent by a dropped packet in the queue has the exponential distribution with the parameter $\mu(1-gp)$.

4.2. Served Packet Time Distributions

Similar to previous consideration we can derive $W^{(\text{serv})}(x)$ and $\omega^{(\text{serv})}(s)$:

$$\omega^{(\text{serv})}(s) = \frac{\mu p}{s + \mu p}, \tag{10}$$

$$W^{(\text{serv})}(x) = \begin{cases} 1 - e^{-\mu p x}, & x > 0, \\ 0, & x \leqslant 0. \end{cases}$$

For $W^{(\text{serv})}_{(\text{wait})}(x)$ and $\omega^{(\text{serv})}_{(\text{wait})}(s)$:

$$\omega^{(\text{serv})}_{(\text{wait})}(s) = \frac{1}{p^{(\text{serv})}} \left(p_0^- + \frac{p p_1^-}{(1-gp)} \frac{\mu(1-gp)}{(s + \mu(1-gp))} \right), \tag{11}$$

$$W^{(\text{serv})}_{(\text{wait})}(x) = \begin{cases} \dfrac{1}{p^{(\text{serv})}} \left(p_0^- + \dfrac{p p_1^-}{1-gp} e^{-\mu(1-gp)x} \right), & x > 0, \\ 0, & x \leqslant 0. \end{cases}$$

4.3. Arbitrary Packet Waiting Time Distribution

As we denoted before, the $W_{(\text{wait})}(x)$ is the probability that an arbitrary packet will stay in the queue for a time more than x (waiting time distribution function of an arbitrary packet), and $\omega_{(\text{wait})}(s)$ Laplase-Stieltjes transformation (LST) of this probability :

$$\omega_{(\text{wait})}(s) = p^{(\text{loss})}\omega^{(\text{loss})}(s) + p^{(\text{serv})} + \omega^{(\text{serv})}_{(\text{wait})}(s) = p_0^- + \frac{p_1^-}{(1-g)}\frac{\mu(1-gp)}{(s+\mu(1-gp))}, \quad (12)$$

$$W_{(\text{wait})}(x) = \begin{cases} p_0^- + \dfrac{pp_1^-}{1-g}\left(1 - e^{-\mu(1-gp)x}\right), & x > 0, \\ 0, \quad x \leqslant 0. \end{cases}$$

4.4. Mean Waiting Time Characteristics

With help of (9)–(12) we can find mean characteristics:

$$w^{(\text{loss})} = \frac{1}{\mu(1-gp)}, \quad w_{(\text{wait})} = \frac{p_1^-}{(1-g)}\frac{1}{\mu(1-gp)},$$

$$w^{(\text{serv})}_{(\text{wait})} = \frac{1}{p^{(\text{serv})}}\frac{pp_1^-}{(1-gp)}\frac{1}{\mu(1-gp)}.$$

5. Conclusion

The one-server queuing system with general input flow, infinite buffer, exponential service time, renovation and repeated service was considered in this work. Some steady-state time characteristics for dropped, served and arbitrary packets were derived.

In future authors suppose to consider renovation with repeat service for other system: several input flows, thresholds, hysteric policy [13], bunker [14].

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Временные характеристики системы массового обслуживания с обновлением и повторным обслуживанием

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Данная работа посвящена временным характеристикам однолинейной системы массового обслуживания с рекуррентным входящим потоком заявок, экспоненциальным распределением времени обслуживания заявки на приборе, накопителем неограниченной ёмкости. Введён механизм обновления с повторным обслуживанием — заявка, находящаяся на приборе, в момент окончания обслуживания либо с некоторой вероятностью покидает систему, либо с дополнительной вероятностью остаётся в системе, при этом сбрасывая из накопителя все находящиеся в нём другие заявки.

Предполагая, что известно стационарное распределение заявок по цепи Маркова, вложенной по моментам поступления, внимание уделено временным характеристикам рассматриваемой системы — стационарному распределению времени пребывания заявки в системе (обслуженной или сброшенной), а также средним временным характеристикам — среднее время обслуживания заявки на приборе, среднее время ожидания начала обслуживания, среднее время, проведённое сброшенной заявкой в накопителе, среднее время пребывания в системе произвольной заявки.

Ключевые слова: обновление, повторное обслуживание, рекуррентный входящий поток, временные характеристики, обслуженная заявка, сброшенная заявка.